Contagious Bank Runs and Buyer of Last Resort*

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30 May 2019

Abstract

In a crisis, central banks and private investors can find it difficult, if not impossible, to tell whether banks facing runs are insolvent or merely illiquid. In a global-games framework, we show that the lack of information leads to distressed asset prices and restricts central banks' ability to act as a lender of last resort. Under aggregate uncertainty, financial contagion and price volatility emerge as multiple-equilibria phenomena despite global-games refinements. We explain how a central bank can use a buyer-of-last-resort policy to contain contagion and stabilize asset prices while breaking even ex ante—even without information on individual banks' solvency.

Keywords: Buyer of last resort, Global games, Bank runs, Asset price volatility

JEL Classification: G01, G11, G21

*The authors thank Toni Ahnert, Thorsten Beck, Christoph Bertsch, Elena Carletti, Mark Carey (discussant), Fabio Castiglionesi, Ben Craig (discussant), Eric van Damme, Tobias Dieler (discussant), Murray Frank (discussant), Xavier Freixas, Ivanov Ivan (discussant), Xuewen Liu, Frederic Malherbe, Rafael Matta (discussant), Roberto Robatto (discussant), Jean-Charles Rochet, Javier Suarez, Wolf Wagner, Lucy White, and Andrew Winton (discussant) for their comments and discussion. Helpful comments from WEAI/IBEFA summer conference, 13th FDIC Bank Research Conference, CICF 2015 conference, 2016 CEPR 1st Annual Spring Symposium, 2016 Fixed Income and Financial Institutions conference, 2016 FIRS conference, 2016 UECE Lisbon Meetings in Game Theory and Applications, 2nd Chapman Conference on Money and Finance, 1st Bristol Workshop on Banking and Financial Intermediation, 2017 IWFSAS Conference, 6th Workshop in Macro Banking and Finance, 2018 OFR and Fed Atlanta Financial Stability Conference, 16th Paris December Finance Meeting, 2019 Chicago Financial Institutions Conference, seminar at Tilburg University, seminar in Riksbank, seminar at University of Zurich, seminar at Bank of Canada, seminar at Federal Reserve Board, seminar at IMF, seminar at Carlos III University of Madrid, and seminar at Universitat Pompeu Fabra are also gratefully acknowledged. Both authors have benefited from the support from National Natural Science Foundation of China (Ref. No. 71803024).

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1 Introduction

The events of the 2007-2009 financial crisis highlight the importance of liquidity risk. During the crisis, market liquidity evaporated, and asset prices dropped sharply. At the same time, funding liquidity dried up, and even well-capitalized banks found it difficult to roll over their short-term debt. To tackle the dual-illiquidity problem, central banks were creative in providing facilities for liquidity support, and certain policy interventions deviated from the classic lender-of-last-resort (LoLR) policy formulated by Bagehot (1873). As observed by Mehrling (2010, 2012), the Fed has increasingly become a dealer of last resort in its crisis management. Instead of lending directly to banks, the Fed boosted the market liquidity of banks' assets which in turn increased banks' ability to raise funding.[1] The central bank's efforts were also supplemented by asset purchase programs, such as Public-Private Investment Program (PPIP), which aimed to rejuvenate markets for mortgage-backed securities. The European Central Bank adopted a similar practice: through Outright Monetary Transactions (OMT), the central bank pledged to purchase from European banks assets that were otherwise illiquid in markets. It is fair to say that to provide liquidity support, public authorities increasingly focus on boosting the market liquidity of assets rather than only on lending directly to financial institutions.

In this paper, we model the two-way feedback between distressed asset prices and bank runs and show that a buyer-of-last-resort intervention can be effective. Central to our model is the observation that in crisis times, it can be difficult—if not impossible—to distinguish illiquid banks from insolvent ones.[2] We show that, in the presence of aggregate uncertainty, the information constraint creates a vicious cycle between falling asset prices and contagious bank runs, and that multiple equilibria arise despite global-games refinements. The information friction that creates the financial fragility also limits the effectiveness of traditional LoLR policies: an informationally constrained central bank will not be able to lend only to the solvent-but-illiquid banks as suggested by Bagehot. By contrast, an asset purchase program that we interpret as a buyer-of-last-resort intervention can short-circuit the feedback loop and stabilize the financial system. We show that the intervention can be designed so that the buyer of last resort (BoLR) makes no expected loss even without information on individual banks' solvency.

The lack of information about a bank's solvency generates two-way feedback between bank runs and distressed asset prices in the following way. When private asset buyers cannot dis-

[1] For example, on top of the usual LoLR policies such as open market operations and the discount window, the Fed launched novel emergency liquidity assistance (ELA) programs such as Primary Dealer Credit Facility (PDCF), Term Securities Lending Facility (TSLF), and Term Asset-Backed Securities Loan Facility (TALF). Via these programs, the central bank substituted illiquid assets of private institutions with liquid assets, by accepting a wide range of collateral.

[2] The information constraint is widely recognized both in practice and in the academic literature. It is considered a main challenge for central banks to act as lenders of last resort. See, e.g., Freixas et al. (2004).

tinguish assets sold by illiquid banks from those sold by insolvent banks, the price they offer will reflect the average asset quality between these two. As a result, an illiquid bank will be unable to recoup a fair value for its assets on sale. In a global-games framework, we show that creditors' expectation of low asset prices due to this information friction can deprive a solvent bank of its short-term funding. Each creditor, anticipating the liquidation loss caused by other creditors' early withdrawals, chooses to join the run himself. However, it is the run and the forced liquidation, which pools the illiquid bank with insolvent ones, that lead to the decline in asset prices in the first place.[3]

Financial contagion and price volatility emerge once we introduce aggregate uncertainty. In particular, distinct equilibrium outcomes (i.e., different asset prices and different numbers of bank runs) can arise for the same realization of bank fundamentals. This is because economic agents can now coordinate their beliefs about the aggregate state. In particular, when asset buyers observe more bank runs, they revise their beliefs about the aggregate state downwards. To break even given their deteriorating beliefs, they have to reduce their bid for the banks' assets. The depressed asset price, in turn, precipitates runs at more banks. Pessimistic expectations can arise and justify themselves. This leads to a vicious cycle between collapsing asset prices and contagious bank runs. Formally, this is captured by the multiple equilibria of the model.[4]

The multiple equilibria leave scope for policy intervention. Intuitively, a central bank can provide a backstop for the prices of banks' assets and thereby break down the two-way feedback between falling asset prices and bank runs. When committing unilaterally to such price support, however, a central bank will make expected losses and raise concerns that it is providing a public bail-out. To avoid this unpopular solution, we propose a design where a BoLR and banks mutually commit to an agreement for the BoLR to purchase a bank's assets for a pre-specified price when the bank experiences runs.[5] In making her offer before the aggregate state is realized, the BoLR need not concern herself with the solvency of individual banks but need only focus on setting a 'fair price' for banks' assets. From an ex-ante perspective, the price allows the BoLR to break even across possible posterior beliefs about the aggregate state and prevents contagious bank runs that are driven by pessimistic beliefs.

[3]While our model focuses on asset sale and prices, the information constraint and its implication for financial fragility would also apply to collateralized borrowing, where the lack of information can contribute to both an increase of haircut and bank runs.

[4]The belief-driven runs in our model are different from those in papers such as Diamond and Dybvig (1983) because the beliefs about the aggregate state have to be rationalized by the observed number of runs, which depends on fundamentals in the global-games framework.

[5]This setting resembles the proposal in King (2017) that banks should pre-position their assets with the central bank for emergency liquidity support. The proposal has been implemented by the Bank of England in its Sterling Monetary Framework and by the Reserve Bank of Australia in its Committed Liquidity Facility. We discuss this in more details in Section 4.

The main novelty of our paper is to introduce into global-games-based bank run models a realistic information constraint: it is difficult to distinguish illiquid banks from insolvent ones during crisis times. The resulting feedback between distressed asset prices and bank runs, together with our policy analyses, contributes to the literature in three ways. First, regarding the market liquidity of assets, we show that given the lack of information, bank runs depress asset prices even when the supply of cash is perfectly elastic. This contrasts with the cash-in-the-market approach that assumes limited market participation and a fixed supply of liquidity. Second, regarding the funding liquidity risk of banks, multiple equilibria in the form of contagious bank runs emerge in our model even if the noise of private signals in the global-games framework approaches zero. This contrasts with existing global-games papers that rely on precise public signals to generate multiplicity. Finally, from a policy perspective, we show that an effective liquidity intervention resembles a buyer-of-last-resort practice, providing a formal theory to interpret the recent developments in central bank policy practices. We show that a well-designed BoLR intervention can limit financial fragility and let the regulator break even at the same time, without requiring information about individual banks' solvency.

Related literature: Our paper contributes to the literature on public liquidity intervention and global-games-based bank run models. Central bank liquidity injection in a global-games framework was first studied by Rochet and Vives (2004). The authors considered a single-bank setup and derived a unique threshold equilibrium that featured solvent-but-illiquid banks as in Bagehot (1873). The authors further assume that banks' fundamentals are perfectly observable to the central bank and suggest that the central bank can act as a LoLR by lending directly and only to solvent banks. In a multiple-bank setup, we generalize Rochet and Vives (2004) by introducing information constraints, endogenous liquidation value, and aggregate uncertainty. We focus on systemic crises instead of runs on individual banks and show that the uniqueness achieved by the global-games refinement does not survive the introduction of aggregate uncertainty. We emphasize that emergency liquidity assistance programs need to take into account information constraints, and provide an explanation for central banks' deviation from traditional lender-of-last-resort policies.

In terms of predicting feedback between market liquidity and funding liquidity, our model is related to Brunnermeier and Pedersen (2009) and Liu (2016).[6] Liu (2016) studies the interaction between bank runs and rising interbank market rates. While the driving mechanism in Liu (2016) is limited participation in the interbank market, we show that in the presence of asymmetric information, multiple equilibria emerge even if the supply of liquidity is perfectly

[6]Goldstein (2005) and Leonello (2018) also use the method of global games to model the two-way feedback between different financial markets. Their studies focus on how bank runs interact with currency crisis and sovereign bond crisis, respectively.

elastic.[7] In a non-bank setup that has no coordination failures, Brunnermeier and Pedersen (2009) emphasize a margin constraint on a speculator who supplies liquidity to a financial market with limited participation. In their model, asset prices are volatile because the selling and buying of assets are not synchronized. By contrast, we emphasize the funding liquidity risk caused by equilibrium bank runs and that asymmetric information on asset qualities causes asset illiquidity.[8]

Eisenbach (2017) also studies financial fragility caused by aggregate uncertainty using global games. Assuming observable aggregate states, the author suggests using contingent liabilities to maintain both financial stability and the disciplinary power of runs. Our model, by contrast, emphasizes that incomplete information on the aggregate state leads to multiple equilibria and financial fragility. We also show that the information about the aggregate state does not necessarily help in central banks' liquidity injection.

Our paper relates to the broader literature that uses the global-games refinement to study bank runs, e.g., Morris and Shin (2000), Rochet and Vives (2004), Goldstein and Pauzner (2005), and Morris and Shin (2016), but we relax the common assumption of exogenous liquidation losses.[9] This simplifying assumption implicitly excludes the possibility of bank runs to affect asset prices, even though bank failures often put downward pressure on asset prices in reality and despite that the mechanism being central to theories such as Allen and Gale (1998) and Gromb and Vayanos (2002). We introduce the impact of runs on asset prices in the framework of global games. We emphasize that buyers' lack of knowledge about the aggregate state and the inability to distinguish illiquid banks from insolvent ones result in a downward spiral between runs and declining asset prices. In the broader literature of global games, Angeletos and Werning (2006) and Ozdenoren and Yuan (2008) also introduce endogenous asset prices and predict the co-existence of price volatility and multiple equilibria. In generating multiple equilibria, both papers emphasize the impact of endogenous market price on the precision of public signals. In contrast, we study a case where asset prices directly affect players' payoffs in coordination games. In this context, we show that even if the price is endogenous, one can still have a unique equilibrium, which disappears only upon the introduction of aggregate uncertainty.

[7]Liu (2016) also discusses a policy intervention which is modeled as an ex-post net transfer from the central bank to private institutions, conditional on central bank's observation of bad state. In contrast, we emphasize that the intervention should be pre-emptive: BoLR intervention should be announced before the realization of the aggregate uncertainty and can still be effective even if the central bank does not observe the aggregate state.

[8]Using historical data, Fohlin et al. (2016) empirically document the feedback between market and funding illiquidity, providing evidence that information asymmetry on asset qualities contributes to the vicious cycle.

[9]For example, Rochet and Vives (2004) assume an exogenous fire-sale discount; Goldstein and Pauzner (2005) assumes unit liquidation value; and Morris and Shin (2016) assume an exogenous haircut of 100%.

We suggest an asset purchase program with price support can break down the feedback loop between distressed asset prices and bank runs.[10] This policy suggestion is consistent with recent contributions that promote asset purchase programs as a way to avoid credit crunch and to reduce financial instability. For example, Bolton et al. (2009, 2011) emphasize that a price support reduces adverse selection in asset markets, which helps to maintain efficient origination and distribution of risky assets. Tirole (2012) also points out that an asset purchase program can limit adverse selection by cleaning the market of its lowest-quality assets, which will jumpstart frozen asset markets and facilitate the financing of new investment. Diamond and Rajan (2011) suggest that a bank that faces liquidity risk has incentives to hold rather than sell its illiquid assets, because its incentives are distorted by the limited liability. Therefore, an asset purchase program that moves illiquid asset from leveraged and distressed institutions will avoid unnecessary asset fire sales. Bhattacharya and Nyborg (2013) show that a menu of asset purchase programs can screen banks of heterogenous qualities, making liquidity injection more efficient. On the empirical side, Acharya et al. (2017) argue that the asset purchase program of ECB stabilized markets better than its lending facilities did.

The paper proceeds as follows. Section 2 lays out a model in a laissez-faire market. We analyze the equilibrium of the model in Section 3. Section 4 extends the model by introducing a buyer of last resort. We extend the policy discussion in Section 5 and conclude in Section 6.

2 Model setup

We consider a three-date ($t = 0, 1, 2$) economy with two banks ($i = 1, 2$).[11] There are two groups of active players: a continuum of wholesale creditors to banks, and a large number of secondary-market asset buyers. All players are risk neutral.

2.1 Banks

Banks are identical at $t = 0$. Each of them holds a unit portfolio of long-term assets and finances the portfolio with equity E, retail deposits F, and short-term wholesale debt $1 - E - F$. We consider banks as contractual arrangements among claim holders, designed to fulfill the function of liquidity transformation. Therefore, banks in our model are passive, with given loan portfolios and liability structures.

[10]Since central banks directly purchasing *risky* assets can be an efficient intervention in our model, our paper is also related to Koulischer and Struyven (2014) and Choi et al. (2017). For the design of central bank lending programs, both papers show that central banks should accept a broad range of collateral in their liquidity injection.

[11]This two-bank setup is parsimonious and rich enough to illustrate our findings. A version of the model with N-bank can be provided upon request.

A bank i's assets generate a random cash flow $\tilde{\theta}^i \sim U(\underline{\theta}_s, \overline{\theta})$, where $i = 1, 2$. The realization of the cash flow is not only affected by the idiosyncratic risk of the bank, but also by a systematic risk factor s. The systematic risk, as indicated by the subscript of the lower bound, determines the distribution of both banks' cash flows. There are two possible aggregate states, $s = G$ and $s = B$. With $\underline{\theta}_G \geq \underline{\theta}_B$, State G is assumed to be more favorable. All players hold a prior belief that State G and B occur with probabilities α and $1-\alpha$ respectively. Note that the upper bound of banks' cash flows is assumed to be the same across the two aggregate states. This reflects the fact that banks hold mostly debt claims whose highest payoffs are capped by their face values. Once the aggregate state s is realized, the two banks' cash flows are determined by their idiosyncratic risks and are assumed to be independently and identically distributed. The fundamentals of the banking sector can be represented by a vector $\boldsymbol{\theta} \equiv (\theta^1, \theta^2)$.

On the liability side, we assume that retail deposits are fully protected by deposit insurance, and this financial safety net is provided to banks free of charge. Therefore, retail depositors will hold their claims passively to maturity and demand only a gross risk-free rate which we normalize to 1. On the other hand, banks' wholesale debt is risky, demandable, and raised from a continuum of creditors of mass 1. Provided that a bank does not fail, the wholesale debt contract promises a gross interest rate $r_D > 1$ if a wholesale creditor waits till $t = 2$, and qr_D if the wholesale creditor withdraws early at $t = 1$. Here, $q < 1$ reflects the penalty for the early withdrawal. A bank run occurs if a positive mass of wholesale creditors withdraw funds from their bank at $t = 1$. For the ease of presentation, we denote by D_1 the total amount of debt that a bank needs to repay at $t = 1$ if *all* wholesale creditors withdraw early, and by D_2 the total amount of debt that a bank needs to repay at $t = 2$ if *no* wholesale creditor withdraws early.

$$D_1 \equiv (1 - E - F)qr_D$$
$$D_2 \equiv (1 - E - F)r_D + F$$

We make the following three parametric assumptions.

$$D_2 > \underline{\theta}_s \tag{1}$$
$$F > D_1 \tag{2}$$
$$q > \frac{1}{2} + \frac{\underline{\theta}_G}{2D_2} \tag{3}$$

Inequality (1) states that banks are not risk-free, and there is a positive probability of bankruptcy even in State G. Inequality (2) suggests that banks' retail debt exceeds their short-term wholesale debt, which is a realistic scenario and helps to simplify the analysis of bank run games.[12] Finally, inequality (3) states that the penalty for early withdrawal is only moderate.[13] While we do not endogenize banks' capital structure (therefore taking q, D_1, and D_2 as given), as long as the optimal capital structure satisfies the aforementioned conditions, all of our results apply.

We assume that if a wholesale run happens at $t = 1$, a bank has to liquidate its assets in a secondary market and sell them to outside asset buyers. As early liquidation is costly in this model, we assume that a bank will sell its assets if and only if it faces a bank run.[14]

2.2 The bank run game

A bank run game of complete information can have two strict equilibria: all short-term debt holders withdraw from the bank, and no one withdraws. To refine the multiplicity, we take the global-games approach pioneered by Carlsson and Van Damme (1993) and assume that creditors observe noisy signals of banks' cash flows. At the beginning of $t = 1$, both the systematic risk (aggregate state s) and banks' idiosyncratic risks (cash flow $\tilde{\theta}^i$) have been realized, but the information is not fully revealed to players. We assume that wholesale creditors hold claims in *both banks* and observe independent noisy signals for the banks' cash flows.[15] Specifically, a representative creditor j privately observes a vector of noisy signals $\boldsymbol{x}_j = (x_j^1, x_j^2)$, where $x_j^i = \theta^i + \epsilon_j^i$ is his signal of Bank i's realized cash flow θ^i. Noise ϵ_j^i is drawn from a uniform distribution with support $[-\epsilon, \epsilon]$. For simplicity, we assume that noises are independent across banks as well as across creditors. We also focus on a limiting case where ϵ approaches zero.

After receiving his signals $\boldsymbol{x}_j$, creditor j has two possible actions at each bank: to wait till $t = 2$ or to withdraw early at $t = 1$. We assume that creditors play a bank run game with each other in both banks simultaneously and focus on *threshold strategies that are symmetric across all creditors for both banks*.[16,17] That is, any creditor j withdraws from any bank i if and only if $x_j^i < x^*$. As a consequence, an equilibrium bank run will happen if and only if a bank's cash

[12]Despite the rapid growth of wholesale funding, most commercial banks and bank holding companies are still financed more by retail deposits than wholesale debt. For example, Cornett et al. (2011) document that the median core deposit to asset ratio for US commercial banks was 67.88% over the period from 2006 to 2009.

[13]For example, when $\underline{\theta}_G = \underline{\theta}_B = 0$, the condition states that $q > 1/2$. That is, by withdrawing early, a wholesale creditor will not lose more than a half of the face value of his claim. The moderate penalty for early withdrawal is in line with banks' role as liquidity providers as suggested by Diamond and Dybvig (1983).

[14]Diamond and Rajan (2011) also provide an exposition why banks protected by the limited liability prefer not to sell their asset until runs happen, in which case the sale is too late and causes bank failures.

[15]It is not uncommon for institutional investors to hold demandable debt claims in multiple banks. A similar setup is analyzed by in Goldstein and Pauzner (2004).

[16]In the finance application of global games, the threshold equilibrium is of primary interest. For example, see Morris and Shin (2004) and Liu (2016). Following Vives (2014) and Angeletos and Lian (2016), we also show in Appendix B that the restriction to threshold strategies is without loss of generality.

[17]Once state s is realized, both banks' cash flows are independently and identically distributed. As creditors are ex-ante homogenous and banks are also assumed to have the same capital structure, there is no loss of generality to focus on symmetric strategies.

flow falls below a critical level θ^*. We show in the limiting case where $\epsilon \to 0$, the critical cash flow θ^* converges to x^*.[18]

As in standard global-games models, creditors formulate posterior beliefs about banks' fundamentals $\boldsymbol{\theta}$ and the fraction of creditors who will withdraw early in each bank. Different from the standard setup, creditors in our model also form rational expectations about the number of bank runs and anticipate the impact of this number on the equilibrium asset price in the secondary asset market.

A wholesale creditor's payoff from a bank depends both on his withdrawal decision and on the bank's solvency. The creditor will receive D_1 if he withdraws early and the bank does not fail at $t = 1$; he will receive D_1/q, if he waits and the bank stays solvent at $t = 2$. In the case of failure, a bank incurs a bankruptcy cost C, which is a constant and can be interpreted as the legal cost of bankruptcy. We further assume C to be sufficiently high such that if the wholesale creditor waits and the bank fails at either $t = 1$ or $t = 2$, the wholesale creditor will receive a zero payoff and a senior deposit insurance company obtains the residual value of the bank.[19] Finally, we assume that the creditor can obtain an arbitrarily small reputational benefit by running on a bank that fails at $t = 1$.[20]

2.3 Secondary asset market

When facing withdrawals at $t = 1$, banks have to liquidate their long-term assets in a secondary asset market. We assume that a large number of identical, deep-pocketed buyers participate in the market and that they are called into action only when a run happens. When no bank run occurs, the asset buyers will not have the opportunity to move, and the game between wholesale creditors and asset buyers ends. The buyers observe neither the aggregate state s nor private signals about banks' cash flows. Thus, they cannot determine the exact quality of assets on sale. They can, however, observe the outcome of creditors' bank run game (i.e., the number of banks forced into liquidation) and can infer the quality of bank assets from the observation.

Buyers competitively bid for banks' assets on sale when observing any positive number of runs. We denote the number of bank runs by M, with $M \in \{1, 2\}$. A strategy for an asset buyer is a price schedule $\boldsymbol{P} = (P_1, P_2)$ that specifies a price P_M for a unit of a bank's assets

[18]For its tractability, it is common to study the limiting case in the literature. For example, see Liu and Mello (2011). In our model, the limiting case also allows for a clear-cut definition of a bank run. For a given equilibrium and a realized cash flow θ^i, either all creditors withdraw or no one withdraws from the bank. That a fraction of creditors withdraw becomes a zero probability event.

[19]As it will be clear from the analysis, this case is off equilibrium.

[20]The reputational benefit may come from the fact that the creditor makes a "right decision". Rochet and Vives (2004) argue that the vast majority of wholesale deposits are held by collective investment funds, whose managers are compensated if they build a good reputation, and penalized otherwise. As we will show later, wholesale creditors receiving this small reputational payoff is also off equilibrium.

in the contingency that M bank runs are observed. Since the buyers are homogenous, their equilibrium strategy will be symmetric. An equilibrium strategy $\boldsymbol{P}^* = (P_1^*, P_2^*)$ can be viewed as the market demand for bank assets.

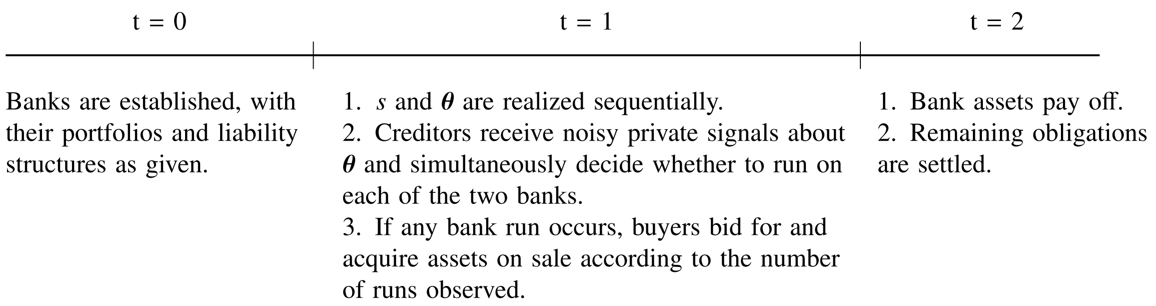
The price schedule $\boldsymbol{P}$ offered by an asset buyer will aggregate all information available to her. First, the buyer understands the creditors' bank run game and knows that the quality of assets on sale must be below an equilibrium threshold θ^*. Second, the buyer updates her beliefs about the aggregate state s. After the aggregate state is realized, both bank's cash flows are i.i.d., so that more bank runs (i.e., more cases where $\theta^i < \theta^*$) suggest that State B is more likely. It should be noted that buyers' belief about s is endogenous to creditors' strategy.

Finally, asset buyers bid competitively based on their beliefs. Therefore, upon the contingency where M runs have occurred, P_M^* in the equilibrium strategy profile must leave the buyers breaking even in expectation.

2.4 Timing

The timing of the game is summarized in Figure 1. Events at $t = 1$ take place sequentially.

Figure 1: Timing of the game in a laissez-faire market



3 Equilibrium analysis in a laissez-faire market

We now solve the model presented in Section 2. As it is a dynamic game with incomplete information, we apply the concept of Perfect Bayesian Equilibrium.

Definition. *A PBE of our dynamic game is characterized by an equilibrium strategy profile* $(x^*, \boldsymbol{P}^*)$ *and a system of beliefs. (i) Each creditor plays a threshold strategy: to withdraw from a bank if and only if his private signal about the bank's cash flow is lower than the threshold* x^**. Asset buyers purchase banks' assets on sale according to a price schedule* $\boldsymbol{P}^* = (P_1^*, P_2^*)$*, where* P_M^* *is the asset price given creditors' threshold* x^* *and the observation of* M *bank runs,* $M \in \{1, 2\}$*. (ii) Each creditor forms beliefs about the realized cash flows* θ *for each bank, and calculates the ex-post distribution of early withdrawals in each bank, conditional on his private* *signals and other players' equilibrium strategies. Based on the observed number of bank runs, the buyers then form beliefs about the qualities of banks' assets on sale and beliefs about the realized aggregate state, conditional on the creditors' equilibrium strategy. (iii) The strategy profile described in (i) is sequentially rational given the beliefs described in (ii).*

For an equilibrium $(x^*, \boldsymbol{P}^*)$ and a fundamental $\boldsymbol{\theta}$, *an equilibrium outcome* in this laissez-faire market can feature no bank runs and no asset liquidation (which we denote by *No Run*), or be summarized by a duplex (M, P^*_M), where $M \in \{1, 2\}$ is the number of bank runs and P^*_M is the prevailing market price for banks' assets. When multiple equilibria exist, different equilibrium outcomes can emerge for the same fundamental, capturing the concept of financial fragility.

It takes three steps to establish an equilibrium $(x^*, \boldsymbol{P}^*)$. We start with asset buyers who move last and characterize their beliefs and optimal actions—taking creditors' equilibrium strategy as given (Section 3.1). Because buyers make zero profits in expectation, we determine the competitive asset price in the case of M runs as a response to creditors' equilibrium threshold strategy. Buyers understand the bank run game and know $x^* = \theta^*$ when $\epsilon \to 0$, so we denote the competitive price by $\mathbb{P}_M(\theta^*)$.

We then solve the global games played by wholesale creditors who move first (Section 3.2). We construct a representative creditor j's posterior beliefs about $\boldsymbol{\theta}$ and the other creditors' withdrawal decisions. Furthermore, forward-looking creditors foresee the equilibrium outcome in the secondary asset market and expect the price to be $\mathbb{P}_M(\theta^*)$ when anticipating M runs. Based on the expected asset price and creditor j's beliefs, we calculate his best response to the threshold strategy x^*. For symmetric equilibria, we derive a condition that an equilibrium critical cash flow θ^* must satisfy.

Finally, we establish the existence of the equilibrium by solving for θ^* as a fixed point, which, in turn, pins down the threshold strategy x^* and the equilibrium price schedule $\boldsymbol{P}^*$. For illustrative purposes, we proceed in steps and analyze two contrasting cases. We start with a baseline case with no aggregate uncertainty ($\underline{\theta}_B = \underline{\theta}_G$) and derive a unique equilibrium (Section 3.3). We then introduce aggregate uncertainty ($\underline{\theta}_B < \underline{\theta}_G$) and show that multiple equilibria can emerge. For an intermediate range of fundamentals, while the equilibrium price schedule is unique, creditors' equilibrium switching strategy can have multiple thresholds. As a result, the equilibrium outcome cannot be determined, which has the natural interpretation of financial contagion and price volatility (Section 3.4).

3.1 Competitive bidding in the secondary asset market

In this section, we solve the asset buyers' bidding game. That is, given the creditors' strategy, what will the secondary-market asset prices be?

Asset buyers observe neither cash flows $\boldsymbol{\theta}$ nor state s. Nevertheless, they form rational beliefs about the quality of assets on sale. In a Perfect Bayesian Equilibrium, buyers who believe creditors using a symmetric switching threshold x^* understand that a bank run happens if and only if the bank's cash flow is lower than θ^*. Asset buyers also Bayesian update their beliefs about the aggregate state s. Given their beliefs of creditors' equilibrium strategy and the observation of M bank runs, we can calculate buyers' posterior beliefs about state s as follows (details in Appendix A.1):

$$\omega_M^B(\theta^*) \equiv Prob(s = B|\theta < \theta^*, M) = \frac{\left(\theta^* - \underline{\theta}_B\right)^M}{\left(\theta^* - \underline{\theta}_B\right)^M + \kappa\left(\theta^* - \underline{\theta}_G\right)^M}, \tag{4}$$

$$\omega_M^G(\theta^*) \equiv Prob(s = G|\theta < \theta^*, M) = \frac{\kappa\left(\theta^* - \underline{\theta}_G\right)^M}{\left(\theta^* - \underline{\theta}_B\right)^M + \kappa\left(\theta^* - \underline{\theta}_G\right)^M}, \tag{5}$$

where $M \in \{1, 2\}$ and κ is a constant and defined as

$$\kappa \equiv \frac{\alpha}{1-\alpha}\left(\frac{\overline{\theta} - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_G}\right)^2.$$

When buyers bid competitively for banks' assets on sale, the equilibrium of the secondary market requires buyers' bid to be equal to the expected asset quality. Otherwise, undercutting would happen among the buyers. Specifically, when M bank runs occur, the competitive price offered by the homogeneous buyers can be written as follows.

$$\mathbb{P}_M(\theta^*) = E_s\left[E_\theta\left(\theta\middle|\theta < \theta^*, M\right)\right] = \omega_M^B(\theta^*) \cdot E_\theta\left(\theta\middle|\theta < \theta^*, s = B\right) + \omega_M^G(\theta^*) \cdot E_\theta\left(\theta\middle|\theta < \theta^*, s = G\right)$$

For a given aggregate state s, the buyers perceive the average quality of the asset on sale to be

$$E_\theta\left(\theta\middle|\theta < \theta^*, s\right) = \int_{\underline{\theta}_s}^{\theta^*} \theta \cdot \frac{1}{\theta^* - \underline{\theta}_s} d\theta = \frac{\underline{\theta}_s + \theta^*}{2}, \quad s \in \{B, G\}.$$

Therefore, the competitive asset price can be written explicitly as follows:

$$\mathbb{P}_M(\theta^*) = \omega_M^B(\theta^*) \cdot \frac{\underline{\theta}_B + \theta^*}{2} + \omega_M^G(\theta^*) \cdot \frac{\underline{\theta}_G + \theta^*}{2} = \frac{E_s\left(\underline{\theta}_s|M\right) + \theta^*}{2}. \tag{6}$$

Expression $E_s(\underline{\theta}_s|M) = \omega_M^B(\theta^*) \cdot \underline{\theta}_B + \omega_M^G(\theta^*) \cdot \underline{\theta}_G$ represents the expected lower bound of θ, based on the observation of M runs.

It is worth noticing that creditors' strategy affects the secondary market price in two ways. First, x^* and the associated θ^* directly determine the types of assets on sale, with asset quality following a uniform distribution on $[\underline{\theta}_s, \theta^*]$. Second, θ^* affects buyers' perception of the aggregate state. For a given number of runs, a more optimistic strategy on the creditors' side (i.e., a lower x^*) is associated with a more pessimistic perception of the aggregate state s (i.e., a higher ω_M^B). Both channels imply that higher x^* and θ^* are associated with higher asset prices.

It should be pointed out that a candidate equilibrium price must belong to $[\underline{P}, qD_2)$, where $\underline{P} \equiv (\underline{\theta}_B + D_2)/2$. Intuitively, if the price is greater than qD_2, early liquidation will not hurt a bank's solvency so that its creditors would not run in the first place.[21] On the other hand, as all fundamentally insolvent banks will be liquidated, the average asset quality is guaranteed to be no lower than $(\underline{\theta}_B + D_2)/2$. This restricts the set of candidate equilibria and will facilitate the solution of the bank run game in the next section.

Lemma 1. *When asset buyers believe that creditors follow a switching threshold x^* and that a bank fails if and only if $\theta < \theta^*$, an equilibrium asset price is characterized by equation (6) given an observation of $M \in \{1, 2\}$ bank runs. The price cannot be greater than or equal to qD_2, nor can it be smaller than $\underline{P}$.*

Proof. See Appendix A.2. □

It follows from $\mathbb{P}_M(\theta^*) \geq \underline{P}$ that banks do not fail at $t = 1$. This is because $\mathbb{P}_M(\theta^*) \geq \underline{P} > D_1$, so that banks can always repay their $t = 1$ liabilities.[22] Runs on the intermediate date, however, do accelerate bank failures because liquidation losses will lead to a higher probability of $t = 2$ bankruptcy. Specifically, while a partial liquidation can generate sufficient cash to pay early withdrawals and the bank does not immediately fail at $t = 1$, the cash flow from the residual portfolio will be insufficient to cover the remaining liabilities at $t = 2$, making a bank that is otherwise solvent fail at $t = 2$.[23] It is also worth noticing that parametric assumption (3) guarantees $qD_2 > \underline{P}$, so that the set of candidate equilibrium prices is non-empty.

3.2 Bank run game

We now turn to the creditors' bank run game and derive the condition that an equilibrium critical cash flow θ^* needs to satisfy. We do so by examining a representative creditor j's best response—running on a bank if and only if his private signal is below $\hat{x}$—to other players' equilibrium strategy $(x^*, \boldsymbol{P}^*)$.

[21] For an asset price equal to qD_2, one can show that any run will reduce a bank's asset and liabilities by the same amount, resulting in a neutral impact of runs on the solvency of the bank.

[22] Note that for $q < 1$, inequality (2) implies $D_2 > 2D_1$, because $D_2 = D_1/q + F > D_1 + F > 2D_1$.

[23] Similar to Morris and Shin (2016), even if a bank survives $t = 1$ runs, it would be doomed to fail at $t = 2$. The funding liquidity risk is captured by higher ex-ante probability of bank failure and the fact the survival threshold is higher than the solvency threshold. This feature of no interim date failure also emerges in Ahnert et al. (2018).

For a realized fundamental $\boldsymbol{\theta}$, an equilibrium switching threshold x^* results in runs for any bank with $\theta^i < \theta^*$. Because creditor j receives sufficiently accurate signals of both banks' fundamentals, he perfectly foresees the number of runs.[24] Accordingly, creditor j rationally expects the asset price to be $\mathbb{P}_M(\theta^*)$ when he foresees M runs, $M \in \{1, 2\}$. As we assume creditors play symmetric strategies across the two banks, the analysis of creditor j's withdrawal decisions in any of the two banks is the same. As a result, we suppress the index i of banks and focus our discussion on a representative bank.

We start with establishing the existence of two dominance regions. First, there exists a θ^L such that a bank with cash flow $\theta \in [\underline{\theta}_s, \theta^L)$ will always fail at $t = 2$, independently of the fraction of creditors who run. So it is a dominant strategy for creditor j to withdraw. Similarly, provided $\overline{\theta} > F/(1 - D_1/\underline{P})$, there exists a θ^U such that a bank with $\theta \in (\theta^U(\mathbb{P}_M(\theta^*)), \overline{\theta}]$ will always survive at $t = 2$, independently of the fraction of creditors who run. It is, therefore, a dominant strategy for creditor j to wait. We show in Appendix B that $\theta^L = D_2$ and that θ^U has an upper bound $F/(1 - D_1/\underline{P})$.

To solve for the best response of creditor j in the intermediate range $[\theta^L, \theta^U(\mathbb{P}_M(\theta^*))]$, we derive his payoffs for the actions 'wait' and 'withdraw' as functions of the fraction of other creditors who withdraw early. When L fraction of creditors withdraw early, a bank will face a liquidity demand of LD_1, $L \in [0, 1]$, and need to liquidate a λ fraction of its assets at a price $\mathbb{P}_M(\theta^*)$.

$$\lambda(\theta^*, M) = \frac{LD_1}{\mathbb{P}_M(\theta^*)} \in [0, 1)$$

Note that λ is between 0 and 1, since we have established in Lemma 1 that $\mathbb{P}_M(\theta^*)$ must be higher than $\underline{P}$, which is higher than D_1. After liquidating a fraction λ of its assets, the bank will fail at $t = 2$ if and only if the value of its remaining assets is lower than its remaining liabilities.

$$[1 - \lambda(\theta^*, M)] \cdot \theta < F + (1 - L)(1 - E - F)r_D \tag{7}$$

In other words, a bank will fail at $t = 2$ if and only if the fraction of creditors who withdraw exceeds a critical value L^c.

$$L > \frac{\mathbb{P}_M(\theta^*) \cdot [\theta - F - (1 - E - F)r_D]}{[q\theta - \mathbb{P}_M(\theta^*)]\,(1 - E - F)r_D} = \frac{\mathbb{P}_M(\theta^*) \cdot (\theta - D_2)}{D_1 \cdot [\theta - \mathbb{P}_M(\theta^*)/q]} \equiv L^c(\theta, \theta^*, M) \in [0, 1] \tag{8}$$

Creditor j's payoff, therefore, depends on the actions of other creditors, in particular, the fraction of withdrawals L. Depending on L, creditor j's payoffs from playing 'withdraw' or 'wait' are tabulated as follows.

[24] Recall that we define a run in a bank when a positive mass of creditors who made withdrawals in that bank. Consequently, creditor j's decision alone has no impact on the bank run outcome.

	$L \in [0, L^c]$	$L \in (L^c, 1]$
withdraw	D_1	D_1
wait	D_1/q	0

Note that if the creditor withdraws, his payoff will always be $W_{run}(L) = D_1$. Instead, if he waits, his payoff depends on the action of other creditors.

$$W_{wait}(L) = \begin{cases} D_1/q & L \in [0, L^c] \\ 0 & L \in (L^c, 1] \end{cases}$$

Defining the difference between the creditor's payoffs of "wait" and "withdraw" as $DW(L) \equiv W_{wait}(L) - W_{run}(L)$, we have the following expression.

$$DW(L) = \begin{cases} (1-q)D_1/q & L \in [0, L^c] \\ -D_1 & L \in (L^c, 1] \end{cases}$$

The strong strategic complementarity in creditors' game is clear. In a perfect information benchmark, creditor j strictly prefers 'wait' ('withdraw') if L is marginally lower (higher) than L^c, so that the slope of his best-response function tends to infinity when L approaches L^c. In fact, the bank run game of perfect information has two equilibria in which either all creditors withdraw or all creditors wait. We refine the multiplicity using the technique of global games.

With noisy signals of banks' fundamentals, the analysis resembles a standard global-games procedure except for the endogenous asset price. We outline the analysis here, and interested readers can refer to Appendix B for full details. As a first step, we have established the existence of lower and upper dominance regions. Second, we characterize creditor j's posterior belief about L when the bank's fundamental is out of the dominance regions. Finally, we show that creditor j's best response to the other creditors' threshold strategy is a threshold strategy too. In a symmetric equilibrium, the critical cash flow θ^* must satisfy the following condition:

$$\theta^* = \frac{D_2 - D_1}{1 - qD_1/\mathbb{P}_M(\theta^*)} \quad \text{and} \quad M \in \{1, 2\}. \tag{9}$$

We summarize these results in Lemma 2.

Lemma 2. *When creditors receive noisy signals* $\boldsymbol{x}$ *and expect the asset price to be* $\mathbb{P}_M(\theta^*)$*, the only equilibrium of the bank run game that survives iterated elimination of dominated strategies is a threshold equilibrium characterized by a critical signal* x^**. The corresponding bank critical cash flow* θ^* *is given by equation* (9)*. In the limiting case* $\epsilon \to 0$, $\theta^* = x^*$.

Proof. See Appendix B. □

A few comments are in order. First, a $t = 2$ failure happens when $L > L^c$, because the partial liquidation at $t = 1$ incurs a liquidation loss. When a sufficiently large number of creditors withdraw, the bank will be forced to liquidate prematurely a significant share of it assets, and the remaining assets will generate insufficient cash flows to meet the remaining liabilities. The creditors who withdraw therefore can impose negative externalities on creditors who wait.

Second, our model differs from classic global-games-based bank run models because creditors in our model are forward-looking and understand the impact of their decisions to run on asset prices. While the critical fraction L^c only depends on the bank's fundamental and an exogenous asset price in a classic model, L^c in our model is also a function of the threshold θ^* and the number of bank runs M, because both affect the endogenous asset price $\mathbb{P}_M(\theta^*)$.

Finally, equation (9) characterizes a symmetric threshold equilibrium if it exits. We still need to establish the existence of the equilibrium given the endogenous asset price, which is the focus of Section 3.3 and Section 3.4.

3.3 Unique equilibrium without aggregate uncertainty

We start with a baseline case where $\underline{\theta}_B = \underline{\theta}_G = \underline{\theta}$. With no aggregate uncertainty, banks are only exposed to the idiosyncratic risks of their cash flows. With cash flows independently distributed, the failure of one bank carries no information for the other banks' fundamentals. The asset buyers, therefore, will offer a single price P independent of the number of runs observed. In other words, their strategy features a price schedule $\boldsymbol{P} = (P_1, P_2) = (P, P)$, so that the demand for banks asset is perfectly elastic.

As discussed in Section 3.1, a candidate equilibrium price P^* must satisfy the zero-profit condition (6). Without aggregate uncertainty, $E_s(\underline{\theta}_s|M)$ degenerates to $\underline{\theta}$, and the condition becomes

$$P^* = \mathbb{P}(\theta^*) = \frac{\theta^* + \underline{\theta}}{2}, \tag{10}$$

which is no longer a function of M. As creditors anticipate the single asset price P^*, equation (9) that defines the critical fundamental θ^* becomes

$$\theta^* = \frac{D_2 - D_1}{1 - qD_1/P^*}. \tag{11}$$

An equilibrium of the game, if exists, is a solution of the system of two equations (10) and (11). In the absence of aggregate uncertainty, we show that there exist a unique pair of a critical fundamental $\theta^* \in [\theta^L, \theta^U(P^*)]$ and an asset price $P^* \in [\underline{P}, qD_2)$, which jointly solve the system of equations and are independent of the number of runs observed. We present the closed-form solution in Appendix A.3 and summarize the results in Proposition 1.

Proposition 1. *Without aggregate uncertainty, there exists a unique PBE characterized by* $(x^*, \boldsymbol{P}^*)$*, with* $\boldsymbol{P}^* = (P^*, P^*)$ *and* $x^* = \theta^*$ *when noise* $\epsilon \to 0$*. A bank run happens if and only if the bank's fundamental is below* θ^**, and the bank's assets will be sold for price* P^**.*

Proof. See Appendix A.3. □

The equilibrium is unique and stable despite the two-way feedback between bank runs and asset prices. Intuitively, if creditors take a more optimistic strategy than the equilibrium one, they will rationally anticipate the asset buyers to bid a lower price P^* according to equilibrium condition (10). The lower P^*, however, implies an aggravated coordination problem by equilibrium condition (11). This, in turn, restores the equilibrium threshold strategy.[25]

A few comments are in order regarding the unique equilibrium. First, in the absence of aggregate uncertainty, the equilibrium outcome of the bank run game is qualitatively similar to the classic global-games-based bank run models such as Rochet and Vives (2004) and Vives (2014). That is, a bank with $\theta \in [D_2, \theta^*)$ can repay its debt if no bank run happens, but will fail in equilibrium because of premature asset liquidation. The uniqueness, however, obtains only because we examined a special case without aggregate uncertainty. Given a single aggregate state, creditors' strategy cannot affect buyers' perception of the aggregate state, which makes equation (6) reduce to (10) and only one price rationalizable. The introduction of aggregate uncertainty will open up the possibility for players to coordinate on different beliefs about the aggregate state, generating price volatility and contagious runs in the form of multiple equilibria.

Second, regarding the asset market, it is worth noticing that our model does not feature asset fire sales. Since the buyers pay the expected payoff of the asset given their information set, no welfare loss emerges due to the change of ownership of the asset.[26]

Finally, the classic LoLR policies that aim to target only solvent-but-illiquid banks will be infeasible when central banks do not possess accurate information on individual banks' solvency. An uninformed central bank cannot offer a better price than the private market participants—at least not without incurring expected losses.

[25] It is worth noticing that the information asymmetry in our model does not generate standard adverse selection problems where lower prices are associated with lower average qualities. Since banks in our model are forced into asset liquidation rather than strategically choose to do so, a lower asset price in our model is associated with a *higher* average quality.

[26] This differs from classic views of asset fire sales, such as in Shleifer and Vishny (1992). When the seller of the asset can make better use of the capital as compared to the potential buyers, the asset sale itself generates a negative impact on social welfare.

3.4 Multiple equilibria with aggregate uncertainty

In this section, we characterize the equilibrium of the fully-fledged model with both idiosyncratic and aggregate risks. This generates an extra complication as compared to the last section. Asset buyers can now form beliefs about the aggregate state s, and their beliefs have to be rationalized by the number of bank runs generated by the creditors' equilibrium strategies.

To construct an equilibrium, we first derive a critical cash flow θ^*_M and the corresponding asset price P^*_M, for a given number of runs $M \in \{1, 2\}$.[27] Upon the observation of M runs, asset buyers update their belief about the quality of asset on sale, and their competitive bidding leads to an asset price $P^*_M = \mathbb{P}_M(\theta^*_M)$ as specified by equation (6). On the other hand, expecting M runs and an asset price P^*_M, creditors follow their threshold strategy, which implies a critical cash flow θ^*_M formulated by equation (9). Creditors' strategy and belief need to be consistent in the sense that exactly M bank runs should happen according to the critical cash flow θ^*_M. We establish in Lemma 3 that the pair (θ^*_M, P^*_M) is unique for a given $M \in \{1, 2\}$.

Lemma 3. *In the presence of aggregate uncertainty, for a given $M \in \{1, 2\}$, there exists a unique pair of an asset price $P^*_M \in [\underline{P}, qD_2)$ and a critical cash flow $\theta^*_M \in [\theta^L, \theta^U(P^*_M)]$ that jointly solve the system of equations (6) and (9).*

Proof. See Appendix A.4. □

The critical cash flows θ^*_M and asset prices P^*_M exhibit monotonicity with respect to M. Intuitively, asset buyers form more pessimistic beliefs about the aggregate state s when observing more bank runs. As a result, they will offer a lower price, since the expected asset quality is lower in State B. This, in turn, pushes up the threshold cash flow for a bank to survive a run.

Lemma 4. *When more runs happen, asset buyers bid less, and a bank needs to generate a higher critical cash flow to survive runs. That is, $P^*_2 < P^*_1$, and $\theta^*_2 > \theta^*_1$.*

Proof. See Appendix A.5. □

To better understand the monotonicity in Lemma 4, one may consider a hypothetical case where the aggregate state is observable. Following the same reasoning of Proposition 1, one can derive a unique critical cash flow θ^*_s and price P^*_s, conditional on that State s has realized, with $\theta^*_G < \theta^*_B$, and $s \in \{G, B\}$. However, when the aggregate state is unobservable, asset buyers have to form posterior beliefs about s. Each (θ^*_M, P^*_M) pair is associated a unique posterior belief $\omega^B_M \in (0, 1)$ that is rationalized by the observation of M bank runs. If $(\theta^*_M, \boldsymbol{P^*})$ can be sustained

[27] Since the creditors' strategy needs to be consistent with buyers' bid and can, in principle, changes with the number of runs M, we index the corresponding critical cash flow θ^* by M.

as an equilibrium for a fundamental $\boldsymbol{\theta}$, the fundamental and the equilibrium strategy must lead to exactly M bank runs. As M increases, the buyers' posterior belief worsens, generating the monotonicity.

Lemma 5. *When the aggregate state s is perceived to be G or B with probability 1, there exists a unique PBE characterized by $(\theta^*_s, \boldsymbol{P}^*_s)$, with $\boldsymbol{P}^*_s = (P^*_s, P^*_s)$, $s \in \{G, B\}$. It holds that $\theta^*_G < \theta^*_1 < \theta^*_2 < \theta^*_B$ and $P^*_G > P^*_1 > P^*_2 > P^*_B$.*

Proof. See Appendix A.6. □

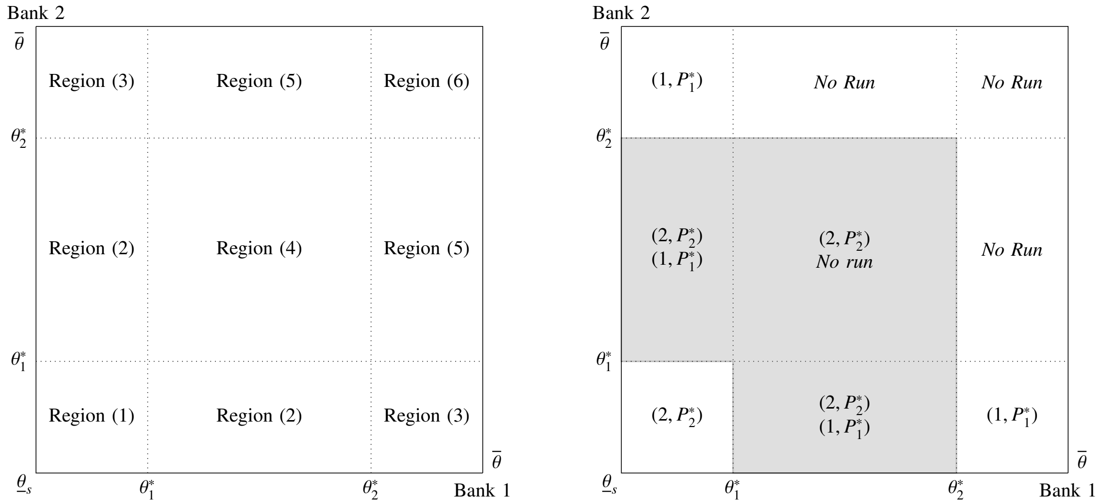
Lemma 3 and 4 do not fully characterize the equilibrium of the model, because M is part of the equilibrium outcome and cannot be treated as given. Contrast to classic bank run models à la Rochet and Vives (2004), the equilibrium of our model can vary with banks' fundamentals.[28] In fact, for certain fundamentals, creditors can form distinct rational expectations about the number of runs M and coordinate on one of multiple threshold strategies. As a result, multiple equilibria can emerge.

We now give a complete characterization of the equilibrium of our model. As $\theta^*_1 < \theta^*_2$, the critical cash flows make a unique partition of the set of bank fundamentals as shown in Panel (a) of Figure 2. For the ease of exposition, we dub creditors' threshold strategy 'to run on a bank if and only if the private signal about the bank's fundamental is below θ^*_1' the *optimistic strategy*, and the threshold strategy 'to run on a bank if and only if the private signal about the bank's fundamental is below θ^*_2' the *pessimistic strategy*. Noticing that buyers' equilibrium price schedule $\boldsymbol{P}^* = (P^*_1, P^*_2)$ does not change across different regions, we characterize the equilibrium strategy and the equilibrium outcomes of the game as follows.

- In Region (1), where both banks' cash flows are below θ^*_1, the only *PBE* is that creditors play the *pessimistic strategy*. Consequently, two bank runs happen, and the asset buyers purchase bank assets for the price P^*_2.
- In Region (2), where one bank's fundamental is between θ^*_1 and θ^*_2 and the other bank's is below θ^*_1, two equilibria can emerge. If creditors take the *optimistic strategy*, only the weaker bank will fail, which leads to the asset price of P^*_1. However, if creditors take the *pessimistic strategy*, both banks will fail, and the asset price will be P^*_2.
- In Region (3), where one bank's fundamental is below θ^*_1 and the other bank's is greater than θ^*_2, we have a unique equilibrium that creditors play the *optimistic strategy*. Consequently, only the weaker bank fails, resulting the asset price of P^*_1.

[28] In the classic models, the equilibrium is independent of a bank's fundamental, even though the equilibrium outcome can change with the fundamental.

Figure 2: Equilibrium outcomes for given regions of fundamentals



(a) Partition of bank fundamentals

(b) Equilibrium outcomes ($N = 2$)

The equilibrium outcome for given fundamentals is indicated in Panel (b). Depending on whether the creditors play the *optimistic* or *pessimistic* strategy, the banking sector's fundamentals lead to the following equilibrium outcomes: $(2, P_2^*)$ in Region (1); $(1, P_1^*)$ or $(2, P_2^*)$ in Region (2); $(1, P_1^*)$ in Region (3); No Run or $(2, P_2^*)$ in Region (4); and No Run in Region (5) and (6). Shaded areas indicate regions where multiple equilibrium outcomes can emerge for the same fundamentals.

- In Region (4), where both banks' cash flows are between θ_1^* and θ_2^*, two equilibria can emerge. If creditors take the *pessimistic strategy*, the banking sector will experience two bank runs, and the asset buyers purchase banks' assets for P_2^*. If creditors play the *optimistic strategy*, no bank will fail, and no asset liquidation will occur.
- In Region (5), where one bank's fundamental is between θ_1^* and θ_2^* and the other bank's is greater than θ_2^*, the unique PBE is that creditors play the *optimistic strategy*. Consequently, no bank will fail, and no asset liquidation will occur.
- In Region (6), where both banks' fundamentals exceed θ_2^*, two equilibria can be sustained. The creditors can justify either the *optimistic* or the *pessimistic strategy*. But the equilibrium outcome is unique, with no bank run and no asset sale.

For an intermediate range of fundamentals, i.e., Region (2) and Region (4), multiple equilibrium outcomes can emerge for the same fundamental. This contrasts with the full determinacy (conditional on fundamentals) of classic global-games-based bank run models. The multiplicity is driven by asset buyers' belief about the aggregate state. When creditors pessimistically expect the low asset price P_2^*, they will set their threshold at θ_2^*. Their pessimistic strategy will generate two bank runs and justify itself. We emphasize the financial fragility in Proposition 2.

Proposition 2. *For any realization of banks' fundamentals, a PBE exists. When the fundamentals can generate M runs, a PBE is characterized by* $(\theta_M^*, \boldsymbol{P}^*)$*, with* $\boldsymbol{P}^* = (P_1^*, P_2^*)$ *and*

$P^*_M = \mathbb{P}_M(\theta^*_M)$, $M \in \{1,2\}$. Multiple equilibria emerge, when one bank's fundamental belongs to $[\theta^*_1, \theta^*_2]$ and the other bank's below θ^*_2. In this intermediate range of fundamentals, both creditors' optimistic strategy and optimistic strategy can be rationalized.

The existence of multiple equilibria implies financial fragility in the form of contagious runs and price volatility. For example, suppose that one bank's fundamental is below θ^*_1 whereas the other bank's fundamental is between θ^*_1 and θ^*_2. Two distinct equilibrium outcomes, $(1, P^*_1)$ and $(2, P^*_2)$, can emerge in equilibrium. When creditors play the *pessimistic strategy*, the stronger bank will experience '*contagious bank runs*' given the expectation that the weaker bank will fail. We indicate the equilibrium outcomes in Panel (b) of Figure 2 and characterize the regions of fundamentals where both banks can fail (a scenario that we dub as '*systemic bank runs*') in Corollary 1.

Corollary 1. *Systemic bank runs can happen when both banks' fundamentals are below θ^*_2. In particular, systemic runs are driven by the creditors' pessimistic beliefs when one bank's fundamental belongs to $[\theta^*_1, \theta^*_2]$ and the other bank's is below θ^*_2.*

Corollary 1 shows that extreme financial fragility can emerge in a laissez-faire market. Even if the banking sector's fundamental is strong, e.g., both banks' fundamentals only marginally below the critical cash flow θ^*_2, runs can still happen to both banks if the creditors play the *pessimistic strategy*.

When the same fundamental allows for multiple equilibrium outcomes, there is scope for policy intervention. We consider liquidity intervention that will limit the range of fundamentals where systemic bank runs can happen, because systemic crises are particularly detrimental to the economy.[29] Since systemic bank runs can happen when both banks' fundamentals are below θ^*_2, we consider intervention that will diminish this region.

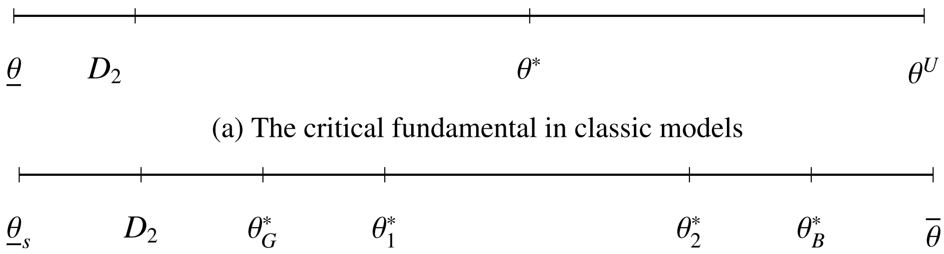
Our model can be seen as a hybrid model of fundamental and belief-driven runs. In the intermediate range of fundamentals, i.e., Regions (2) and (4), creditors' multiple equilibrium strategies can be rationalized by the resulting asset price and number of runs, whereas only a unique equilibrium strategy can be rationalized in Regions (1), (3), and (5). The multiplicity or uniqueness of the equilibrium depends on the fundamental. This feature differs from models such as Diamond and Dybvig (1983), in which creditors can coordinate on a belief about each other's strategy so that multiple equilibrium strategies exist independent of fundamentals. The equilibrium in our model also differs from classic global-games-based bank run models. Creditors in classic models such as Rochet and Vives (2004) and Vives (2014) cannot coordinate

[29] Systemic banking crises can put payment system in danger, are more difficult to resolve, and can put significant pressure on the fiscal balance of the government.

their strategies, so they have a unique equilibrium strategy, with the threshold independent of fundamentals.

Our model generalizes classic global-games models by introducing endogenous asset prices, aggregate states and multiple banks. The generalization can be best understood from Figure 3. In the classic models, the refinement generates a unique threshold as shown in Panel (a). In contrast, the introduction of aggregate states G and B creates two corresponding thresholds θ_G^* and θ_B^*. The difference between θ_G^* and θ_B^* creates a range for possible multiple equilibria. For this intermediate range of fundamentals, multiple equilibria can arise and be ranked according to the number of observed runs and buyers' beliefs about the aggregate state.

Figure 3: The comparison between our model and classic global-games models



(a) The critical fundamental in classic models

(b) Critical fundamentals in our model

4 The buyer of last resort

An intuitive way to break the two-way feedback between falling asset prices and bank runs is to introduce a backstop for the price of banks' assets, which will short-circuit the feedback loop. In this section, we consider two possible designs for the BoLR intervention along this line of reasoning. We start by introducing a buyer of last resort who unilaterally offers a price support. That is, the BoLR commits to purchasing banks' assets for a single price regardless of the number of runs. Such a design with unilateral commitment resembles the asset purchase program as initially envisaged under the Troubled Asset Relief Program (TARP).[30] We show that the price support can stabilize the financial system only if the BoLR is willing to accept expected losses. If the role of the BoLR is assumed by a public authority (e.g., a central bank or a treasury), the expected losses imply a transfer from the public to banks' creditors.[31] This intervention is thus similar in nature to a bail-out and can be an unpopular solution to the financial fragility. To address this limitation, we propose an alternative design where the BoLR's

[30]Eventually, the TARP fund was mostly disbursed to recapitalize banks via the purchase of their preferred shares. The asset purchase was carried out by the Public-Private Investment Program (PPIP) at a smaller scale, with $18.6 billions of asset purchased. See Baird Webel (2013) for details.

[31]The expected losses, of course, also imply that private entities have no incentive to provide the price support.

commitment to the offered price is matched by banks' commitment to pledge their assets to the BoLR for her liquidity support. The idea behind the mutual commitment is that the public liquidity support should be commensurate with the regulatory obligations of private entities. This setup resembles the recent policy developments such as banks pre-positioning their assets with the Bank of England for its liquidity support, and banks paying upfront a premium for the liquidity support of the Reserve Bank of Australia under its committed liquidity facility (CLF). We show that the BoLR can limit financial fragility and breaks even from an ex-ante perspective, even though she is no better informed than private asset buyers.

Formally, we consider a buyer of last resort who can intervene by committing at $t = 0$ to purchasing bank assets for a single price at $t = 1$, in case any bank is forced into an asset sale then. We assume that the BoLR's offer is publicly observable. The intervention does not target any particular bank but aims directly at banks' assets. The intervention is also pre-emptive, in the sense that the BoLR's offer is made before the realization of the systematic risk s and idiosyncratic risks $\boldsymbol{\theta}$, and therefore before the observation of actual bank runs. As a result, the intervention requires neither information on the (in)solvency of individual banks nor the knowledge of the aggregate state. Nevertheless, when making the offer, the BoLR needs to form rational expectations about the possible number of runs and the associated probabilities of such events. The BoLR is assumed to possess full commitment power and will not revoke her offer ex-post.

4.1 BoLR with unilateral commitment

We start with a design where the BoLR offers a single price P_U and *unilaterally* commits to the price independently of the number of runs.[32] That is, the BoLR purchases banks' assets for P_U, even if two bank runs occur and interim-date asset buyers are only willing to purchase banks' assets for P_2^*.

To see the stability effect, we start by analyzing the benefit and cost of a BoLR intervention that features a price support $P_U \in (P_2^*, P_1^*)$.[33] Under this intervention, creditors understand that banks will be able to sell their assets to the BoLR for the price P_U even if two bank runs happen. Consequently, the price P_2^* cannot be part of their rational expectation, and θ_2^* can no longer be sustained as an equilibrium critical cash flow of the bank run game. As a result, systemic bank runs will occur if and only if both banks' fundamentals are below

[32]Notice that, instead of providing a price schedule, the BoLR commits to a single price that does not depend on the number of bank runs. Otherwise, the BoLR's offer will also lead to multiple equilibria and financial contagion for the same reason as in Section 3.

[33]Since P_2^* is the lowest possible price in a laissez-faire market, the BoLR's intervention will be ineffective in terms of promoting financial stability if she sets $P_U \leq P_2^*$.

$\theta^*(P_U) = (D_2 - D_1)/(1 - qD_1/P_U) \in (\theta_1^*, \theta_2^*)$. This contrasts with the laissez-faire market where systemic bank runs happen when both banks' fundamentals are below θ_2^*.

Such an intervention, however, must involve the BoLR making expected losses—at least under a reasonable assumption that a bank will sell its assets to buyers that offer the highest bid. Since $P_U < P_1^*$, a bank will sell its assets to the interim-date asset buyers if it is the only bank that experiences a run. The BoLR, therefore, expects to purchase banks' assets only when runs happen to both banks. By our analysis in Section 3, we know that when two bank runs occur, the price allowing the BoLR to break even must be P_2^*. Purchasing assets for $P_U \in (P_2^*, P_1^*)$ will lead to expected losses.

If the BoLR increases P_U above P_1^*, the critical cash flow that triggers bank runs will fall below θ_1^*. The range of bank fundamentals where systemic runs can happen will be further reduced. However, this benefit of greater financial stability can be obtained only if the BoLR makes even greater losses as compared to the case where she offers $P_U \in (P_2^*, P_1^*)$. We establish in Proposition 3 that the BoLR cannot reduce financial fragility and avoid expected losses at the same time by unilaterally committing to a price support.

Proposition 3. *If a BoLR unilaterally commits to purchasing banks' assets on sale at $t = 1$ for a price $P_U > P_2^*$, she will limit the range of fundamentals where systemic bank runs can happen, but will always make expected losses.*

Proof. See Appendix A.7. □

4.2 BoLR with mutual commitment

To show that a BoLR can limit financial fragility and break even in expectation at the same time, we now consider an alternative design where the banks commit to selling their assets to the BoLR in case they face runs. Formally, in exchange for the BoLR's price support, each bank enters at $t = 0$ a binding agreement with the BoLR that when experiencing runs, the bank will raise liquidity from the BoLR for a specified price P_A per unit of its assets. We assume in this section that both banks have chosen to enter the agreement with the BoLR at $t = 0$. Consequently, a bank will not sell its assets to the interim-date asset buyers even if it is the only bank that experiences runs. We show in Appendix C that banks have incentives to do so.[34]

We start by deriving a price P_A^* that allows the BoLR to break even in expectation, and then analyze the equilibrium outcomes when the BoLR offers such a price P_A^*. We show that P_A^* must be smaller than P_1^*. On the other hand, for the price to limit financial fragility, P_A^* must be higher than P_2^*.

[34] In particular, we consider a game where both banks contract with the BoLR is a Nash equilibrium. In other words, we can implement the BoLR intervention that features mutual commitment.

Lemma 6. *A price P_A^* that allows a BoLR to break even and to limit financial fragility must satisfy $P_A^* \in (P_2^*, P_1^*)$.*

Proof. See Appendix A.8. □

Having narrowed down the region in which the BoLR's break-even price P_A^* lies, we now derive P_A^* and the corresponding critical cash flow θ_A^*. Compared to the game analyzed in Section 3, where creditors need to form rational expectations about the intermediate-date price of banks' assets, the creditors have learned the price offered by the BoLR from the mutual agreement by the time of their move. Therefore, we solve the model backward and start with the creditors' bank run game at $t = 1$. For a price $P_A \in (P_2^*, P_1^*)$, the bank run game can be solved by a standard global-games procedure with an exogenous asset price. Following our analysis in Appendix B, we have the following critical cash flow

$$\theta^*(P_A) = \frac{D_2 - D_1}{1 - qD_1/P_A}. \tag{12}$$

We then move one step backward to formulate the BoLR's expected payoff from offering a price P_A under the mutual agreement. When the BoLR makes her offer, the uncertainties about s and M have not yet been resolved. Denote the BoLR's expected payoff by $V(P_A)$. We have

$$\begin{aligned} V(P_A) &= E_s\left(E_M\left(M \cdot C_2^M \cdot \pi(P_A|s)\right)\right) \\ &= \sum_{s=G,B} Pr(s) \cdot \left(\sum_{M=1}^{2} Pr(\theta < \theta^*(P_A)|s)^M \cdot Pr(\theta > \theta^*(P_A)|s)^{2-M} \cdot C_2^M \cdot M \cdot \pi(P_A|s)\right), \end{aligned} \tag{13}$$

where $M \in \{1, 2\}$, and $\pi(P_A|s) = [\underline{\theta}_s + \theta^*(P_A)]/2 - P_A$ is the BoLR's expected payoff from purchasing assets from one bank for the price P_A in a given aggregate state s. Correspondingly, $M \cdot \pi(P_A|s)$ denotes the BoLR's payoff of purchasing assets from M banks. Since $Pr(\theta < \theta^*(P_A)|s)^M \cdot Pr(\theta > \theta^*(P_A)|s)^{2-M}$ is the probability that M and only M banks are forced into liquidation in a given aggregate state s, the expression in the parentheses denotes the BoLR's expected payoff in the state s. The BoLR will break even in expectation when offering a price P_A^* such that $V(P_A^*) = 0$.

Offering P_A at $t = 0$ allows the BoLR to *break even across possible posterior beliefs of state s*. By contrast, asset buyers who bid ex-post at $t = 1$ have to *break even within a given belief of state s*. To highlight the difference, we define $\Pi_M(P)$ as the expected payoff from purchasing one unit of bank assets for a price P in the contingency of M bank runs. Denote the corresponding critical cash flow by $\theta^*(P)$. We can calculate

$$\Pi_M(P) = \omega_M^B(\theta^*(P)) \cdot \pi(P|s = B) + \omega_M^G(\theta^*(P)) \cdot \pi(P|s = G),$$

where $\omega_M^s(\theta^*(P))$ is the posterior belief about state $s \in \{G, B\}$ upon the observation of M bank runs when the bank's assets are sold for a price P. One can verify that $\Pi_M(P)$ strictly decreases in P.[35] By our analysis in Section 3, we know that asset buyers who bid at $t = 1$ will offer a unique price P_M^* such that $\Pi_M(P_M^*) = 0, \forall M \in \{1, 2\}$. In other words, they always break even for a *realized* M and the associated belief $\omega_M^s(P_M^*)$. The BoLR, on the other hand, can offer a P_A^* to break even *across different numbers of runs* and thereby across possible posterior beliefs about the aggregate state s. To see this, we use the definition of $\Pi_M(P)$ to re-arrange $V(P_A)$ into the following form:

$$V(P_A) = \sum_{M=1}^{2} \left(\sum_{s=G,B} Pr(s) \cdot Pr(\theta < \theta^*(P_A)|s)^M \cdot Pr(\theta > \theta^*(P_A)|s)^{2-M} \right) C_2^M \cdot M \cdot \Pi_M(P_A). \quad (14)$$

The expression in parentheses is the BoLR's perceived probability that M bank runs will occur, when she offers a price P_A. Note that $\Pi_M(P_A)$ is the BoLR's expected payoff in the contingency where M runs happen. Importantly, the BoLR does not require $\Pi_M(P_A^*) = 0$ but instead $V(P_A^*) = 0$. In fact, the BoLR expects to make losses in the contingency $M = 2$ and to profit in the contingency $M = 1$. Since $\Pi_1(P_2^*) > 0$ and $\Pi_2(P_1^*) < 0$, we know that $V(P_A) > 0$ for $P_A = P_2^*$ and that $V(P_A) < 0$ for $P_A = P_1^*$.[36] By continuity, there exists a $P_A^* \in (P_2^*, P_1^*)$ that allows the BoLR to break even. We establish in Proposition 4 that P_A^* is also unique.

Proposition 4. *If a BoLR and banks mutually commit to an agreement for the BoLR to purchase assets from any bank that experiences runs at $t = 1$, there exists a unique price P_A^* that allows the BoLR to break even ex ante. Compared to the ex-post market prices, we have $P_A^* \in (P_2^*, P_1^*)$.*

Proof. See Appendix A.9. □

Given the unique P_A^*, there exists a unique θ_A^* given by equation (12). Since $P_A^* < P_1^* < qD_2$, we know $\theta_A^* > D_2$. Therefore, the proposed intervention only mitigates banks' funding liquidity risk and has no impact on the pure insolvency risk.

To appreciate the stability effect, consider the case where both banks' cash flows are only marginally below θ_2^*. We know that one possible equilibrium outcome in a laissez-faire market is that both banks fail and the prevailing asset price drops to P_2^*. Once the BoLR pre-commits

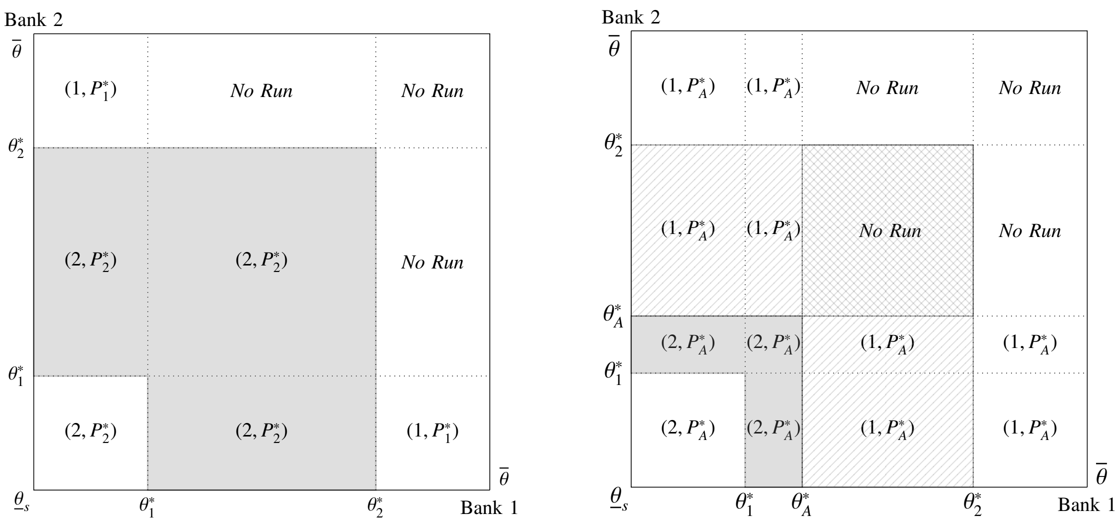
[35]Intuitively, by bidding a higher price, one decreases the expected payoff in three ways. First, a higher bid increases the cost of acquiring a piece of assets, which directly reduces her payoff. Second, a higher P also alleviates banks' illiquidity risk, making fewer banks sell for liquidity reasons. As a result, they face a pool of assets with deteriorating quality where more banks are selling assets because of fundamental insolvency. Both channel reduces $\pi(P|s)$, $s = \{G, B\}$. Finally, conditional on the observation of M bank runs, a higher P leads to a perception that State B is more likely, i.e., $\omega_M^B(P)$ increasing in P, which further reduces $\Pi_M(P)$.

[36]By Lemma 4, we have $P_2^* < P_1^*$. Since $\Pi_M(P)$ strictly decreases in P and $\Pi_M(P_M^*) = 0$, we know $\Pi_1(P_2^*) > 0$ and $\Pi_2(P_1^*) < 0$.

to the price $P_A^* > P_2^*$, the pessimistic equilibrium can no longer be sustained. Because creditors expect price not to go below P_A^*, the switching threshold θ_2^* can no longer be rationalized. As a result, no bank run happens under the BoLR intervention. This example illustrates that promoting financial stability does not always involve the BoLR purchasing banks' assets. A credible commitment to price P_A^* is sufficient to eliminate the inefficient equilibrium that is belief-driven. For such fundamentals, the BoLR improves the allocation by eliminating coordination failures, in a similar way that deposit insurance works in Diamond and Dybvig (1983). This feature resembles the consequences of ECB's outright monetary transactions (OMT) program, in which case the market stabilized even though no actual transactions happened under the program. We illustrate this by the cross-hatched area in Panel (b) of Figure 4.

The BoLR can also reduce the risk of systemic bank runs when she actually purchases assets from banks. To see this, suppose that creditors play the pessimistic strategy. The BoLR will prevent systemic bank runs in states where one bank's fundamental is below θ_A^* while the other's belongs to the interval $[\theta_A^*, \theta_2^*]$. The BoLR saves the bank that has a relatively strong fundamental which would otherwise fail due to pessimistic beliefs and contagion. This is illustrated by the single-hatched area in Panel (b) of Figure 4. We summarize these results in Proposition 5.

Figure 4: The effects of the BoLR intervention under creditors' pessimistic strategy



(a) Allocation under creditors' pessimistic strategy (b) Allocation under the BoLR intervention

Under creditors' pessimistic strategy, the range of fundamentals where contagious bank runs can happen under the BoLR intervention is indicated by the gray area in Panel (b). This contrasts the gray area in Panel (a) where contagious runs can happen in a laissez-faire market. The range of fundamentals where the BoLR promotes financial stability without purchasing banks' assets is indicated by the cross-hatched area in Panel (b); and the range of fundamentals where the BoLR promotes stability by actually purchasing banks' assets is indicated by the single-hatched area in Panel (b).

Proposition 5. *If a BoLR and banks mutually commit to an agreement for the BoLR to purchase assets from any bank that experiences runs at $t = 1$ for price P_A^*, the BoLR will limit the range of fundamentals where systemic runs can happen and will break even in expectation at the same time. The intervention eliminates belief-driven contagious bank runs when one bank's cash flow belongs to $[\theta_A^*, \theta_2^*]$ and the other bank's is below θ_2^*.*

The BoLR differs from asset buyers who bid ex-post because she has commitment power. As mentioned, the interim-date buyers are constrained by ex-post break-even conditions and must make no expected losses given any realized number of bank runs. In fact, if an asset buyer has no commitment power but offers a price $P = P_A^*$, the buyer would expect to make a loss when two bank runs happen and will thus revoke the offer. To break even from this ex-post perspective, the asset buyer has to lower her offered price, so as to decrease the loss from purchasing assets with $\theta \in [\underline{\theta}_s, P)$ and to increase the profit from purchasing assets with $\theta \in [P, \theta^*(P))$. The lack of commitment power, therefore, leads to a lower asset price which in turn results in more bank runs. By contrast, a BoLR with the commitment power can choose not to react to the outcome of bank run games and stick to a single asset price. The BoLR does not need to break even for a given observed number of bank runs, but to break even across possible contingencies of $M = 1, 2$.[37] This allows the BoLR to break the vicious cycle between bank runs and asset sales that are fueled by pessimistic beliefs in market.

4.3 Central banks as the BoLR

As the BoLR expects to break even, it can be in principle a private institution, or more realistically, a large number of private institutions that participate in a decentralized market to provide liquidity insurance.[38] When a public authority acts as the BoLR, the intervention with mutual commitment differs from a public bailout, since the BoLR does not incur any expected loss and therefore makes no net transfer to banks' creditors. We believe that a public authority such as a central bank can be a natural candidate to act as a BoLR, because committing to a pre-specified price can be easier for a central bank than for private institutions.

Central banks can hold the commitment power to offer a price support for at least three reasons. First, central banks have different objective functions as compared to private asset buyers. Negative externalities from bank failures (such as the threat to the payment system, the loss of soft information on borrowers) are not taken into account by private asset buyers but

[37] The role of commitment power in improving allocation is central to classic models such as Holmstrom and Tirole (1998), where banks that commit to providing credit line contracts can break even across ex-post verifiable states. Our model differs from such a setup, because State s is ex-post unobservable, and that the BoLR breaks even across posterior beliefs about s.

[38] In this case, the commitment power of both the BoLR and banks must be due to a binding agreement enforceable by a court.

are major concerns to central banks. In the presence of such externalities, P_A^* can be ex-post optimal for the central bank even if the observed number of runs suggests expected losses from the trade itself. Second, central banks are not subject to the same stark bankruptcy constraints as private institutions and can sit on temporary losses caused by short-term price fluctuations. Similarly, central banks do not face pressure to lower its bid for banks' assets to increase the financial returns from the intervention, which could be the case for a private BoLR as it needs to compensate its shareholders. Third, central banks can employ commitment devices such as establishing financial stability funds to signal its commitment to support the asset price.[39]

Our modeling of the BoLR intervention with mutual commitment is broadly consistent with the suggestion of a former Governor of the Bank of England that a central bank should act as a "pawnbroker for all seasons (PFAS)" that commits to providing liquidity insurance to banks in times of crisis (King (2017)). His proposal is that banks should be required to pre-position their assets with the central bank for emergency liquidity assistance, which is similar, in our model, to a bank committing to selling its assets to the BoLR in the case of runs.[40] In our model, when making a commitment to the BoLR, a bank de facto purchases liquidity insurance: it forgoes the possibility of selling its assets for a more favorable price in the case of idiosyncratic runs but is insured against the risk of contagion in a systemic crisis driven by creditors' pessimistic beliefs. We have shown that banks' commitment to participating in the liquidity insurance scheme at $t = 0$ is essential for the BoLR to break even. This commitment is also emphasized by King (2017), who suggests that banks should be "required to take out insurance in the form of pre-positioned collateral with the central bank" and that the provision of liquidity insurance should be "mandatory and paid for upfront". King (2017) argues that banks should pre-position their assets for the central bank to "assess the collateral in normal times and not, as happened during the crisis, be forced to make snap judgements about collateral when the storm arrives." According to our model, banks' commitment in the form of pre-positioning their assets is also crucial for the central bank to avoid being cornered into a situation where it is forced to intervene only in the severest crisis.

An alternative way for the BoLR to break even is for it to charge banks upfront for the liquidity support—in the form of a liquidity insurance premium. In our setup, the premium will compensate the BoLR for her losses in states where she purchases assets from both banks. In return, each bank can receive from the BoLR a guaranteed price support for its assets and can

[39]A similar observation can be made about deposit insurance. Most countries require banks to pay deposit insurance premium ex-ante into a deposit insurance fund, which adds to the credibility of deposit insurance schemes.

[40]See King (2017) for detailed discussion. In fact, the Bank of England has implemented asset pre-positioning as a part of its Sterling Monetary Framework. By the spring of 2015, £469 billions of bank assets had been pre-positioned with the central bank, with an average haircut of 33%. In January 2019, the central bank published detailed guidelines regarding the procedures for private institutions to pre-position their illiquid assets. See Bank of England (2019).

still keep the option to sell its assets to the intermediate-date asset buyers. Such a design resembles the Committed Liquidity Facility (CLP) of the Reserve Bank of Australia. To benefit from the central bank's liquidity support under the facility, a bank needs to pay ex-ante a premium of 15 basis points for the amount of liquidity committed by the central bank. In return, the central bank contractually commits to entering repo transactions with the participating bank, should runs happen to it.[41]

4.4 Trade-offs of BoLR interventions

While the BoLR intervention with unilateral commitment leads to expected losses, the BoLR intervention with mutual commitment eliminates contagion and asset price volatility, but only at a cost. The equilibrium outcomes under price P_A^* are dominated by those associated with the *optimistic strategy* of creditors. Intuitively, since the BoLR breaks even across different posterior beliefs associated with different numbers of runs, her offer P_A^* must be lower than P_1^*. Therefore, compared to the equilibrium outcomes generated by creditors' optimistic strategies, allocations can deteriorate for certain fundamentals. This potential downside is indicated by the dotted area in Panel (a) of Figure 5.[42] The BoLR intervention, however, may still be a sensible solution to banks' liquidity problem, as the intervention limits the set of states where *systemic bank runs* can happen, making that set shrink from $[\underline{\theta}_s, \theta_2^*] \times [\underline{\theta}_s, \theta_2^*]$ to $[\underline{\theta}_s, \theta_A^*] \times [\underline{\theta}_s, \theta_A^*]$.[43]

In fact, a trade-off emerges when it comes to the design of the BoLR intervention. When the intervention is designed with unilateral commitment from the BoLR, financial stability improves unambiguously. The downside, however, is that the BoLR will make expected losses. The design of mutual commitment, on the other hand, allows the BoLR to break even in expectation, but only at a cost of less improvement in financial stability.

5 Further policy discussion

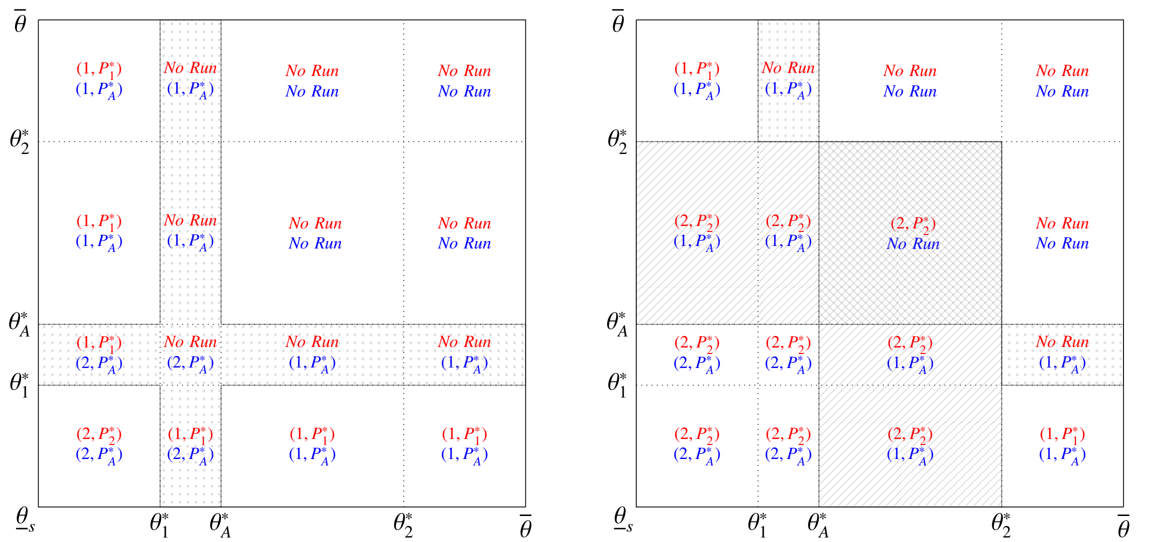
In this section, we extend our policy discussion by comparing the BoLR intervention with traditional LoLR policies. We then go beyond the proposed BoLR policy and consider in

[41]See Reserve Bank Australia (2018, 2019) for details. The need for the Reserve Bank of Australia to provide liquidity insurance is partly driven by the shortage of high quality liquid assets such as domestic sovereign bonds that banks can hold to satisfy the 100% liquidity coverage ratio (LCR) as required by Basel III. The CLP is designed to fill the gap. By the end of 2018, a total amount of AU$ 248 billions of central bank liquidity support was committed through the facility to eligible deposit-taking institutions.

[42]Notice that when creditors' beliefs are pessimistic, there can still be deterioration in allocation for certain fundamentals. This is because the BoLR's break-even price must be smaller than P_1^*, so that $\theta_A^* > \theta_1^*$. Such potential deterioration is indicated by the dotted area in Panel (b) of Figure 5.

[43]In this sense, we agree with Goodhart and Huang (2005) that in designing of ELA policies, the regulator should focus more on the systemic risk than on individual banks.

Figure 5: The potential downside of the BoLR intervention with mutual commitment



(a) Allocation under creditors' pessimistic strategy (b) Allocation under creditors' optimistic strategy

The equilibrium outcomes of the laissez-faire market is indicated in red, while the equilibrium outcomes under the BoLR intervention with mutual commitment is indicated in blue. Panel (a) and (b) depict the equilibrium outcomes under creditors' optimistic and pessimistic strategies respectively. The hatched area indicates regions of fundamentals where the BoLR policy promotes financial stability; and the dot-shaded areas indicate the opposite.

general the design of emergency liquidity assistance (ELA) programs in light of the information constraint. We conclude the section by evaluating the proposed BoLR policy in the presence of frictions other than information constraints.

5.1 BoLR v.s. LoLR

We believe BoLR policies can be more effective than the traditional LoLR policies for two reasons. First, from an operational point of view, a bank that fails because of illiquidity cannot be distinguished from one that fails because of fundamental insolvency, which leaves central banks informationally constrained and makes it hard to follow Bagehot's principles. When practicing BoLR policies, instead of assessing the solvency of each bank, central banks only need to know the distribution of the quality of the assets to be purchased, which can be less informationally demanding. Second, by aiming to lend only to banks that are solvent but illiquid, a LoLR sets a policy objective that is both ambitious and limited. It is ambitious because the policy aims to eliminate inefficient liquidation completely. But at the same, the policy aims only at individual banks and does not pay enough attention to systemic stability. A BoLR, on the other hand, can focus on preventing contagion and systemic crises by maintaining the price stability of systemic bank assets (e.g., sovereign bonds in Europe and mortgage-related assets in the U.S.). To make the contrast clear, we compare BoLR and classic LoLR policies as suggested by Bagehot side by side in Table 1.[44]

Table 1: Comparison between BoLR policies and classic LoLR policies

	Classic Lender of Last Resort (LoLR)	**Buyer of Last Resort (BoLR)**
Direct target	Individual financial institutions	(Systemic) bank assets
Policy channels	Funding liquidity	Market liquidity
Eligible assets	'Good' collateral (usually government bonds)	A wide range of assets
Information required	Info on individual banks' solvency	Valuation of securities to purchase
Policy objective	Avoiding inefficient liquidation of individual banks	Preventing systemic meltdowns

5.2 Information constraints and emergency liquidity assistance

Up to now, we discussed the stability effect of one particular liquidity assistance program that we interpret as a BoLR policy. We now take a broader view and try to answer the question: how should the information constraint affect the design of liquidity assistance programs in general? We do so by examining two dimensions of information constraints: the granularity of available information, and whether the information can be costlessly and credibly communicated to market participants.

Note that in our model, the policy intervention can in principle be based on four information sets, ranked according to the level of granularity: (1) precise information on banks' idiosyncratic cash flows, i.e., the realization of $\boldsymbol{\theta}$, (2) the range of idiosyncratic cash flows as reflected by the number of bank runs, (3) the aggregate state, i.e., the realization of s, and (4) no information on any realized states.[45] Having discussed the BoLR intervention that requires no information on the realized states, we now analyze the other three scenarios in turn.

Let's start with a benchmark case where the information on $\boldsymbol{\theta}$ is available. In this case, it is possible for a central bank to lend directly to the solvent-but-illiquid banks. But such public intervention would not be necessary if the central banks can communicate the information to the market. This is because once the private buyers learn banks' fundamentals, the asset price will be $P = \theta$ in a competitive market and no inefficient liquidation will occur. In other words, the central bank can eliminate market inefficiency by eradicating its root in information friction. A more interesting case arises when the central bank observes θ but cannot perfectly communicate the information to private market participants. As pointed out by Angeletos et al. (2006), if an informed central bank aims to defend a regime (e.g., saving a solvent-but-illiquid

[44]Regarding the timing of intervention, Bagehot (1873) does suggest that a LoLR should make clear in advance her readiness to lend to troubled institutions that fulfill solvency and collateral conditions, but the lending is conditional on the actual occurrence of runs.

[45]We consider the number of runs represents more granular information set than observing s, because based on the number of runs, players can update their beliefs about the aggregate state. But the opposite is not true.

bank) by sending a costly signal of θ, the ex-post intervention itself can create fragility in the form of multiple equilibria. Therefore, a more efficient solution will be lending directly to the solvent-but-illiquid bank.

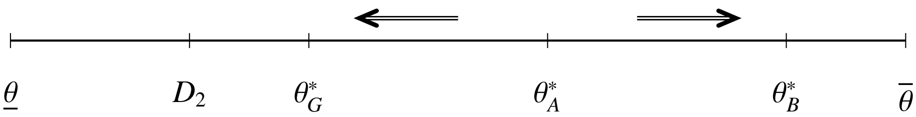
What if the intervention is based on the number of observed bank runs? In this case, the regulator cannot outperform the market—at least not without incurring expected losses. This is because other than the coordination failure, the only friction in our model is the information friction. As private buyers also observe M, the efficiency of the policy intervention will be bounded by the market allocation.

Finally, it can be particularly relevant to assess interventions conditional on the aggregate state s. If s is interpreted as a macro-economic variable, it is not unreasonable to assume that a central bank is better informed than the rest of the economy. If the central bank purchases bank assets after learning s, it will need to condition the price on the aggregate state to avoid expected losses (i.e., offering P_B^* and P_G^* in state B and G respectively). The offered price will generate thresholds θ_B^* and θ_G^*. The ex-post intervention creates the following trade-off. When $s = G$, the policy intervention boosts asset prices and saves banks with $\theta \in (\theta_G^*, \theta_A^*)$ from illiquidity. However, when $s = B$, the policy intervention exacerbates liquidity problems by pushing the critical cash flow all the way up to θ_B^*.[46] We depict the comparison in Figure 6. It should be clear that ex-post intervention conditional on s does not necessarily dominate the ex-ante BoLR intervention with mutual commitment. We summarize the result in Proposition 6.

Proposition 6. *Suppose that the BoLR knows the realized aggregate state $s \in \{G, B\}$. There exists $\overline{\theta}^c$ such that ex-post interventions, like disclosing s or purchasing bank assets for price P_s^*, are dominated by pre-committing to price P_A^* if and only if $\overline{\theta} > \overline{\theta}^c$.*

Proof. See Appendix A.10. □

Figure 6: Intervention conditional on the aggregate state s



The effectiveness of ex-post interventions will be further compromised when the knowledge of s cannot be costlessly communicated to private market participants. Indeed, if the regulator

[46] When the information on s can be perfectly communicated to the private market participants, the intervention is equivalent to disclosing the State s, as private buyers will offer the same price P_s^*, $s \in \{G, B\}$. The disclosure faces the same trade-off: If the state is good, the reassuring disclosure can calm down the market and save banks from illiquidity; but if the state is unfavorable, acknowledging a crisis will result in even more runs by pushing asset prices further down.

must communicate s in a separating equilibrium, efficiency will be lower than the case where the communication is costless, since the regulator will incur the cost of signaling. On the other hand, if a separating equilibrium cannot be sustained and the regulator does not directly purchase banks' assets, the allocation will be the same as in a laissez-faire market.

In sum, we believe that if the main market friction is the information constraint of learning banks' fundamentals $\boldsymbol{\theta}$, a pre-emptive BoLR intervention can be more effective than ex-post interventions. We summarize the policy options for different information sets in Table 2.

Table 2: The design ELA policies in light of information constraints

	Perfect communication	Imperfect communication
θ	Traditional LoLR is feasible. Disclosing θ is equally efficient.	Traditional LoLR is feasible. Disclosing θ can create a 'policy trap'.
M	Interventions cannot to improve over the market allocation.	
s	Purchasing asset for P_s^* in State s. Disclosing s is equally efficient.	Purchasing asset for P_s^* in State s Disclosure in a separating equilibrium: efficiency bounded by the costless disclosure Disclosure in a pooling equilibrium: equivalent to intervention based on M
No Info	BoLR is the only feasible intervention and can be more efficient than ex-post interventions conditional on s	

5.3 The BoLR policy and other market frictions

The key message of our paper is that regulators have to take into account information constraints when designing ELA policies. Admittedly, the information constraints are not the only challenge faced by regulators. In reality, successful policy designs must take into account additional constraints such as the incentive compatibility constraints of market participants (e.g., the moral hazard problem of banks that receive support), budget constraints of regulators, constraints due to limited market participation, and etc. In this section, we discuss briefly the proposed BoLR policy in the light of constraints other than the informational one.

One of the major concerns for ELA policies is that the policy intervention can weaken market discipline and reduce banks' incentives to manage their funding liquidity risk. Indeed, this is the main concern highlighted by papers such as Goodhart (1999), Goodhart and Huang (2005), Repullo (2005), and Freixas et al. (2004). In fact, the moral hazard problem and information constraints are intertwined. It is regulators' inability to distinguish an illiquid bank from an insolvent one that raises the concern that blind intervention will benefit insolvent banks and compromise market discipline. For this reason, a certain degree of moral hazard problem is inevitable given the existence of information constraints.

We believe that compared to traditional LoLR policies as suggested by Bagehot, the BoLR intervention that we propose may suffer less from moral hazard by banks. As the BoLR intervention does not aim to avoid inefficient liquidation of individual banks, it still allows banks with $\theta \in (D_2, \theta_A^*)$ to fail. In other words, those banks are still punished for their mismanagement of risk, which would reduce incentives for risk-taking. By contrast, the traditional LoLR policy that aims to save all banks with $\theta > D_2$ is likely to be associated with greater liquidity support and provides stronger incentives for banks to take on liquidity risks.

Another prominent market friction is limited market participation. As emphasized by Allen and Gale (1994, 2005), the supply and demand of liquidity can be inelastic in the short run, so that small aggregate uncertainty can generate large volatility in asset prices. This calls for ex-post intervention contingent on the aggregate state. Liu (2016) highlights this mechanism in a global-games framework and suggests that central banks should provide liquidity when there is an aggregate shortage. We believe that while the limited market participation can be addressed by ex-post aggregate liquidity injections, information asymmetry can still call for ex-ante intervention such as the BoLR policy.

In sum, our intuition suggests that the proposed BoLR intervention is unlikely to perform poorly or to contradict policies designed for the other frictions

6 Concluding remarks

We have presented a model where falling asset prices and bank runs are mutually reinforcing. In a global-games-based bank run model à la Rochet and Vives (2004), we show that the lack of granular information on individual banks' solvency creates two-way feedback between bank runs and falling asset prices. When creditors anticipate low prices for a bank's assets, they may run on a solvent bank and force the bank into premature liquidation. However, it is the run that pools the solvent bank with the insolvent ones, depresses the bank's asset price, and justifies the creditors' pessimistic expectation. In the presence of aggregate uncertainty, multiple equilibria with financial contagion and volatile asset prices emerge, despite global-games refinements. When asset buyers observe more bank runs, they revise their beliefs about the aggregate state downwards. Their deteriorating beliefs lead to lower asset prices and precipitate runs at more banks, creating a vicious cycle between collapsing asset prices and contagious bank runs.

Our model provides an illustration that market frictions such as information asymmetry can create financial fragility and restrict the set of feasible policy tools at the same time. Without granular information on individual banks' solvency, it is infeasible to target only solvent-but-illiquid banks as suggested by Bagehot's principles. Instead, we show that buyer-of-last-resort policies can break the vicious cycle between falling asset prices and contagious bank runs. When unilaterally providing a price support for banks' assets, a BoLR can unambiguously promote financial stability but will always incur expected losses. To avoid the unpopular solution, we propose a design where the BoLR and banks mutually commit to an agreement for the BoLR to purchase banks' assets when runs happen. This feature of mutual commitment can also be found in the recent developments of emergency liquidity assistance programs at the Bank of England and the Reserve Bank of Australia. We show that BoLR interventions with mutual commitment allow the central bank to reduce the risk of systemic bank runs and to break even in expectation at the same time, even without information on individual banks' solvency.

We also take a broader view to examine the design of emergency liquidity assistance programs in light of information constraints. That is, how effective different ELA programs can be, given the lack of granular information and the challenges for central banks to communicate the information to private market participants. We show that the BoLR intervention in our model is least informationally demanding and yet can outperform alternative programs. In particular, pre-emptive intervention based on a minimum amount of information does not necessarily underperform policies that are conditional on the knowledge of the realized aggregate states.

References

Acharya, V. V., Pierret, D., and Steffen, S. (2017). Lender of last resort versus buyer of last resort: Evidence from the european sovereign debt crisis. Technical report, Social Science Research Network.

Ahnert, T., Anand, K., Gai, P., and Chapman, J. (2018). Asset encumbrance, bank funding and fragility. Technical Report 52, European Systemic Risk Board.

Allen, F. and Gale, D. (1994). Limited market participation and volatility of asset prices. *The American Economic Review*, 84(4):933–955.

Allen, F. and Gale, D. (1998). Optimal financial crises. *The Journal of Finance*, 53(4):1245–1284.

Allen, F. and Gale, D. (2005). From cash-in-the-market pricing to financial fragility. *Journal of the European Economic Association*, 3(2-3):535–546.

Angeletos, G.-M., Hellwig, C., and Pavan, A. (2006). Signaling in a global game: Coordination and policy traps. *Journal of Political Economy*, 114(3):452–484.

Angeletos, G.-M. and Lian, C. (2016). *Incomplete Information in Macroeconomics*, volume 2. Elsevier.

Angeletos, G.-M. and Werning, I. (2006). Crises and prices: Information aggregation, multiplicity, and volatility. *The American Economic Review*, 96(5):1720–1736.

Bagehot, W. (1873). *Lombard Street: A Description of the Money Market*.

Baird Webel (2013). Troubled asset relief program (tarp): Implementation and status. https://fas.org/sgp/crs/misc/R41427.pdf.

Bank of England (2019). Loan collateral: Guidance for participants in the sterling monetary framework.

Bhattacharya, S. and Nyborg, K. G. (2013). Bank Bailout Menus. *Review of Corporate Finance Studies*, 2(1):29–61.

Bolton, P., Santos, T., and Scheinkman, J. A. (2009). Market and public liquidity. *The American Economic Review*, 99(2):594–599.

Bolton, P., Santos, T., and Scheinkman, J. A. (2011). Outside and inside liquidity. *The Quarterly Journal of Economics*, 126(1):259–321.

Brunnermeier, M. K. and Pedersen, L. H. (2009). Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6):2201 –2238.

Carlsson, H. and Van Damme, E. (1993). Global games and equilibrium selection. *Econometrica*, 61:989–1018.

Choi, D. B., Santos, J. A. C., and Yorulmazer, T. (2017). A theory of collateral for the lender of last resort. Technical report, Social Science Research Network, Rochester, NY.

Cornett, M. M., McNutt, J. J., Strahan, P. E., and Tehranian, H. (2011). Liquidity risk management and credit supply in the financial crisis. *Journal of Financial Economics*, 101(2):297–312.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *The Journal of Political Economy*, 91(3):401–419.

Diamond, D. W. and Rajan, R. G. (2011). Fear of fire sales, illiquidity seeking, and credit freezes. *The Quarterly Journal of Economics*, 126(2):557–591.

Eisenbach, T. M. (2017). Rollover risk as market discipline: A two-sided inefficiency. *Journal of Financial Economics*, 126(2):252–269.

Fohlin, C., Gehrig, T., and Haas, M. (2016). Rumors and runs in opaque markets: Evidence from the panic of 1907. Technical report, SSRN.

Freixas, X., Rochet, J.-C., and Parigi, B. M. (2004). The lender of last resort: A twenty-first century approach. *Journal of the European Economic Association*, 2(6):1085–1115.

Goldstein, I. (2005). Strategic Complementarities and the Twin Crises. *The Economic Journal*, 115(503):368–390.

Goldstein, I. and Pauzner, A. (2004). Contagion of self-fulfilling financial crises due to diversification of investment portfolios. *Journal of Economic Theory*, 119(1):151–183.

Goldstein, I. and Pauzner, A. (2005). Demand-deposit contracts and the probability of bank runs. *The Journal of Finance*, 60(3):1293–1327.

Goodhart, C. A. and Huang, H. (2005). The lender of last resort. *Journal of Banking & Finance*, 29(5):1059–1082.

Goodhart, C. A. E. (1999). Myths about the lender of last resort. *International Finance*, 2(3):339–360.

Gromb, D. and Vayanos, D. (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, 66:361–407.

Holmstrom, B. and Tirole, J. (1998). Private and public supply of liquidity. *Journal of Political Economy*, 106(1):1.

King, M. A. (2017). *End of alchemy: money, banking, and the future of the global economy*.

Koulischer, F. and Struyven, D. (2014). Central bank liquidity provision and collateral quality. *Journal of Banking & Finance*, 49:113–130.

Leonello, A. (2018). Government guarantees and the two-way feedback between banking and sovereign debt crises. *Journal of Financial Economics*, 130(3):592–619.

Liu, X. (2016). Interbank market freezes and creditor runs. *The Review of Financial Studies*, 29(7):1860–1910.

Liu, X. and Mello, A. S. (2011). The fragile capital structure of hedge funds and the limits to arbitrage. *Journal of Financial Economics*, 102(3):491–506.

Mehrling, P. (2010). *The New Lombard Street: How the Fed Became the Dealer of Last Resort*. Princeton University Press, Princeton, N.J.

Mehrling, P. (2012). Three principles for market-based credit regulation. *American Economic Review*, 102(3):107–112.

Morris, S. and Shin, H. S. (2000). Rethinking multiple equilibria in macroeconomic modeling. *NBER Macroeconomics Annual*, 15:139–161.

Morris, S. and Shin, H. S. (2004). Liquidity black holes. *Review of Finance*, 8(1):1–18.

Morris, S. and Shin, H. S. (2016). Illiquidity component of credit risk. *International Economic Review*, 57(4):1135–1148.

Ozdenoren, E. and Yuan, K. (2008). Feedback effects and asset prices. *The Journal of Finance*, 63(4):1939–1975.

Repullo, R. (2005). Liquidity, risk taking, and the lender of last resort. *International Journal of Central Banking*.

Reserve Bank Australia (2018). CLF operational notes. https://www.rba.gov.au/mkt-operations/resources/tech-notes/clf-operational-notes.html.

Reserve Bank Australia (2019). Domestic market operations. https://www.rba.gov.au/mkt-operations/dom-mkt-oper.html#tiotb3ls.

Rochet, J.-C. and Vives, X. (2004). Coordination failures and the lender of last resort: Was bagehot right after all? *Journal of the European Economic Association*, 2(6):1116–1147.

Shleifer, A. and Vishny, R. W. (1992). Liquidation values and debt capacity: A market equilibrium approach. *The Journal of Finance*, 47(4):1343–1366.

Tirole, J. (2012). Overcoming adverse selection: How public intervention can restore market functioning. *American Economic Review*, 102(1):29–59.

Vives, X. (2014). Strategic complementarity, fragility, and regulation. *The Review of Financial Studies*, 27(12):3547–3592.

Appendix A Proofs to lemmas and propositions

Appendix A.1 Asset buyers' posterior beliefs

Proof. We derive the functional form of $\omega_M^B(\theta^*)$, where $M \in \{1, 2\}$, for an illustration. Asset buyers evaluate the posterior probability of $s = B$ as $\omega_M^B(\theta^*)$ conditional on their belief that the equilibrium critical cash flow of the bank run game to be θ^* and their observation of M bank runs.

By Bayes' rule, we have

$$\omega_M^B(\theta^*) \equiv Prob\left(s = B \middle| \theta < \theta^*, M\right) = \frac{Prob\,(s = B, \theta < \theta^*, M)}{Prob\,(\theta < \theta^*, M)}$$
$$= \frac{Prob(s = B) \cdot Prob(\theta < \theta^*, M|s = B)}{Prob(s = B) \cdot Prob(\theta < \theta^*, M|s = B) + Prob(s = G) \cdot Prob(\theta < \theta^*, M|s = G)},$$

Here, $Prob(\theta < \theta^*, M|s = B)$ calculates the probability that M banks' cash flows be lower than θ^* and rest $2 - M$ banks' cash flows be higher than θ^*, conditional the distribution of banks' assets to be $U(\underline{\theta}_B, \overline{\theta})$. In particular, $M = 2$ means that both banks' cash flows are lower than θ^*, systemic bank runs occur. We can write $\omega_M^B(\theta^*)$ in details as

$$\omega_M^B(\theta^*) = \frac{(1-\alpha)\left(\frac{\theta^*-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_B}\right)^M \left(\frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}_B}\right)^{2-M}}{(1-\alpha)\left(\frac{\theta^*-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_B}\right)^M \left(\frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}_B}\right)^{2-M} + \alpha\left(\frac{\theta^*-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_G}\right)^M \left(\frac{\overline{\theta}-\theta^*}{\overline{\theta}-\underline{\theta}_G}\right)^{2-M}}$$
$$= \frac{(1-\alpha)\left(\frac{\theta^*-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_B}\right)^M}{(1-\alpha)\left(\frac{\theta^*-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_B}\right)^M + \alpha\left(\frac{\theta^*-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_G}\right)^M \left(\frac{\overline{\theta}-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_G}\right)^{2-M}} = \frac{\left(\theta^*-\underline{\theta}_B\right)^M}{\left(\theta^*-\underline{\theta}_B\right)^M + \kappa\left(\theta^*-\underline{\theta}_G\right)^M}.$$

For simplicity, we define $\kappa \equiv \frac{\alpha}{1-\alpha}\left(\frac{\overline{\theta}-\underline{\theta}_B}{\overline{\theta}-\underline{\theta}_G}\right)^2$ as an exogenous parameter, which does not depend on the endogenous threshold θ^*.

□

Appendix A.2 Proof of Lemma 1

Proof. Since buyers' equilibrium bid cannot be negative, an equilibrium price $\mathbb{P}_M(\theta^*)$, $M \in \{1, 2\}$, if exits, must be in one of the three regions: $[0, \underline{P})$, $[\underline{P}, qD_2)$, or $[qD_2, +\infty)$. We show that it cannot be greater than or equal to qD_2, nor can it be lower than $\underline{P}$.

Suppose $\mathbb{P}_M(\theta^*) \geq qD_2$ for any number of runs $M \in \{1, 2\}$ observed, then it is not sequentially rational for the wholesale creditors to withdraw from a solvent bank, i.e., $\theta > D_2$. To see this, one can take the perspective of a representative creditor j. Even when all other creditors withdraw, the bank needs to liquidate no more than D_1/qD_2 fraction of its asset, for $\mathbb{P}_M(\theta^*) \geq qD_2$. While the bank's $t = 2$ liability drops to F, its residual cash flow is $(1 - \frac{D_1}{\mathbb{P}_M(\theta^*)})\theta \geq (1 - \frac{D_1}{qD_2})D_2 \geq F$ as $\theta \geq D_2$. As a result, by running on the bank, creditor j will only incur a penalty for early withdrawal. This implies that whenever a run happens when $\mathbb{P}_M(\theta^*) \geq qD_2$, the bank must be fundamentally insolvent with $\theta < D_2$. Therefore, buyers must expect asset quality to be lower than $(D_2 + \underline{\theta}_G)/2$, which is in turn lower than qD_2 given our parametric assumption (3). Buyers would make a loss by offering $\mathbb{P}_M(\theta^*) \geq qD_2$, a contradiction.

An equilibrium price $\mathbb{P}_M(\theta^*)$ cannot be smaller than $\underline{P}$ either. Note that when a bank is fundamentally insolvent with cash flow $\theta < D_2$, it is a dominant strategy for its wholesale creditors to run, independently of the asset price. To see this, notice that if $\mathbb{P}_M(\theta^*) \geq D_1$ and the bank does not fail at $t = 1$, a creditor is better off to run and receive D_1 than to wait and receive 0.[47] On the other hand, if $\mathbb{P}_M(\theta^*) < D_1$, a creditor will receive a zero payoff for his claim whether he runs or not, but can still obtain an arbitrarily small reputational benefit by running on a bank that is doomed to fail. This implies that runs must happen to those banks with $\theta < D_2$, and the expected quality of assets on sale is at least $(\underline{\theta}_B + D_2)/2 = \underline{P}$. As asset buyers break even with their competitive bidding, the price they offer must be greater than or equal to $\underline{P}$.

□

Appendix A.3 Proof of Proposition 1

Proof. We characterize the solution to the baseline case without aggregate uncertainty. Inserting (10) into (11), we obtain the following quadratic function, which θ^* must satisfy.

$$(\theta^*)^2 - \left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right]\theta^* - (D_2 - D_1)\,\underline{\theta} = 0, \tag{A.15}$$

The solution(s) of this quadratic function is given by

$$\theta^*_{+,-} = \frac{\left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right] \pm \sqrt{\left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right]^2 + 4\,(D_2 - D_1)\,\underline{\theta}}}{2}$$

[47] Note that asset sale will never increase an insolvent bank's solvency as we just proved that $\mathbb{P}_M(\theta^*) \geq qD_2$ could never happen.

From (10), we also have

$$P^*_{+,-} = \frac{\theta^*_{+,-} + \underline{\theta}}{2} = \frac{\left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right] \pm \sqrt{\left[(D_2 - D_1) + 2qD_1 - \underline{\theta}\right]^2 + 4\left(D_2 - D_1\right)\underline{\theta} + 2\underline{\theta}}}{4} \tag{A.16}$$

P^* and θ^* can be futher rearranged into

$$P^*_{+,-} = \frac{\Psi \pm \sqrt{\Psi^2 - 8qD_1\underline{\theta}}}{4}, \quad \theta^*_{+,-} = \frac{\Psi \pm \sqrt{\Psi^2 - 8qD_1\underline{\theta}} - 2\underline{\theta}}{2}$$

In these expressions, we denote $\Psi \equiv (D_2 - D_1) + 2qD_1 + \underline{\theta}$ for simplicity. In the following proof, we will work with the asset prices $P^*_{+,-}$ first, as the analysis tends to be simpler from this angle.

We check that only the positive root of P^*_+ belongs to the interval $[\underline{P}, qD_2)$, so that it is the unique equilibrium price of our baseline case.

$$P^* = \frac{\Psi + \sqrt{\Psi^2 - 8qD_1\underline{\theta}}}{4} = P^*_+. \tag{A.17}$$

We first verify that both roots monotonically increase in q. For the negative root P^*_-, it can be checked that the cross derivative $\partial^2 P^*_-/(\partial q \partial \underline{\theta}) > 0$, and $\partial P^*_-/\partial q = 0$ when $\underline{\theta} = 0$.[48] Therefore, we obtain that $\partial P^*_-/\partial q > 0$ for all $\underline{\theta} > 0$. Direct calculation shows that P^*_- is smaller than D_1 when $q = 1$. As $D_1 < \underline{P}$, we conclude that P^*_- is less than $\underline{P}$ for all $q \in (0, 1)$ and cannot be part of the equilibrium of our baseline case. Indeed, Lemma 1 restrict the set of the candidate equilibrium price.

For the positive root P^*_+, we can also check that $\partial^2 P^*_+/(\partial q \partial \underline{\theta}) < 0$, and $\partial P^*_+/\partial q > 0$ when $\underline{\theta} = (2q - 1)D_2$. Therefore, we obtain that $\partial P^*_+/\partial q > 0$ for all $\underline{\theta} < (2q - 1)D_2$.[49] Furthermore, one can verify that $P^*_+ = \underline{P}$ when $q = 0$,[50] so that $P^*_+ > \underline{P}$ for any $q > 0$ and $\underline{\theta} < (2q - 1)D_2$. On the other hand, we can directly check that $P^*_+ < qD_2$ when parametric assumption (3) holds. Consequently, only the positive root P^*_+ defined in expression (A.17) can be the equilibrium asset price of our baseline case.

We aslo check that the critical cash flow θ^*_+ associated with the price P^*_+ is the unique equilibrium critical cash flow of our baseline case.

$$\theta^* = \frac{\Psi + \sqrt{\Psi^2 - 8qD_1\underline{\theta}} - 2\underline{\theta}}{2} = \theta^*_+ \tag{A.18}$$

[48] The detailed calculation of the derivatives in this proof can be provided upon request.

[49] Note that this is actually parametric assumption (3) in the baseline case. The assumption degenerates to $q > \frac{1}{2} + \frac{\underline{\theta}}{2D_2}$ as $\underline{\theta}_G = \underline{\theta}_B = \underline{\theta}$.

[50] Note that in this baseline case, $\underline{P} = (\underline{\theta} + D_2)/2$

We need to show this θ^* indeed belongs to the intermediate region $[\theta^L, \theta^U(P^*)]$. To show $\theta^* > \theta^L = D_2$, note that the inequality can be explicitly written as $\sqrt{\Psi^2 - 8qD_1\underline{\theta}} > 2D_2 + 2\underline{\theta} - \Psi$. To see it indeed holds, we can square both sides and rearrange the terms to get $(D_2 + \underline{\theta})\Psi - (D_2 + \underline{\theta})^2 - 2qD_1\underline{\theta} > 0$. By the definition of Ψ, the inequality can be simplified to $2qD_2 - D_2 - \underline{\theta} > 0$, which is guaranteed again by assumption (3).

For the part $\theta^* < \theta^U(P^*)$, we rewrite the equilibrium condition (10) as $\theta^* = 2P^* - \underline{\theta}$. Therefore, $\theta^* < \theta^U(P^*)$ is equivalent to

$$(2P^* - \underline{\theta}) < \frac{F}{1 - D_1/P^*},$$

or equivalently $2(P^*)^2 - (2D_1 + \underline{\theta} + F)P^* + D_1\underline{\theta} < 0$. Using the closed-form solution of P^*, we can rewrite the inequality as follows.

$$(1-q)D_1\left[\frac{1-2q}{q}\frac{\Psi + \sqrt{\Psi^2 - 8qD_1\underline{\theta}}}{4} + \underline{\theta}\right] < 0 \tag{A.19}$$

It can be verified that the term inside the square bracket of (A.19) increases in $\underline{\theta}$, and it equals zero when $\underline{\theta} = (2q-1)D_2$. Use assumption (3), this term must be negative for $0 < \underline{\theta} < (2q-1)D_2$, and this proves $\theta^* < \theta^U(P^*)$.

To conclude, we prove the pair θ^* and P^* characterized by (A.18) and (A.17) is unique and satisfy our equilibrium requirements. There exists a unique PBE $(\theta^*, \boldsymbol{P}^*)$ for this baseline case.

□

Appendix A.4 Proof of Lemma 3

Proof. Suppose all other creditors take a θ^* as the critical threshold of their switching strategy. A creditor j rationally anticipates that M ($M \in \{1, 2\}$) runs will occur after receiving signals of the 2 banks' cash flows. He understands asset buyers' bidding game and expects the equilibrium asset price to satisfy equation (6). His best response to all other player' strategies is then derived in Appendix B. For θ^* to be a symmetric equilibrium of the bank run game, it must satisfy equation (9).

To ease discussion, we transform equation (6) and (9) to the following system of two equations, and let the price P^*_M be the argument being analyzed.

$$P^*_M = \omega^B_M(\theta^*(P^*_M))\frac{\underline{\theta}_B + \theta^*(P^*_M)}{2} + \omega^G_M(\theta^*(P^*_M))\frac{\underline{\theta}_G + \theta^*(P^*_M)}{2}, \quad M \in \{1, 2\} \tag{A.20}$$

and

$$\theta^* = \frac{D_2 - D_1}{1 - qD_1/P_M^*} \equiv \theta^*(P_M^*), \tag{A.21}$$

To prove Lemma 3, it is then equivalent to prove the existence and uniqueness a pair of $\theta^* \in [\theta^L, \theta^U(P_M^*)]$ and $P_M^* \in [\underline{P}, qD_2)$ as the solution of (A.20) and (A.21). We argue it in two steps.

Step 1: We prove there exists a unique $P_M^* \in [\underline{P}, qD_2)$ associated with each number of runs M, satisfying the system of equations (A.20) and (A.21). Combing the two equations, we define asset buyers' expected profit of bidding P_M when observing M runs as

$$\Pi_M(P_M) \equiv \omega_M^B(\theta^*(P_M))\left(\frac{\underline{\theta}_B + \theta^*(P_M)}{2} - P_M\right) + \omega_M^G(\theta^*(P_M))\left(\frac{\underline{\theta}_G + \theta^*(P_M)}{2} - P_M\right). \tag{A.22}$$

Then P_M^* must satisfy the zero-profit condition $\Pi_M(P_M^*) = 0$. As $\omega_M^B(\theta^*(P_M)) + \omega_M^G(\theta^*(P_M)) = 1$, we can further express $\Pi_M(P_M)$ as follows.

$$\Pi_M(P_M) = \frac{\theta^*(P_M) + \underline{\theta}_G}{2} - \frac{\underline{\theta}_G - \underline{\theta}_B}{2}\omega_M^B(\theta^*(P_M)) - P_M$$

Take the first order derivative with respect to P_M, we obtain

$$\frac{d\Pi_M(P_M)}{dP_M} = \frac{1}{2}\frac{d\theta^*(P_M)}{dP_M} - \frac{\underline{\theta}_G - \underline{\theta}_B}{2}\frac{d\omega_M^B(\theta^*(P_M))}{dP_M} - 1. \tag{A.23}$$

From (A.21), it is straightforward to check that $d\theta(P_M)/dP_M < 0$. And it can be verified that

$$\frac{d\omega_M^B(\theta^*(P_M))}{dP_M} = -\frac{d\theta^*(P_M)}{dP_M}\frac{M\cdot\kappa\cdot\left(\theta^*(P_M) - \underline{\theta}_B\right)^{M-1}\cdot\left(\theta^*(P_M) - \underline{\theta}_B\right)^{M-1}\cdot\left(\underline{\theta}_G - \underline{\theta}_B\right)}{\left[\left(\theta^*(P_M) - \underline{\theta}_B\right)^M + \kappa\left(\theta^*(P_M) - \underline{\theta}_G\right)^M\right]^2} > 0.$$

Consequently, $\Pi_M(P_M)$ monotonically decreases in P_M.

The asset market zero-profit condition in each contingency M can be rewritten as follows.

$$\Pi_M(P_M^*) = \omega_M^B(\theta^*(P_M^*))\,\pi(P_M^*|B) + \omega_M^G(\theta^*(P_M^*))\,\pi(P_M^*|G) = 0 \tag{A.24}$$

In this expression, we denote $\pi(P_M|s) \equiv [\underline{\theta}_s + \theta^*(P_M)]/2 - P_M$ as a buyer's profit in State s when her offered price is P_M.

Independently of State s, a buyer makes a profit if $P_M = \underline{P}$. To see so, a sufficient condition for $\pi(\underline{P}|s) > 0$ is $\theta^*(\underline{P}) > D_2$, which is implied by assumption (3).

$$\pi(\underline{P}|s) = \frac{\theta^*(\underline{P}) + \underline{\theta}_s}{2} - \underline{P} = \frac{\theta^*(\underline{P}) + \underline{\theta}_s}{2} - \frac{D_2 + \underline{\theta}_B}{2} > 0$$

It can also be checked that a buyer makes a loss if $P_M = qD_2$. Note that $\theta^*(qD_2) = D_2$ and

$$\pi(qD_2|s) = \frac{\theta^*(qD_2) + \underline{\theta}_s}{2} - D_2 = \frac{D_2 + \underline{\theta}_s}{2} - D_2 < 0.$$

With the posterior beliefs $\omega_M^B(\theta^*(P_M))$ and $\omega_M^G(\theta^*(P_M))$ positive and smaller than one,[51] we have $\Pi_M(\underline{P}) > 0$ and $\Pi_M(qD_2) < 0$. As a result, there exists a $P_M^* \in [\underline{P}, qD_2)$ such that $\Pi_M(P_M^*) = 0$. The uniqueness of such P_M^* is guaranteed by the monotonicity of $\Pi_M(P_M)$, which we just established.

Step 2: The corresponding $\theta^* = \theta^*(P_M^*)$ defined in (A.21) is indeed between θ^L and $\theta^U(P_M^*)$. This step is a direct replication of the second part of Appendix A.3, thus we suppress it for simplicity. The detailed derivation can be provided upon request.

Finally, it should be pointed out that the integer M in (A.20) and (A.21) defines a unique system of equations and a unique pair of θ^* and P_M^* for each M, $M \in \{1, 2\}$. For each number $M \in \{1, 2\}$ of bank runs, there always exists a unique equilibrium price P_M^* that clears the asset market, and there always exists a unique equilibrium critical cash flow of bank runs θ^* when creditors anticipating the number of runs being M and the asset price being P_M^*. To avoid ambiguity, we label this θ^* as θ_M^*, that is, the critical threshold associated with M runs, $M \in \{1, 2\}$.

□

Appendix A.5 Proof of Lemma 4

Proof. In this proof, we borrow some of the notations from Appendix A.4. We will work with the critical cash flow θ as our argument in the following discussion, as the analysis tends to be simpler from this angle. One can easily get $P^*(\theta) = qD_1\theta/[\theta - (D_2 - D_1)]$ as the inverse of $\theta^*(P)$ from (9).

For any value $\theta \in [D_2, \overline{\theta}]$, the proof hinges on the monotonicity of two ratios.

$$\frac{\omega_M^B(\theta)}{\omega_M^G(\theta)} = \frac{(\theta - \underline{\theta}_B)^M}{\kappa(\theta - \underline{\theta}_G)^M} \qquad \text{and} \qquad \frac{\pi(\theta|G)}{\pi(\theta|B)} = \frac{(\underline{\theta}_G + \theta)/2 - P^*(\theta)}{(\underline{\theta}_B + \theta)/2 - P^*(\theta)}$$

where $M \in \{1, 2\}$. The former one is a ratio of posterior beliefs about state when observing M runs, and the latter one is a ratio of buyers' profit across two states. Note that here we reformulate buyers' profit in State s as $\pi(\theta|s) \equiv (\underline{\theta}_s + \theta)/2 - P^*(\theta)$.

[51] This result is guaranteed by $\theta(P_M) > D_2 > \underline{\theta}_G > \underline{\theta}_B$, which can be calculated directly.

As an inverse function of $\theta^*(P)$, It is obvious that $\partial P^*(\theta)/\partial\theta < 0$. Then, both ratios strictly monotonically decrease in θ when $\theta > D_2 > \underline{\theta}_s$ as

$$\frac{d}{d\theta}\left(\frac{\omega_M^B(\theta)}{\omega_M^G(\theta)}\right) = -\frac{1}{\kappa}\cdot\frac{M\cdot(\theta-\underline{\theta}_B)^{M-1}\cdot(\underline{\theta}_G-\underline{\theta}_B)}{\left(\theta-\underline{\theta}_G\right)^{M+1}} < 0$$

$$\frac{d}{d\theta}\left(\frac{\pi(\theta|G)}{\pi(\theta|B)}\right) = -\frac{[1/2-\partial P^*(\theta)/\partial\theta]\,(\underline{\theta}_G-\underline{\theta}_B)}{2\left[\frac{\theta+\underline{\theta}_B}{2}-P^*(\theta)\right]^2} < 0$$

Furthermore, notice that $(\theta-\underline{\theta}_B)/(\theta-\underline{\theta}_G) > 1$ for $\theta > D_2$, therefore

$$\kappa\cdot\frac{\omega_1^B(\theta)}{\omega_1^G(\theta)} = \left(\frac{\theta-\underline{\theta}_B}{\theta-\underline{\theta}_G}\right) < \left(\frac{\theta-\underline{\theta}_B}{\theta-\underline{\theta}_G}\right)^2 = \kappa\cdot\frac{\omega_2^B(\theta)}{\omega_2^G(\theta)}. \tag{A.25}$$

As a result, for any value $\theta > D_2$, $\frac{\omega_1^B(\theta)}{\omega_1^G(\theta)} < \frac{\omega_2^B(\theta)}{\omega_2^G(\theta)}$ holds.

With the above preliminary result established, we then prove our main result by contradiction. Suppose the equilibrium critical cash flows are such that $\theta_1^* = \theta^*(P_1^*) \geq \theta_2^* = \theta^*(P_2^*)$.[52] By the monotonicity of $\pi(\theta|G)/\pi(\theta|B)$, we have

$$\frac{\pi(\theta_1^*|G)}{\pi(\theta_1^*|B)} \leq \frac{\pi(\theta_2^*|G)}{\pi(\theta_2^*|B)}. \tag{A.26}$$

By the equilibrium condition (A.24) from Appendix A.4, we have

$$\frac{\pi(\theta_1^*|G)}{\pi(\theta_1^*|B)} = -\frac{\omega_1^B(\theta_1^*)}{\omega_1^G(\theta_1^*)} \quad \text{and} \quad \frac{\pi(\theta_2^*|G)}{\pi(\theta_2^*|B)} = -\frac{\omega_2^B(\theta_2^*)}{\omega_2^G(\theta_2^*)}.$$

Together with (A.26), the two equations above imply that

$$\frac{\omega_2^B(\theta_2^*)}{\omega_2^G(\theta_2^*)} \leq \frac{\omega_1^B(\theta_1^*)}{\omega_1^G(\theta_1^*)}.$$

By inequality (A.25), we further have

$$\frac{\omega_2^B(\theta_2^*)}{\omega_2^G(\theta_2^*)} \leq \frac{\omega_1^B(\theta_1^*)}{\omega_1^G(\theta_1^*)} < \frac{\omega_2^B(\theta_1^*)}{\omega_2^G(\theta_1^*)}.$$

Then by the monotonicity of $\omega_2^B(\theta)/\omega_2^G(\theta)$, we must have $\theta_2^* > \theta_1^*$, a contraction.

Therefore, $\theta_2^* > \theta_1^*$ must hold. It follows directly $P_2^* < P_1^*$ from the monotonicity of $P^*(\theta)$.

□

[52] Note that we prove in Lemma 3, those critical cash flows exist and are unique in $[D_2, \theta^U] \subset [D_2, \overline{\theta}]$.

Appendix A.6 Proof of Lemma 5

Proof. When $s = G$ or B with probability 1, then the secondary market equilibrium is characterized by

$$P_s^* = \frac{\underline{\theta}_s + \theta_s^*}{2}$$

Rationally anticipating the equilibrium price, the bank run equilibrium solves

$$\theta_s^* = \frac{D_2 - D_1}{1 - qD_1/P_s^*}$$

where $s = G$ (or B) in the above equations. Solving this system of equations, we obtain the equilibrium critical cash flow θ_s^* and the equilibrium asset price P_s^*, where $s = G$ or $s = B$.

We can directly follow the proof of Appendix A.3 to show $P_s^* \in [\underline{P}, qD_2)$ and $\theta_s^* \in [\theta^L, \theta^U(P_s^*)]$ for both $s = G$ and B. The corresponding PBE when $s = G$ or B with certainty is as being characterzed in Lemma 5.

Moreover, we can also follow directly the proof of Appendix A.5 to show $\theta_G^* < \theta_1^* < \theta_2^* < \theta_B^*$ as the ratio of posterior beliefs about state satisfying the following order.

$$1 > \frac{\omega_2^B(\theta)}{\omega_2^G(\theta)} > \frac{\omega_1^B(\theta)}{\omega_1^G(\theta)} > 0$$

for any $\theta \in [D_2, \overline{\theta}]$. Note that such ratio is 1 (0) when state equals to B (G) with certainty.

□

Appendix A.7 Proof of Proposition 3

Proof. The proof for the case where $P_U \in (P_2^*, P_1^*)$ has been discussed in the main text. We prove, in this appendix, for the case where $P_U \geq P_1^*$.

Suppose $P_U \in [P_1^*, qD_2)$, wholesale creditors rationally anticipate that banks will always sell assets to the BoLR upon runs. By Lemma 1, a global-games based bank run game of creditors is also well defined when banks' asset price belongs to this region.[53] The equilibrium critical cash flow of the bank run game is then $\theta^*(P_U) = (D_2 - D_1)/(1 - qD_1/P_U)$. As $qD_2 > P_U \geq P_1^*$, we have $D_2 < \theta^*(P_U) \leq \theta_1^* < \theta_2^*$ by the monotonicity of $\theta^*(P)$. Compared to the market equilibrium outcome in Corollary 1, the range of bank fundamentals where bank runs can happen shrinks from $[\underline{\theta}_s, \theta_2^*]$ to $[\underline{\theta}_s, \theta^*(P_U)]$. The BoLR then limits the range of systemic bank runs by offering $P_U \in [P_1^*, qD_2)$.

[53] If $P_U = P_1^*$, we make an innocuous assumption that a bank will sell its assets to the BoLR.

The BoLR will make expected losses by committing to such a price. Since $P_U \geq P_1^*$, a bank will always sell its assets to the BoLR upon a run. Recall our analysis in Section 3, the price that allows the asset buyers to break even when M bank runs occur must be P_M^*, where $M \in \{1, 2\}$. We established in Appendix A.4 that an asset buyer incurs expected losses when bidding a price $P > P_M^*$ in the contingency of M runs. By offering a price P_U that is greater than any market equilibrium bid P_M^*, the BoLR, now as the only asset buyer, will always make losses ex-post for any realized number of runs. She will also take losses from an ex-ante perspective.

Suppose $P_U \geq qD_2$, wholesale creditors still rationally anticipate that banks will always sell assets to the BoLR upon runs. The asset price is now outside the region $[\underline{P}, qD_2)$ defined by Lemma 1, we cannot directly apply the equilibrium outcomes of global game as in the previous case. We have to analyze the creditors' bank run game for $P_U \geq qD_2$.

We start with the complete information bank run game. Note first, observing such a P_U, it is a dominant strategy for wholesale creditors to 'wait' in a bank with a cash flow $\theta > D_2$. Indeed, even if all wholesale creditors other than a creditor j choose to 'withdraw' at $t = 1$, such a bank can still fully repay its retail deposits F at $t = 2$.

$$\left(1 - \frac{D_1}{P_U}\right)\theta \geq \left(1 - \frac{D_1}{qD_2}\right)D_2 = F.$$

Therefore, the representative creditor j will optimally choose to 'wait' and obtain D_2 at $t = 2$. On the other hand, if all creditors other than the creditor j choose to 'wait' till $t = 2$, the creditor j will also 'wait' as the bank is fundamentally solvent. So the range of fundamentals where bank runs can happen further shrinks from θ_2^* to a value that is below or equal to D_2.

Second, it is also a dominant strategy for creditors to choose to 'withdraw' from a bank with a cash flow $\theta < F/(1 - D_1/P_U)$. If all creditors other than a creditor j choose to 'wait' to $t = 2$, the bank has a residual value that is less than D_2. So the creditor j will optimally choose to 'withdraw' at $t = 1$ to obtain D_1 instead of zero at $t = 2$. On the other hand, if all creditors other than a creditor j choose to 'withdraw' at $t = 1$, the residual value of the bank's portfolio becomes

$$\left(1 - \frac{D_1}{P_U}\right)\theta < \left(1 - \frac{D_1}{P_U}\right)\frac{F}{1 - D_1/P_U} = F.$$

As a result, the creditor j should still choose to 'withdraw' at $t = 1$ to obtain D_1 instead of zero by waiting to $t = 2$. Note that the region $[\underline{\theta}_s, F/(1 - D_1/P_U)]$ may not exist if the BoLR offers a price that is too high.[54] For instance, if P_U approaches to infinity, $F/(1 - D_1/P_U)$ approaches to F, a value that is not necessarily larger than $\underline{\theta}_s$. To avoid this additional complication, we restrict that P_U belongs to the region $[qD_2, D_2\underline{\theta}_s/(\underline{\theta}_s - F))$ if $F < \underline{\theta}_s$.

[54] One can check that $F/(1 - D_1/P_U)$ approaches to D_2 from the left as P_U approaches to qD_2 from the right. So this region must exist for some P_U by Assumption (1).

Third, when a bank's cash flow belongs to the intermediate region $[F/(1-D_1/P_U), D_2]$, creditors' actions exhibit strategic substitution. It is known that when information is complete, a game of strategic substitution has a unique equilibrium.[55] Such an equilibrium must involve bank runs and corresponding asset sales to the BoLR. Indeed, if all other creditors choose to 'wait', a bank will become bankruptcy at $t = 2$ as its cash flow is less than D_2, and wholesale creditors will obtain 0. A representative creditor j would be better off by choosing to 'withdraw' instead of 'wait'. All creditors choose to 'wait' must not be an equilibrium in the complete information setup.

When introducing noisy private signals about a bank's fundamentals to the creditors, we have a global game with strategic substitution instead of strategic complementarity. The creditors, again, receive sufficiently accurate signals, so that the fundamental uncertainty disappears. The creditors are almost sure about which region the bank's fundamentals lie in. So it is still a creditors' dominant strategy to 'withdraw' in the region $[\underline{\theta}_s, F/(1-D_1/P_U)]$, and a dominant strategy for them to 'wait' in region $[D_2, \overline{\theta}]$. For the intermediate range $[F/(1-D_1/P_U), D_2]$, the strategic uncertainty still exists. A threshold equilibrium could still exist depending on the creditor's payoff function and the precision of signals.[56]

Given the above analysis, the benefit of the BoLR intervention when the BoLR's offered price $P_U \geq qD_2$ can be seen easily: the range of bank fundamentals where systemic runs can happen is reduced, with the critical cash flow below or equal to D_2. To see the BoLR makes expected losses in this case. Note that with a price support $P_U \geq qD_2$, a bank with a fundamental $\theta \in [\underline{\theta}_s, F/(1-D_1/P_U)] \subset [\underline{\theta}_s, D_2]$ will sell its assets to the BoLR and it will never sell its assets if θ is, instead, higher than D_2 . This implies that the BoLR's expected payoff must be strictly lower than

$$\frac{D_2 + E\left(\underline{\theta}_s\right)}{2} - P_U \leq \frac{D_2 + \underline{\theta}_G}{2} - qD_2 < 0.$$

The last inequality follows from Assumption (3). Therefore, we have established that the BoLR cannot expect to break even by offering a price $P_U \geq qD_2$.

□

Appendix A.8 Proof of Lemma 6

Proof. With the binding agreement between the BoLR and the banks, banks are obliged to sell their assets to the BoLR upon runs. The proof that the BoLR will incur expected losses by

[55] For a reference in the recent literature, see Angeletos and Lian (2016).

[56] One could refer to Morris and Shin (2005) for detailed discussion.

offering $P_A \geq P_1^*$ follows exactly Appendix A.7. On the other hand, note that the BoLR cannot improve the market equilibrium outcomes by offering a P_A that is lower than or equal to P_2^*, the lowest possible market equilibrium asset price.

□

Appendix A.9 Proof of Proposition 4

Proof. We first establish the existence of a price $P_A^* \in (P_2^*, P_1^*)$ that makes the BoLR break even in expectation in this mutual commitment case. Such a price P_A^*, if exists, must solve the system of equations consisting of (14) and (12),

$$V(P_A^*) = \sum_{M=1}^{2} \left(\sum_{s=G,B} Pr(s) \cdot \Lambda_M^s(P_A^*) \right) M \cdot C_2^M \cdot \Pi_M(P_A^*) = 0$$

$$\theta^*(P_A^*) = \frac{D_2 - D_1}{1 - qD_1/P_A^*}.$$

Note that $M \in \{1,2\}$ and $\Lambda_M^s(P_A^*) \equiv Pr(\theta < \theta^*(P_A^*)|s)^M \cdot Pr(\theta > \theta^*(P_A^*)|s)^{2-M}$. Recall that we established in Appendix A.4 that $d\Pi_M(P)/dP < 0$, $\forall M \in \{1,2\}$. An asset buyer's expected payoff strictly decreases in his bid P when M bank runs are observed. We also established that $P_1^* > P_2^*$ in Appendix A.5. An asset buyer bids less in equilibrium when observing more runs. Combine the two results, we obtain $\Pi_2(P_1^*) < 0$ and $\Pi_1(P_2^*) > 0$. Further notice that $\Lambda_M^s(P_A) \in (0,1)$ when $P_A \in (P_2^*, P_1^*) \subset [\underline{P}, qD_2)$. So we have $sgn\,(V(P_A)) = sgn\,(\Pi_M(P_A))$. Consequently, we have $V(P_1^*) < 0$ and $V(P_N^*) > 0$. It can be seen that $\Pi_M(P)$ is continuous in P when $P \in [\underline{P}, qD_2)$. So $V(P_A)$ is also continuous in P_A when $P_A \in (P_2^*, P_1^*)$. By the continuity, there exists a $P_A^* \in (P_2^*, P_1^*)$ such that $V(P_A^*) = 0$.

To establish the uniqueness of P_A^*, we can rewrite $V(P_A)$ as the following parsimonious form .

$$V(P_A) = \sum_{s=G,B} Pr(s) \cdot \left(\frac{\theta^*(P_A) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \cdot 2 \cdot \pi(P_A|s)$$

where $\pi(P_A|s) = \left[\underline{\theta}_s + \theta^*(P_A)\right]/2 - P_A$. We have

$$\frac{dV(P_A)}{dP_A} = \sum_{s=G,B} Pr(s) \cdot 2 \cdot \frac{1}{\bar{\theta} - \underline{\theta}_s} \frac{d}{P_A} \left[\left(\theta^*(P_A) - \underline{\theta}_s\right) \cdot \left(\frac{\underline{\theta}_s + \theta^*(P_A)}{2} - P_A \right) \right],$$

$\forall P_A \in (P_2^*, P_1^*)$. For the term inside the bracket, we have

$$\frac{d}{P_A} \left[\left(\theta^*(P_A) - \underline{\theta}_s\right) \cdot \left(\frac{\underline{\theta}_s + \theta^*(P_A)}{2} - P_A \right) \right] = \frac{d\theta^*(P_A)}{dP_A} \left(\theta^*(P_A) - P_A\right) - \left(\theta^*(P_A) - \underline{\theta}_s\right) < 0,$$

$\forall P_A \in (P_2^*, P_1^*)$. The inequality holds because $\theta^*(P_A) = (D_2 - D_1)/(1 - qD_1/P_A) > P_A$, $\forall P_A \in (P_2^*, P_1^*) \subset [\underline{P}, qD_2)$.

To conclude, there exists a unique $P_A^* \in (P_2^*, P_1^*)$ such that the BoLR breaks even in expectation by offering this price support to the banks.

□

Appendix A.10 Proof of Proposition 6

Proof. We derive a condition to guarantee the expected social costs under the mutual agreement between the BoLR and the banks is lower than that under regulatory disclosure policy. To proceed, we assume regulatory disclosure takes the following form: when s realizes, the regulator observe it and perfectly communicates this information to the players. Accordingly, the equilibrium under regulatory disclosure is the same as the one characterized in Appendix A.6. Banks sell their assets to ex-post asset buyers for a price P_s^* and run occurs when a bank's fundamental is less than θ_s^* depends on the disclosure to be $s = G$ or B. Recall that we must have $\theta_G^* < \theta_1^* < \theta_A^* < \theta_2^* < \theta_B^*$.

In our model, when a successful bank run occurs, the bank fails at $t = 2$ because its remaining asset value is insufficient to repay F. In case of failure, a bankruptcy cost C is incurred. We denote the expected social losses under the BoLR's intervention as SC_{BoLR} and under the regulatory disclosure as SC_{RD}. Those costs can be calculated as

$$SC_{BoLR} = 2\left[\alpha\frac{\theta_A^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} + (1-\alpha)\frac{\theta_A^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right]C, \quad SC_{RD} = 2\left[\alpha\frac{\theta_G^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} + (1-\alpha)\frac{\theta_B^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right]C$$

Lastly, we derive a sufficent condition under which $SC_{BoLR} < SC_{RD}$.

$$2\left[\alpha\frac{\theta_A^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} + (1-\alpha)\frac{\theta_A^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right]C < 2\left[\alpha\frac{\theta_G^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} + (1-\alpha)\frac{\theta_B^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right]C$$

This inequality can be further rearranged into

$$\frac{\underline{\theta}_G - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_G} < \frac{\alpha\theta_G^* + (1-\alpha)\theta_B^* - \theta_A^*}{\alpha(\theta_A^* - \theta_G^*)} \tag{A.27}$$

As a result, from (A.27), we can find a $\overline{\theta}^c$ such that when

$$\overline{\theta} > \underline{\theta}_G + \frac{\alpha(\theta_A^* - \theta_G^*)}{\alpha\theta_G^* + (1-\alpha)\theta_B^* - \theta_A^*}(\underline{\theta}_G - \underline{\theta}_B) = \overline{\theta}^c, \tag{A.28}$$

the expected social costs under the BoLR intevention is lower. □

Appendix B Equilibrium condition of the bank run game

In this appendix, we show that a representative creditor j's best response to other creditors' symmetric threshold strategy 'to withdraw if the private signal lower than x^*' is also a threshold strategy. We establish a condition that a symmetric equilibrium must satisfy. Given creditors' belief about the asset price, this symmetric equilibrium is the only one that survives iterated elimination of dominated strategies. In the analysis, we integrate forward-looking asset prices into a standard bank run game with incomplete information.[57] Appendix A.1-A.3 elaborate the three steps outlined in Section 3.2.

Appendix B.1 Lower and upper dominance regions

First, denoting the lower dominance region by $[\underline{\theta}, \theta^L)$, we prove its existence by construction. By definition, when a bank's cash flow belongs to this region, a representative creditor j would be better off to withdraw even if all other creditors wait. This is the case if and only if inequality (7) holds for $L = 0$ so that $\theta < F + (1 - E - F)r_D = D_2$. In other words, the bank is fundamentally insolvent and will fail at $t = 2$ even if no premature liquidation takes place at $t = 1$. In this case, a creditor, if chooses to wait, will receive a zero payoff because of the bank failure, but will receive D_1 if he withdraws early.[58] Therefore, if creditor j is sure that $\theta < D_2$ given his signal, his best action is to withdraw, independently of his belief about the other creditors' actions. As a result, we establish a dominance region $[\underline{\theta}, \theta^L)$ with $\theta^L = D_2$.

Second, denoting $\theta^U(\mathbb{P}_M(\theta^*)) \equiv F/(1 - D_1/\mathbb{P}_M(\theta^*))$, we show that for an expected asset price $\mathbb{P}_M(\theta^*)$, $M \in \{1, 2\}$, the upper dominance region is $(\theta^U(\mathbb{P}_M(\theta^*)), \overline{\theta}]$. Suppose all other creditors withdraw early (i.e., $L = 1$). The bank will survive if and only if its cash flow $\theta \geq \theta^U(\mathbb{P}_M(\theta^*))$. To see that it is creditor j's dominant strategy to wait, notice that creditor j will receive D_1 if he withdraws, and D_1/q if he waits. Again, when the creditor's signal is accurate, he will be sure that the bank's cash flow is in the upper dominance region, and will choose to wait independently of his belief about other creditors' actions. Note that $\theta^U(\mathbb{P}_M(\theta^*))$ decreases in $\mathbb{P}_M(\theta^*)$, and it has been established in Lemma 1 that the asset price cannot be lower than $\underline{P}$. Therefore, $\theta^U(\mathbb{P}_M(\theta^*))$ has an upper bound $F/(1 - D_1/\underline{P})$. Provided that $\overline{\theta}$ is sufficiently high, $\overline{\theta} > F/(1 - D_1/\underline{P})$, the upper dominance region exists. Different from standard global-games models, θ^U is a function of $\mathbb{P}_M(\theta^*)$, reflecting creditors' rational expectation of asset price.

[57] Our assumption that the noise of private signals diminishes ($\epsilon \to 0$) also guarantees the creditors to anticipate the same equilibrium outcome, in particular, the same number of runs M and the corresponding asset price $\mathbb{P}_M(\theta^*)$.

[58] Recall Lemma 1 that banks will not fail at $t = 1$ as an equilibrium asset price must satisfy $\mathbb{P}_M(\theta^*) > \underline{P} > D_1$.

Appendix B.2 Creditors' beliefs outside the dominance regions

When a bank's realized cash flow is in the intermediate region $[\theta^L, \theta^U(\mathbb{P}_M(\theta^*))]$, creditor j's optimal action depends on his beliefs about other creditors' actions. In this subsection, we characterize such beliefs.

Note first that the fraction of creditors who withdraw early is a function of a bank's fundamental θ and the threshold signal x^*. We denote this fraction by $L = L(\theta, x^*)$ and determine the functional form of $L(\theta, x^*)$. For a realized θ, we have three cases. (1) When $\theta + \epsilon < x^*$, even the highest possible signal is below the threshold x^*. By the definition of the threshold strategy, all creditors will withdraw and $L(\theta, x^*) = 1$. (2) When $\theta - \epsilon > x^*$, even the lowest possible signal exceeds the threshold x^*. All creditors will wait and $L(\theta, x^*) = 0$. (3) When θ falls into the intermediate range $[x^* - \epsilon, x^* + \epsilon]$, the fraction of creditors who withdraw at $t = 1$ is as follows, where x_k denotes the private signal of an arbitrary creditor k other than creditor j.

$$L(\theta, x^*) = Prob\left(x_k < x^* \middle| \theta\right) = Prob\left(\epsilon_k < x^* - \theta \middle| \theta\right) = \frac{x^* - \theta - (-\epsilon)}{2\epsilon} = \frac{x^* - \theta + \epsilon}{2\epsilon} \tag{B.29}$$

$L(\theta, x^*) \in (0, 1)$ would look uncertain from the perspective of creditor j, as the creditor only receives a noisy signal x_j and perceives θ with uncertainty. In particular, creditor j has a posterior belief $\theta \sim U(x_j - \epsilon, x_j + \epsilon)$ conditional his private signal x_j. Depends on the value of x_j, we have five cases.

Case (1) $x_j > x^* + 2\epsilon$: In this case, creditor j is certain that all other creditors must have received signals higher than x^*. As all other creditors choose to wait, creditor j has a posterior belief $Prob(L(\theta, x^*) = 0 | x_j) = 1$ when observing $x_j > x^* + 2\epsilon$.

Case (2) $x^* < x_j \leq x^* + 2\epsilon$: Recall that $\theta \sim U(x_j - \epsilon, x_j + \epsilon)$ conditional x_j. As $x_j > x^*$, it follows $x_j + \epsilon > x^* + \epsilon$, so that we can divide the support of θ into two intervals: $[x_j - \epsilon, x^* + \epsilon]$ and $(x^* + \epsilon, x_j + \epsilon]$. When θ lies in the second interval, all other creditors receive signals higher than x^* and chosen to wait, so that $L = 0$. Creditor j, therefore, assigns the event $L(\theta, x^*) = 0$ with the following posterior probability:

$$Prob\left(L(\theta, x^*) = 0 \middle| x_j\right) = Prob\left(x^* + \epsilon < \theta < x_j + \epsilon \middle| x_j\right) = \frac{x_j + \epsilon - (x^* + \epsilon)}{(x_j + \epsilon) - (x_j - \epsilon)} = \frac{x_j - x^*}{2\epsilon} \in (0, 1]$$

On the other hand, the first interval $[x_j - \epsilon, x^* + \epsilon]$ is a subset of $[x^* - \epsilon, x^* + \epsilon]$ so that $L(\theta, x^*)$ is given by expression (B.29). Creditor j's posterior belief of $L(\theta, x^*)$ can be calculated as

$$Prob\left(L(\theta, x^*) \leq \hat{L} \middle| x_j\right) = Prob\left(\frac{x^* - \theta + \epsilon}{2\epsilon} \leq \hat{L} \middle| x_j\right) = Prob\left(\theta \geq x^* + \epsilon - 2\epsilon\hat{L} \middle| x_j\right),$$

where $\hat{L} \in (0, 1)$. Since $\theta \sim U(x_j - \epsilon, x_j + \epsilon)$ conditional on x_j, we know that θ is still uniformly distributed on $[x_j - \epsilon, x^* + \epsilon]$. And the probability above can be written explicitly as

$$Prob\left(L(\theta, x^*) \leq \hat{L} \middle| x_j\right) = \frac{(x^* + \epsilon) - (x^* + \epsilon - 2\epsilon\hat{L})}{(x^* + \epsilon) - (x_j - \epsilon)} = \frac{2\epsilon\hat{L}}{2\epsilon - (x_j - x^*)} = \frac{\hat{L}}{1 - (x_j - x^*)/2\epsilon}.$$

Therefore, for a signal $x_j \in (x^*, x^* + 2\epsilon]$, creditor j perceives $L(\theta, x^*)$ having a mixed distribution, with a positive probability mass $(x_j - x^*)/2\epsilon$ at $L = 0$ and being uniformly distributed on $(0, 1 - (x_j - x^*)/2\epsilon]$ with density 1.

Case (3) $x_j = x^*$: Creditor j still perceive $L(\theta, x^*)$ as given by expression (B.29) as $\theta \sim U(x^* - \epsilon, x^* + \epsilon)$ conditional on $x_j = x^*$. Creditor j calculates the posterior distribution of $L(\theta, x^*)$ as follows.

$$Prob\left(L(\theta, x^*) \leq \hat{L} \middle| x_j = x^*\right) = Prob\left(\frac{x^* - \theta + \epsilon}{2\epsilon} \leq \hat{L} \middle| x_j = x^*\right)$$
$$= Prob\left(\theta \geq x^* + \epsilon - 2\epsilon\hat{L} \middle| x_j = x^*\right) = \frac{(x^* + \epsilon) - (x^* + \epsilon - 2\epsilon\hat{L})}{(x^* + \epsilon) - (x^* - \epsilon)} = \hat{L}.$$

Therefore, creditor j holds a posterior belief that $L(\theta, x^*) \sim U(0, 1)$, when observing $x_j = x^*$.

Case (4) $x^* - 2\epsilon \leq x_j < x^*$: This case can be analyzed in the same way as Case (2). When observing a signal $x_j \in [x^* - 2\epsilon, x^*)$, creditor j perceives $L(\theta, x^*)$ having a mixed distribution, with a positive probability mass $(x^* - x_j)/2\epsilon$ at $L = 1$ and being uniformly distributed on $[(x^* - x_j)/2\epsilon, 1)$ with density 1.

Case (5) $x_j < x^* - 2\epsilon$: Similar to Case (1), when observing a signal $x_j < x^* - 2\epsilon$, creditor j is certain that all other creditors must have received signals lower than x^*, and therefore, has a posterior belief $Prob(L(\theta, x^*) = 1 | x_j) = 1$.

It worth noticing that creditor j becomes more pessimistic about the proportion of early withdrawals when observing a lower signal. That is, the distribution of $L(\theta, x^*)$ associated with a lower x_j first-order-stochastic dominates one associated with a higher x_j.

Appendix B.3 Threshold equilibrium of the bank run game

Creditor j's expected payoff difference between action 'wait' and 'withdraw' conditional on his signal x_j dictates his action. We denote this expected payoff difference by $E[DW(L)|x_j]$ and calculate it explicitly using the posterior distribution of $L(\theta, x^*)$.

By the definition of θ^*, the following equality must hold.

$$\left(1 - \frac{LD_1}{\mathbb{P}_M(\theta^*)}\right)\theta^* = F + (1 - L)(1 - E - F)r_D \tag{B.30}$$

That is, a bank is on the verge of failure if its fundamental equals θ^*. The critical fundamental θ^* then implies a critical run proportion $L^c(\theta^*, M)$.

$$L^c(\theta^*, M) = \frac{\mathbb{P}_M(\theta^*)(\theta^* - D_2)}{D_1\left[\theta^* - \mathbb{P}_M(\theta^*)/q\right]} \tag{B.31}$$

As we know $\mathbb{P}_M(\theta^*) \in [\underline{P}, qD_2)$ and focus on $\theta \in [\theta^L, \theta^U]$, it holds that $L^c(\theta^*, M) \in (0, 1)$.

In Case (1) $x_j > x^* + 2\epsilon$: Creditor j perceives $L(\theta, x^*) = 0$ with probability 1. As $DW(L) = (1-q)D_1/q$ when $L = 0$, we have

$$E[DW(L)|x_j] = (1-q)\frac{D_1}{q}, \text{ for } x_j > x^* + 2\epsilon.$$

In Case (2) $x^* < x_j \leq x^* + 2\epsilon$: Creditor j believes L has a mixed distribution. Notice that $1-(x_j - x^*)/2\epsilon$ decreases in x_j, with $1-(x_j - x^*)/2\epsilon = 1$ when $x_j = x^*$ and $1-(x_j - x^*)/2\epsilon = 0$ when $x_j = x^* + 2\epsilon$. So there exists a $\overline{x} \in (x^*, x^* + 2\epsilon]$, such that $L^c = 1 - (x_j - x^*)/2\epsilon$. When $x_j \in (\overline{x}, x^* + 2\epsilon]$, we have $1 - (x_j - x^*)/2\epsilon < L^c$ and the expected payoff difference

$$E[DW(L)|x_j] = \frac{x_j - x^*}{2\epsilon}(1-q)\frac{D_1}{q} + \int_0^{1-\frac{x_j - x^*}{2\epsilon}} (1-q)\frac{D_1}{q}dL = (1-q)\frac{D_1}{q}.$$

When $x_j \in (x^*, \overline{x}]$, we have $1 - (x_j - x^*)/2\epsilon > L^c$ and the expected payoff difference

$$\begin{aligned} E[DW(L)|x_j] &= \frac{x_j - x^*}{2\epsilon}(1-q)\frac{D_1}{q} + \int_0^{L^c(\theta^*,M)} (1-q)\frac{D_1}{q}dL + \int_{L^c(\theta^*,M)}^{1-\frac{x_j - x^*}{2\epsilon}} (-D_1)dL \\ &= \frac{D_1}{q} \cdot [L^c(\theta^*, M) - q] + \frac{D_1}{q}\frac{x_j - x^*}{2\epsilon}. \end{aligned}$$

In Case (3) $x_j = x^*$: Creditor j believes $L \sim U(0, 1)$. We have

$$E[DW(L)|x_j] = \int_0^{L^c(\theta^*,M)} (1-q)\frac{D_1}{q}dL + \int_{L^c(\theta^*,M)}^1 (-D_1)dL = \frac{D_1}{q} \cdot [L^c(\theta^*, M) - q]$$

In Case (4) $x^* - 2\epsilon \leq x_j < x^*$: The analysis mirrors that of Case (2). There exists a $\underline{x} \in [x^* - 2\epsilon, x^*)$, such that $(x^* - \underline{x})/2\epsilon = L^c$. When $x_j \in (\underline{x}, x^*)$, we have $(x^* - x_j)/2\epsilon < L^c$. The expected payoff difference can be written as

$$\begin{aligned} E[DW(L)|x_j] &= \int_{\frac{x^* - x_j}{2\epsilon}}^{L^c(\theta^*,M)} (1-q)\frac{D_1}{q}dL + \int_{L^c(\theta^*,M)}^1 (-D_1)dL + \frac{x^* - x_j}{2\epsilon}(-D_1) \\ &= \frac{D_1}{q} \cdot [L^c(\theta^*, M) - q] - \frac{D_1}{q}\frac{x^* - x_j}{2\epsilon}. \end{aligned}$$

When $x_j \in [\theta^* - 2\epsilon, \underline{x}]$, we have $(x^* - x_j)/2\epsilon > L^c$ and

$$E[DW(L)|x_j] = \int_{\frac{x^*-x_j}{2\epsilon}}^{1} (-D_1)dL + \frac{x^* - x_j}{2\epsilon}(-D_1) = -D_1.$$

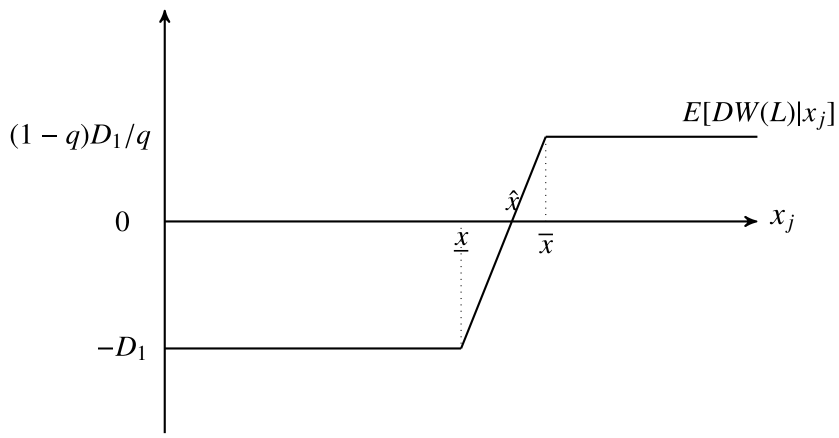
Lastly, in Case (5) $x_j < x^* - 2\epsilon$: Creditor j perceives $L(\theta, x^*) = 1$ with probability 1. As $DW(L) = -D_1$ when $L = 1$, we have

$$E[DW(L)|x_j] = -D_1, \text{ for } x_j < x^* - 2\epsilon.$$

We plot $E[DW(L)|x_j]$ in Figure 7. It is straightforward to see that when $x_j \in (\underline{x}, \overline{x})$, $E[DW(L)|x_j]$ strictly increases in x_j with a slope $r_D/2\epsilon$. As a result, there must exist a unique $\hat{x}$ such that $E[DW(L)|x_j = \hat{x}] = 0$. Therefore, creditor j's best response to the other creditors' threshold strategy is a threshold strategy: to withdraw if $x_j < \hat{x}$ and to wait if $x_j > \hat{x}$. One can derive explicitly the best response of $\hat{x}$ to x^*.

$$\hat{x} = x^* - 2\epsilon \left[L^c\left(\theta^*, M\right) - q\right] \tag{B.32}$$

Figure 7: Payoff differences and the decision to withdraw



In a symmetric equilibrium, it must hold that $\hat{x} = x^*$. Therefore, a condition for a symmetric equilibrium of the bank run game to exist is $L^c(\theta^*, M) = q$. Using (B.31), we can derive the condition explicitly as

$$\theta^* = \frac{D_2 - D_1}{1 - qD_1/\mathbb{P}_M(\theta^*)}. \tag{B.33}$$

Provided that θ^* exists, the fraction of early withdrawal when the bank's fundamental θ happens to be θ^* is given by

$$L(x^*, \theta^*) = \frac{x^* - \theta^* + \epsilon}{2\epsilon}.$$

By the definition of θ^*, the bank will be exactly on the verge of failure when $L(x^*, \theta^*)$ fraction of creditors withdraw. This in turn suggests $L(x^*, \theta^*)$ must exactly equal to $L^c(\theta^*, M)$, the critical run proportion derived in (B.31). Thus, the equilibrium threshold signal x^*, if exist, must satisfy

$$\frac{x^* - \theta^* + \epsilon}{2\epsilon} = \frac{\mathbb{P}_M(\theta^*)(\theta^* - D_2)}{D_1 \left[\theta^* - \mathbb{P}_M(\theta^*)/q\right]} \tag{B.34}$$

In sum, when creditors expecting the asset price to be $\mathbb{P}_M(\theta^*)$, the equilibrium threshold signal x^* and the critical cash flow θ^* jointly satisfy equation (B.33) and (B.34).

It should be noted that θ^*, if exists, must belong to the interval $[x^* - 2\epsilon, x^* + 2\epsilon]$. Otherwise, we would have contradictions. For example, suppose $\theta^* > x^* + 2\epsilon$. When a bank's fundamental happens to be $\theta^* - \epsilon/2$, the bank should fail by the definition of θ^*. Yet, all creditors will receive signals greater than $x^* + \epsilon/2$ and should not withdraw from the bank according to their equilibrium strategy. Therefore θ^* must be no higher than $x^* + 2\epsilon$. Similarly, one can argue that θ^* must be no lower than $x^* - 2\epsilon$. In the limiting case where noise ϵ approaches to zero, θ^* and x^* converge.

Finally, we show that the symmetric equilibrium, if exists in the interval $[\theta^L, \theta^U(\mathbb{P}_M(\theta^*))]$, is the only one that survives iterated elimination of strictly dominated strategies.

We construct a sequence $\{\underline{x}_j\}_{j=0}^{\infty}$ starting by $\underline{x}_0 = \underline{\theta}_s$, $s = G$ or B. We let

$$\underline{x}_{j+1} = \underline{x}_j - 2\epsilon\left[L^c\left(\underline{\theta}_j, \theta^*, M\right) - q\right] \tag{B.35}$$

where

$$L^c(\underline{\theta}_j, \theta^*, M) = \frac{\mathbb{P}_M(\theta^*)(\underline{\theta}_j - D_2)}{D_1\left[\underline{\theta}_j - \mathbb{P}_M(\theta^*)/q\right]}.$$

Note that θ^* enters into the expression of (B.35) because creditors have to form belief that asset price to be $\mathbb{P}_M(\theta^*)$ before they play the global games. It can be checked that $\partial L^c(\underline{\theta}_j, \theta^*, M)/\partial \underline{\theta}_j > 0$ when $\mathbb{P}_M(\theta^*) \in [\underline{P}, qD_2)$. We further let $\underline{\theta}_j$, $j \geq 0$ satisfies the equation $\frac{\underline{x}_j - \underline{\theta}_j + \epsilon}{2\epsilon} = L^c(\underline{\theta}_j, \theta^*, M)$, or equivalently,

$$\underline{\theta}_j + 2\epsilon L^c(\underline{\theta}_j, \theta^*, M) = \underline{x}_j + \epsilon. \tag{B.36}$$

It is easily seen that $\underline{\theta}_j$ increases as $\underline{x}_j$ increases. Given $\underline{x}_0 = \underline{\theta}_s$, $\underline{\theta}_0$ solves the equation $\underline{\theta}_0 + 2\epsilon L^c(\underline{\theta}_0, \theta^*, M) = \underline{x}_0 + \epsilon$. By the monotonicity of L^c with respect to $\underline{\theta}_j$, there exists a unique solution of $\underline{\theta}_0$. The solution is sufficiently close to $\underline{x}_0 = \underline{\theta}_s$ because ϵ is sufficiently small. Recall that we claimed $\theta^* \geq D_2$, if exists, solves $L^c(\theta^*, M) = L^c(\theta^*, \theta^*, M) = q$. By $\underline{\theta}_0 < D_2$ and $\partial L^c(\underline{\theta}_j, \theta^*, M)/\partial \underline{\theta}_j > 0$, we have $L^c(\underline{\theta}_0, \theta^*, M) < q$. Consequently, we obtain

$$\underline{x}_1 = \underline{x}_0 - 2\epsilon\left[L^c\left(\underline{\theta}_0, \theta^*, M\right) - q\right] > \underline{x}_0$$

This in turn means $\underline{\theta}_1 > \underline{\theta}_0$. We can iterate this process and claim both sequences $\{\underline{x}_j\}_{j=0}^N$ and $\{\underline{\theta}_j\}_{j=0}^N$ are increasing for finite number N.

On the other hand, it is easily seen that $\{\underline{\theta}_j\}_{j=0}^\infty$ has an upper limit of θ^*. To see why, because each incremental value from $\underline{\theta}_j$ to $\underline{\theta}_{j+1}$ is small enough, there exists a value of $\underline{\theta}_k$ such that $\underline{\theta}_k = \theta^*$. Then by (B.35), we have $\underline{x}_{k+1} = \underline{x}_k$ as θ^* makes $L^c(\theta^*, \theta^*, M) = q$. Consequently, we have $\underline{\theta}_{k+1} = \underline{\theta}_k = \theta^*$ by (B.36). By iteration, we have $\underline{\theta}_k = \underline{\theta}_{k+1} = \underline{\theta}_{k+2} = ... = \theta^*$, θ^* is the upper bound of the sequence. We then know that both $\{\underline{x}_j\}_{j=0}^\infty$ and $\{\underline{\theta}_j\}_{j=0}^\infty$ are increasing. By (B.36), $\{\underline{x}_j\}_{j=0}^\infty$ has an upper limit of $\theta^* - \epsilon$.

Similarly, we can construct sequences $\{\overline{x}_j\}_{j=0}^\infty$ and $\{\overline{\theta}_j\}_{j=0}^\infty$ starting by $\overline{x}_0 = \overline{\theta}$. It takes the same procedure to show both sequences are decreasing, and with lower bounds $\theta^* - \epsilon$ for the former and θ^* for the latter.

To summarize, $x^* = \theta^*$ when ϵ approaches to zero is the only strategy that survives iterated elimination of strictly dominated strategies.

Appendix C The implementation of the BoLR intervention

In this appendix, we consider an implementation of the BoLR's intervention with mutual commitment presented in Section 4.2. We introduce the following ingredients to our basic model. When a BoLR offers to commit to a price support of $P_A \in (P_2^*, P_1^*)$ at $t = 0$, each of the two banks decides simultaneously whether to accept the BoLR's offer and pledge to sell its assets to the BoLR when runs happen. If the bank refuses the offer, it will sell its assets to the interim-date asset buyers who bid ex-post after runs happen. The agreement is mutually binding for the BoLR and the banks. Banks' creditors and the interim-date asset buyers observe P_A and also know whether a bank has entered the agreement.

We assume that each bank is run by a risk-neutral manager who takes charge of the decision regarding whether to accept the BoLR's offer. The manager of Bank i, makes a binary choice $a_i \in \{A, NA\}$, where 'A' stands for 'accept the offer' and 'NA' denotes the opposite. We further assume that a manager receives a fixed benefit B conditional on his bank being afloat, and chooses a_i to maximize his expected compensation.

The timeline of game with the BoLR's price support and bank managers' decision is depicted in Figure 8. Events at $t = 0$ and $t = 1$ happen sequentially.

Figure 8: Timing of the game with the BoLR's price support and banks' decision

t = 0	t = 1	t = 2
1. Banks are established, with their portfolios and liability structures as given. 2. *A BoLR offers to buy bank assets for a price P_A in case any bank run happens.* 3. *Bank managers simultaneously decide to accept the BoLR's offer or not.*	1. s and θ realize sequentially. 2. Creditors observe P_A and receive noisy private signals about θ and simultaneously decide whether to run on each of the banks. 3. *When experiencing runs, a bank that has accepts the BoLR's offer sells its assets to the BoLR at the pre-specified price P_A; and a bank that has refused the BoLR's offer sells its assets to the interim-date asset buyers.*	1. Bank assets pay off. 2. Remaining obligations are settled.

In an equilibrium of this model, both banks accept the BoLR's offer when $P_A \in (P_2^*, P_1^*)$, and there exists a unique $P_A^* \in (P_2^*, P_1^*)$ that allows the BoLR to breaks even in expectation. Consequently, the game presented in Appendix C implements the BoLR's intervention with mutual commitment discussed in Section 4.2.

Appendix C.1 The equilibrium of subgames

By the end of the initial date, a price P_A offered by the BoLR and a strategy profile (a_i, a_{-i}) of two managers define a subgame $g_{a_i,a_{-i}}(P_A)$. To show both banks accepting the BoLR's offer as an equilibrium, we need only to calculate players' payoffs from two subgames $g_{A,A}(P_A)$ and

$g_{NA,A}(P_A)$. In game $g_{A,A}(P_A)$, both managers accept the BoLR's offer, so both banks will sell assets to the BoLR upon runs happen. In $g_{NA,A}(P_A)$, the manager of Bank i refuses the BoLR's offer, so its bank will sell assets to the buyers who bid ex-post. On the contrary, Bank $-i$ will still sell assets to the BoLR for a unit price P_A. We solve the model backward starting from the two subgames.

In subgame $g_{A,A}(P_A)$, banks' creditors have observed the BoLR's price $P_A \in (P_2^*, P_1^*)$ and learned that both banks accepted the BoLR's offer. The creditors then play a standard global-games-based bank run game in each bank, taking price P_A as given. Applying the procedure in Appendix B, we can derive an equilibrium critical cash flow $\theta^*(P_A) = (D_2 - D_1)/(1 - qD_1/P_A)$.

In subgame $g_{NA,A}(P_A)$, Bank $-i$ accepts the BoLR's offer, whereas Bank i does not. The equilibrium critical cash flow, therefore, remains $\theta^*(P_A)$ for Bank $-i$. Since Bank i refuses the BoLR's offer and will sell its assets to the interim-date asset buyers upon runs, the asset price will be determined by the buyers' competitive bidding. We solve for the equilibrium asset price and the creditors' bank run decision in Bank i following the procedure in Section 3.

We first consider the asset buyers who move *after* observing a run on Bank i. The buyers know that a run occurs if and only if Bank i's cash flow is below a critical value θ^*. They use the number of runs M to update their beliefs about the aggregate state s. Note that the bank run outcome of Bank $-i$ also conveys information about s. We calculate the buyers' posterior beliefs about s when observing a run on Bank i, and M runs in total, $M \in \{1, 2\}$, as the following

$$\omega_M^B(\theta^*, P_A) = \frac{\left(\frac{\theta^* - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right) \cdot \omega_{M-1}^B(P_A)}{\left(\frac{\theta^* - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right) \cdot \omega_{M-1}^B(P_A) + \left(\frac{\theta^* - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_G}\right) \cdot \omega_{M-1}^G(P_A)} = 1 - \omega_M^G(\theta^*, P_A), \tag{C.37}$$

where

$$\omega_1^B(P_A) = Prob(s = B|\theta^{-i} < \theta^*(P_A)) = \frac{\left(\frac{\theta^*(P_A) - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right) \cdot (1 - \alpha)}{\left(\frac{\theta^*(P_A) - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right) \cdot (1 - \alpha) + \left(\frac{\theta^*(P_A) - \underline{\theta}_G}{\bar{\theta} - \underline{\theta}_G}\right) \cdot \alpha} = 1 - \omega_1^G(P_A),$$

$$\omega_0^B(P_A) = Prob(s = B|\theta^{-i} > \theta^*(P_A)) = \frac{\left(\frac{\bar{\theta} - \theta^*(P_A)}{\bar{\theta} - \underline{\theta}_B}\right) \cdot (1 - \alpha)}{\left(\frac{\bar{\theta} - \theta^*(P_A)}{\bar{\theta} - \underline{\theta}_B}\right) \cdot (1 - \alpha) + \left(\frac{\bar{\theta} - \theta^*(P_A)}{\bar{\theta} - \underline{\theta}_G}\right) \cdot \alpha} = 1 - \omega_0^G(P_A).$$

When a run happens on Bank i, and M runs happen in total, the asset buyers bid competitively for Bank i's assets on sale. The equilibrium of the buyers' bidding game results in

$$\mathbb{P}_M(\theta^*) = \omega_M^B(\theta^*, P_A)\frac{\underline{\theta}_B + \theta^*}{2} + \omega_M^G(\theta^*, P_A)\frac{\underline{\theta}_G + \theta^*}{2}, M \in \{1, 2\}. \tag{C.38}$$

Creditors of rationally expect the asset buyers' bid $\mathbb{P}_M(\theta^*)$, so that the equilibrium critical cash flow is given by $\theta^* = (D_2 - D_1)/(1 - qD_1/\mathbb{P}_M(\theta^*))$. Following the same procedure in the proof of Lemma 3, we can prove that for any given $M \in \{1,2\}$, there exists an asset price $P^*_{A,M} \equiv \mathbb{P}_M(\theta^*) \in [\underline{P}, qD_2)$ and a critical cash flow $\theta^*_{A,M} \equiv \theta^* \in [\theta^L, \theta^U(P^*_{A,M})]$ that jointly solve the system of equation (C.38) and equation $\theta^* = (D_2 - D_1)/(1 - qD_1/\mathbb{P}_M(\theta^*))$.[59]

Result 1. *In subgame $g_{NA,A}(P_A)$, Bank $-i$ sells its assets to the BoLR for $P_A \in (P^*_2, P^*_1)$ upon runs and fails if and only if $\theta^{-i} \leq \theta^*(P_A) \in [\theta^L, \theta^U(P_A)]$. Bank i sells its assets to the ex-post asset buyers at $P^*_{A,M} \in [\underline{P}, qD_2)$ upon runs and fails if and only if $\theta^i \leq \theta^*_{A,M} \in [\theta^L, \theta^U(P^*_{A,M})]$, where M indicates the total number of bank runs.*

We have the following result when comparing the equilibrium of sugbame $g_{NA,A}(P_A)$ to that in a laissez-faire market.

Result 2. *Provided $P_A \in (P^*_2, P^*_1)$, when runs happen only to Bank i, we have $\theta^*_{A,1} = \theta^*_1$ and $P^*_{A,1} = P^*_1$; when runs happen to both, we have $\theta^*_{A,2}(P_A) > \theta^*_2$ and $P^*_{A,2} < P^*_2$.*

We first show $\theta^*_{A,1} = \theta^*_1$ and $P^*_{A,1} = P^*_1$. Note that asset buyers' posterior belief about s when observing that runs happen only to Bank i has the same expression in the two cases

$$\omega^B_1(\theta, P_A) = \omega^B_1(\theta) = \frac{(\theta - \underline{\theta}_B)}{(\theta - \underline{\theta}_B) + \kappa(\theta - \underline{\theta}_G)}, \forall \theta \in [D_2, \overline{\theta}].$$

Therefore, both equation (6) and (C.38) have a unique solution $\theta^*_{A,1} = \theta^*_1 \in [D_2, \overline{\theta}]$.

We then prove $\theta^*_{A,2}(P_A) > \theta^*_2$ and $P^*_{A,2} < P^*_2$. Following the proof of Lemma 4, we compare the ratios of posteriors in the two cases. We can express $\omega^s_2(\theta, P_A)$ as

$$\omega^B_2(\theta, P_A) = \frac{\left(\theta - \underline{\theta}_B\right)}{\left(\theta - \underline{\theta}_B\right) + \kappa\left(\frac{\theta^*(P_A) - \underline{\theta}_G}{\theta^*(P_A) - \underline{\theta}_B}\right)\left(\theta - \underline{\theta}_G\right)}, \tag{C.39}$$

$\forall \theta \in [D_2, \overline{\theta}]$. It can be checked that

$$\frac{\omega^B_2(\theta, P_A)}{\omega^G_2(\theta, P_A)} = \frac{\left(\theta - \underline{\theta}_B\right)}{\kappa\left(\frac{\theta^*(P_A) - \underline{\theta}_G}{\theta^*(P_A) - \underline{\theta}_B}\right)\left(\theta - \underline{\theta}_G\right)} > \frac{\left(\theta - \underline{\theta}_B\right)}{\kappa\left(\theta - \underline{\theta}_G\right)} = \frac{\omega^B_2(\theta)}{\omega^G_2(\theta)} \quad \forall \theta \in [D_2, \overline{\theta}].$$

Then $\theta^*_{A,2}(P_A) > \theta^*_2$ directly follows Lemma 4.

Appendix C.2 The equilibrium of bank managers' game

In this section, we move one step backward to analyze the game between the two bank managers regarding whether to accept the BoLR's price support $P_A \in (P^*_2, P^*_1)$. We show that

[59] We use the subscript A here to differentiate the pair of the equilibrium asset price and the equilibrium critical cash flow from those in the laissez-faire market.

both banks accepting the BoLR's offer is a Nash equilibrium of the game. We label the manager of Bank i ($-i$) as Manager i ($-i$). The objective function of Manager i can be expressed as

$$[1 - Pr(\text{Failure of Bank i})] \times B.$$

Our analysis in Appendix C.1 shows that managers' strategy profile (a_i, a_{-i}) determines the price of a bank's assets and the equilibrium critical cash flow of the bank run game. Consequently, a bank's probability of failure also depends on (a_i, a_{-i}). To show both managers accept the BoLR's offer, i.e., $(a_i^*, a_{-i}^*) = (A, A)$, is an equilibrium, we fix Manager $-i$'s action as $a_{-i} = A$ and analyze Manager i's optimal response. Conditional on $a_{-i} = A$, we express Bank i's probability of failure as a function of Manager i's action a_i,

$$Pr(\text{Failure of Bank i}|a_{-i} = A) = \begin{cases} Pr(A, A) & \text{if } a_i = A, \\ Pr(NA, A) & \text{if } a_i = NA. \end{cases}$$

If $Pr(A, A) < Pr(NA, A)$, Manager i's best response to $a_{-i} = A$ is $a_i^* = A$, and the strategy profile $(a_i^*, a_{-i}^*) = (A, A)$ is a Nash equilibrium.

We then derive $Pr(A, A)$ and $Pr(NA, A)$ for a given $P_A \in (P_2^*, P_1^*)$. When Manager i chooses $a_i = A$, Bank i will sell its assets to the BoLR for the price P_A upon runs. The bank will fail if and only if its cash flow is below $\theta^*(P_A)$. Bank i's probability of failure is

$$Pr(A, A) = \sum_{s=G,B} Pr(s) \cdot \left(\frac{\theta^*(P_A) - \underline{\theta}_s}{\overline{\theta} - \underline{\theta}_s} \right).$$

When Manager i chooses $a_i = NA$, Bank i will sell its assets to the interim-date buyers upon runs. Whether runs happens on Bank $-i$ conveys information about the aggregate state s. The asset buyers' bids for Bank i' assets also depends on whether Bank $-i$ experiences runs. Bank i's probability of failure is

$$Pr(NA, A) = \sum_{s=G,B} Pr(s) \cdot \left[\left(\frac{\theta^*_{A,1} - \underline{\theta}_s}{\overline{\theta} - \underline{\theta}_s} \right) \cdot \left(\frac{\overline{\theta} - \theta^*(P_A)}{\overline{\theta} - \underline{\theta}_s} \right) + \left(\frac{\theta^*_{A,2}(P_A) - \underline{\theta}_s}{\overline{\theta} - \underline{\theta}_s} \right) \cdot \left(\frac{\theta^*(P_A) - \underline{\theta}_s}{\overline{\theta} - \underline{\theta}_s} \right) \right].$$

For a given state s, Bank i fails if its cash flow is below $\theta^*_{A,1}$ when there is no run on Bank $-i$. When Bank $-i$ experiences runs, Bank i fails if its cash flow is below $\theta^*_{A,2}$.

Manager i's optimal decision rule as a best response to $a_{-i} = A$ can be expressed as

$$a_i^* = \begin{cases} A & \text{if } Pr(A, A) < Pr(NA, A) \\ NA & \text{otherwise.} \end{cases}$$

The following result characterize a condition under which $(a_i^*, a_{-i}^*) = (A, A)$ is an equilibrium of the bank managers' game.

Result 3. *There exists a unique critical value $P_A^c \in (P_2^*, P_1^*)$ such that for $P_A \in (P_A^c, P_1^*)$, $a_i^* = A$. That is, when the price support P_A is sufficiently high, both bank managers accepting the BoLR's offer is an equilibrium.*

Proof. We now prove Result 3. To pin down the optimal response of Manager i, we define an auxiliary function as

$$\begin{aligned} F(P_A) &= Pr(A,A) - Pr(NA,A) \\ &= \sum_{s=G,B} Pr(s) \cdot \left[\left(\frac{\theta^*(P_A) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) - \left(\frac{\theta^*_{A,1} - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \cdot \left(\frac{\bar{\theta} - \theta^*(P_A)}{\bar{\theta} - \underline{\theta}_s} \right) - \left(\frac{\theta^*_{A,2}(P_A) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \cdot \left(\frac{\theta^*(P_A) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \right]. \end{aligned}$$

We first establish the existence of a $P_A^c \in (P_2^*, P_1^*)$ such that $F(P_A^c) = 0$. That is, when the BoLR offers a price P_A^c to purchase the assets of Bank $-i$, Manager i is indifferent between action A and NA, given $a_{-i} = A$.

To establish this result, first note that inequalities $\lim_{P_A \to P_1^*} \theta^*_{A,2}(P_A) > \theta_2^* > \theta_1^*$ must hold by Result 2. We then show $\lim_{P_A \to P_2^*} \theta^*_{A,2}(P_A) = \theta_2^*$. That is, when the BoLR's price support is as low as P_2^* for Bank $-i$, Bank i will face a critical cash flow θ_2^* and will sell its assets to the interim-date buyers for price P_2^* when experiencing runs. Let $\Pi_2^A(\theta, P_A)$ be the interim-date buyers' payoff when observe two bank runs under the BoLR's price support P_A for Bank $-i$. We know that $\theta = \theta_2^*$ makes $\lim_{P_A \to P_2^*} \Pi_2^A(\theta, P_A) = 0$, since by Lemma 3, we know $\lim_{P_A \to P_2^*} \Pi_2^A(\theta, P_A) = \Pi_2(\theta)$ if and only if $\theta = \theta_2^*$. We have also proved in Result 1 that $\theta^*_{A,2}(P_A)$ is the unique the solution to equation $\Pi_2^A(\theta, P_A) = 0$. Combine these two facts, we obtain $\lim_{P_A \to P_2^*} \theta^*_{A,2}(P_A) = \theta_2^*$. Consequently, we have

$$\begin{aligned} \lim_{P_A \to P_2^*} F(P_A) &= \sum_{s=G,B} \frac{Pr(s)}{(\bar{\theta} - \underline{\theta}_s)^2} \cdot \left[(\theta_2^* - \theta_1^*) \cdot (\bar{\theta} - \theta_2^*) \right] > 0, \\ \lim_{P_A \to P_1^*} F(P_A) &= \sum_{s=G,B} \frac{Pr(s)}{(\bar{\theta} - \underline{\theta}_s)^2} \cdot \left[(\theta_1^* - \underline{\theta}_s) \cdot (\theta_1^* - \theta^*_{A,2}(P_A)) \right] < 0. \end{aligned}$$

By the continuity of $F(P_A)$, there must exist a $P_A^c \in \left(P_2^*, P_1^*\right)$ such that $F(P_A^c) = 0$.

We then establish the monotonicity of $F(P_A)$, with

$$\frac{dF(P_A)}{dP_A} = \sum_{s=G,B} Pr(s) \cdot \left[\frac{d\theta^*(P_A)}{dP_A} \cdot \left((\bar{\theta} - \theta^*_{A,2}(P_A)) + (\theta_1^* - \underline{\theta}_s) \right) - \frac{d\theta^*_{A,2}(P_A)}{dP_A} \cdot \left(\theta^*(P_A) - \underline{\theta}_s \right) \right] < 0.$$

As a result, $F(P_A)$ is monotonically decreasing in P_A, and P_A^c is also unique. Furthermore, we have $F(P_A) < 0$ when $P_A \in (P_A^c, P_1^*)$ and $F(P_A) \geq 0$ when $P_A \in (P_2^*, P_A^c]$, so that Manager i will choose $a_i^* = A$ when $P_A \in (P_A^c, P_1^*)$. We complete the proof of Result 3.

□

Appendix C.3 $P_A^* \in (P_2^*, P_1^*)$ as the BoLR's break-even price

In this section, we establish a sufficient condition to guarantee the existence of a $P_A^* \in (P_A^c, P_1^*)$ such that $V(P_A^*) = 0$. By offering such a price P_A^*, the BoLR can induce both banks to participate in the asset purchase program and still break even in expectation.

By offering a price $P_A \in (P_A^c, P_1^*)$, the BoLR rationally anticipates that both managers will accept her offer. So the BoLR's expected profit function $V(P_A)$ can be expressed by equation (13). In Section 4.2, we prove that $V(P_2^*) > 0$, $V(P_1^*) < 0$ and $dV(P_A)/dP_A < 0$. When $V(P_A^c) > 0$, there exists a unique $P_A^* \in (P_A^c, P_1^*)$ such that $V(P_A^*) = 0$.

Result 4. *When $V(P_A^c) > 0$, there exists a unique $P_A^* \in (P_A^c, P_1^*) \subset (P_2^*, P_1^*)$. By offering such a P_A^*, the BoLR can induce both managers to accept her asset purchase program and still break even in expectation.*