Contagious Bank Runs and Committed Liquidity Support*

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Abstract

In a crisis, regulators and private investors can find it difficult, if not impossible, to tell whether banks facing runs are insolvent or merely illiquid. We introduce such an information constraint into a global-games-based bank run model with multiple banks and aggregate uncertainties. The information constraint creates a vicious cycle between contagious bank runs and falling asset prices and limits the effectiveness of traditional emergency liquidity assistance programs. We explain how a regulator can set up committed liquidity support to contain contagion and stabilize asset prices even without information on banks' solvency, rationalizing some recent developments in policy practices.

Keywords: Committed liquidity support, Global games, Bank runs

JEL Classification: G01, G11, G21

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1 Introduction

The 2007-2009 financial crisis highlights a dual-illiquidity problem. During the crisis, market liquidity evaporated, and asset prices dropped sharply. At the same time, funding liquidity dried up, and even well-capitalized banks found it difficult to roll over their short-term debt. In response to the dual-illiquidity problem, central banks were creative in providing facilities for liquidity support during the crisis and have been experimenting with novel upfront liquidity arrangements afterward. Examples of such arrangements include the asset pre-positioning program of the Bank of England and the committed liquidity facility of the Reserve Bank of Australia. In this paper, we model the two-way feedback between distressed asset prices and contagious bank runs and show how upfront liquidity support can be an effective response to the dual-illiquidity problem, rationalizing the recent policy developments.

Central to our model is the observation that it can be difficult — if not impossible — to distinguish illiquid banks from insolvent ones in crisis times.[1] We show that such an information constraint creates a vicious cycle between falling asset prices and bank runs. When private asset buyers cannot distinguish assets sold by illiquid banks from those sold by insolvent banks, the price they offer would reflect the average asset quality. As a result, a solvent-but-illiquid bank would be unable to recoup a fair value for its assets on sale. In a global-games framework, we show that creditors' expectations of low asset prices due to this information friction can deprive the solvent bank of its short-term funding. Each creditor, anticipating the liquidation loss caused by other creditors' early withdrawals from his bank, chooses to join the run himself. However, it is the run and the forced liquidation — by pooling the illiquid bank with insolvent ones — that lead to the decline in asset prices in the first place.[2]

In a two-bank setting, financial contagion and systemic crisis emerge once we introduce an aggregate risk that affects both banks' fundamentals. We analyze a global game with multiple groups of players (i.e., two distinct groups of creditors for the two banks) and multi-dimensional signals (i.e., in addition to private signals about their own bank's fundamentals, creditors receiving a common signal about the

[1]The information constraint is recognized as one of the main challenges for central banks to provide emergency liquidity assistance. See e.g., Goodhart (1999) and Freixas et al. (2004).

[2]Using historical data, Fohlin et al. (2016) empirically document the feedback between market and funding illiquidity, providing evidence that information asymmetry on asset qualities contributes to the vicious cycle.

other bank's fundamentals as outsiders). We show that coordination failures occur not only among creditors within a bank but also between creditors from different banks. The cross-bank coordination failure is triggered by the expectation of falling asset prices: upon observing more bank runs, the asset buyers' beliefs about the aggregate state deteriorates, which reduces their bids for banks' assets. The lower asset prices, in turn, precipitate runs at more banks. We derive a unique equilibrium despite the two-way feedback between collapsing asset prices and contagious bank runs. The equilibrium features financial contagion: a bank is more likely to experience runs when its creditors perceive the other bank to have weak fundamentals and expect that bank's liquidation to depress asset prices.

The information constraint that creates financial fragility also limits the effectiveness of traditional emergency liquidity assistance programs. An informationally constrained central bank cannot lend only to the solvent-but-illiquid banks as suggested by the classic lender-of-last-resort (LoLR) principles of Bagehot (1873). In particular, in tackling banks' funding problems as runs happen, an informationally constrained central bank would risk rescuing insolvent banks or making losses from the intervention.

We show that upfront liquidity support can contain contagious bank runs even if a central bank holds no information on individual banks' solvency. Intuitively, the central bank can support the price of banks' assets in a pre-committed arrangement, thereby breaking down the two-way feedback between falling asset prices and contagious bank runs. We recommend an arrangement where a regulator and banks mutually commit to an agreement for the regulator to purchase a bank's assets for a pre-specified price when banks experience runs. In making her offer before the aggregate risk is realized, the regulator's price support is neither conditional on an aggregate state nor on the knowledge about the banks' solvency. The pre-specified price allows the regulator to contain the risk of contagion while breaking even across possible posterior beliefs about the aggregate risk from an ex-ante perspective.

Our modeling of the committed liquidity support is broadly consistent with the suggestion of King (2017) that a central bank should act as a 'pawnbroker for all seasons (PFAS)' and commit to providing liquidity insurance to banks in times of crisis. Our theory suggests that liquidity support is the most effective if banks also commit to raising liquidity from the central bank when experiencing runs. This suggested regulatory obligation is in line with King's proposals that, for emergency liquidity assistance, banks should be 'required to take out insurance in the form of pre-positioned collateral with the central bank' and that the provision of liquidity insurance should be 'mandatory and paid for upfront'.

The recent policy practices have indeed seen the implementations of such committed liquidity support. The Bank of England has implemented asset pre-positioning as a part of its sterling monetary framework.[3] Also, in the spirit of the proposals in King (2017), the Reserve Bank of Australia launched the committed liquidity facility with an explicit requirement for banks to commit to the regime. To benefit from the liquidity support of the facility, a bank needs to pay a premium of 15 basis points for the amount of liquidity committed by the central bank. In return, the central bank contractually commits to entering repo transactions with the participating bank, should runs happen to it.[4]

This paper makes three contributions. First, we introduce a relevant information constraint into a global-games-based bank run model: it is difficult to distinguish illiquid banks from insolvent ones during crisis times. We show that the information constraint not only results in a vicious cycle between bank runs and distressed asset prices, but also makes traditional policy interventions ineffective. Second, we analyze a novel global-games setting with multiple groups of players and multi-dimensional signals. In addition to the coordination problem among creditors within a bank, our bank run game also features strategic complementarities between creditors from different banks. Finally, from a policy perspective, we show that committed liquidity support can make an effective intervention, providing a formal theory to interpret some of the recent developments in central bank policy practices.

Related literature: Our paper contributes to the literature on public liquidity intervention and global-games-based bank run models. Central bank liquidity injection in a global-games framework was first studied by Rochet and Vives (2004). The authors consider a single-bank setup and derive a unique threshold equilibrium where a solvent bank can be illiquid. The authors further assume that the bank's fundamentals are perfectly observable to a central bank and suggest that the central bank can act

[3]By the spring of 2015, £469 billion of bank assets had been pre-positioned with the central bank, with an average haircut of 33%. Bank of England (2019) provides detailed guidelines for private institutions to pre-position their illiquid assets.

[4]See Reserve Bank of Australia (2018, 2019) for details. By the end of 2018, a total of AU$ 248 billion of central bank liquidity support was committed through the facility to eligible banks.

as a LoLR, by lending directly to the solvent-but-illiquid bank as suggested by Bagehot (1873). In a two-bank setting, we generalize Rochet and Vives (2004) by introducing information constraints, endogenous liquidation value, and aggregate uncertainty. We focus on systemic crises instead of runs on individual banks and show that upfront liquidity support can mitigate system-wide financial fragility.

Our model predicts a vicious cycle between bank runs and falling asset prices, and strategic complementarities between creditors from different banks. These two features are most related to Liu (2016) and Goldstein et al. (2020), respectively. Liu (2016) studies how limited participation in the interbank market can lead to an interaction between bank runs and rising interbank market rates.[5] Goldstein et al. (2020) allow for cross-bank coordination failures to study the impact of bank heterogeneity on financial stability.[6] Both papers, however, assume that the external providers of liquidity — i.e., the lending banks in Liu (2016) and the asset buyers in Goldstein et al. (2020) — can perfectly observe the distressed bank' fundamentals. By contrast, we emphasize that both private investors and central banks face the information constraint in telling whether creditors run on a bank due to its insolvency or mere illiquidity, and that public liquidity intervention should be designed with the information constraint taken into account.[7]

Our paper also contributes to the debate on the design of emergency liquidity assistance programs. Goodfriend and King (1988) and Freixas et al. (2004) argue that when it is hard to tell whether an illiquid bank is solvent, it can be optimal for central banks to only provide liquidity in open market operations and let the interbank market allocate the liquidity. The others question the view, arguing that the asymmetric information about banks' solvency can also interrupt the functioning of the interbank market, which justifies central banks' direct lending to banks (Flannery (1996), Heider et al. (2015), and Choi et al. (2017)). We study a setting where neither central banks nor private investors possess precise

[5]In a non-bank setup without coordination failures, Brunnermeier and Pedersen (2009) highlights two-way feedback between market and funding illiquidity by emphasizing a margin constraint on a speculator who supplies liquidity. In their model, asset prices are volatile because the selling and buying of assets are not synchronized. By contrast, we emphasize the funding liquidity risk caused by equilibrium bank runs and that the lack of information on asset qualities causes asset illiquidity.

[6]In addition, Goldstein (2005) and Leonello (2018) also feature cross-entity coordination failures. Our approach differs from all the three papers as we solve a model with multiple groups of players, each of whom receives multi-dimensional signals. Also different from the focus of the current paper, Goldstein (2005) and Leonello (2018) examine how bank runs interact with currency crisis and sovereign bond crisis, respectively.

[7]Liu (2016) also discusses a policy intervention, which is modeled as an ex-post net transfer from the central bank to private institutions, conditional on the central bank's observation of bad state. In contrast, we emphasize that the intervention should be pre-emptive: the terms of the intervention should be announced before the realization of the aggregate risk, and such an intervention can still be effective and at zero expected cost even if the central bank does not observe the aggregate state.

information on the solvency of illiquid banks and suggest that setting up upfront liquidity support can be more effective than providing ex-post emergency liquidity assistance as runs happen. We are also the first to study this information constraint in a global-games framework, which is a natural setting since the framework endogenously defines solvent-but-illiquid banks.

In terms of analyzing global games with multi-dimensional signals, our paper is most related to Fujimoto (2014), who studies a game where one group of speculators learn private signals about multiple countries and choose only one country's currency to attack.[8] Assuming a short-selling constraint for the speculators, Fujimoto (2014) shows that the speculators' attack at one country makes them less likely to attack the other countries. By contrast, the multi-dimensional signals in our model with two groups of players generate strategic complementarities between creditors from different banks.

The paper proceeds as follows. Section 2 lays out our model. Section 3 characterizes the model's equilibrium, showing how a vicious cycle between falling asset prices and contagious bank runs can emerge in a laissez-faire market. We then show how committed liquidity support can mitigate such financial fragility. We extend the policy discussion in Section 4 and conclude in Section 5.

2 Model setup

We consider a three-date ($t = 0, 1, 2$) economy with two banks ($i = 1, 2$). There are two types of risk-neutral players: banks' wholesale creditors and secondary-market asset buyers.

2.1 Banks

The two banks are identical at $t = 0$. Each of them holds a unit portfolio of long-term assets and finances the portfolio with equity E, retail deposits F, and short-term wholesale debt $1 - E - F$. We consider banks as contractual arrangements among claim holders to fulfill the function of liquidity transformation. Thus, banks in our model are passive, with given loan portfolios and liability structures.

[8] In their seminal work, Carlsson and Van Damme (1993) allow the state variables that affect players' payoffs to be multi-dimensional and players only observe noisy signals about the multi-dimensional state variables. Oury (2013) analyze when the equilibrium selection does not depend on the distribution of players' noises about the multi-dimensional state variables.

A bank i's assets generate a random cash flow $\tilde{\theta}^i \sim U(\underline{\theta}_s, \overline{\theta})$. The realization of the cash flow is not only affected by the bank's idiosyncratic risk but also by a systematic risk factor s. The systematic risk affects the distribution of both banks' cash flows, particularly their lower bounds. There are two possible aggregate states, $s = G$ and $s = B$. With $\underline{\theta}_G > \underline{\theta}_B > 0$, the distribution of the cash flows in State G first-order stochastic dominates that in State B. All players hold a prior belief that State G and B occur with probabilities α and $1 - \alpha$, respectively. Note that the upper bound of the cash flows remains the same across the two aggregate states, reflecting that banks hold mostly debt claims whose highest payoffs are capped by their face values. Once the aggregate state s is realized, the two banks' cash flows are determined by their idiosyncratic risks and assumed to be independently and identically distributed.

On the liability side, we assume that each bank is financed by retail depositors and its pool of wholesale creditors. The retail depositors are fully protected by deposit insurance, which is provided to the bank free of charge. Therefore, retail depositors passively hold their claims to maturity and demand only a gross risk-free rate that we normalize to 1. On the other hand, each bank's wholesale debt is risky, demandable, and raised from a distinct continuum of creditors of mass 1. Provided that the bank does not fail, the wholesale debt pays a gross interest rate $r_D > 1$ if a wholesale creditor waits till $t = 2$, and qr_D if the wholesale creditor withdraws early at $t = 1$. Here, $q < 1$ reflects the penalty for the early withdrawal.[9] For the ease of presentation, we denote by $D_1 \equiv (1 - E - F)qr_D$ the total amount of debt that a bank needs to repay at $t = 1$ if *all* of its wholesale creditors withdraw early, and by $D_2 \equiv (1 - E - F)r_D + F$ the total amount of debt that a bank needs to repay at $t = 2$ if *none* of its wholesale creditors withdraws early. We make the following parametric assumptions.

$$D_2 > \underline{\theta}_s \tag{1}$$

$$F > D_1 \tag{2}$$

$$q > \frac{1}{2} + \frac{\underline{\theta}_G}{2D_2} \tag{3}$$

[9] Jin et al. (2019) and Capponi et al. (2020) consider the risk of runs in a setting of equity mutual funds and point out that a more flexible contract such as swing pricing can mitigate the risk. The current paper focuses on financial institutions with non-contingent liabilities, typically banks with demandable debt financing.

Inequality (1) states that banks are not risk-free and face a positive probability of bankruptcy even in State G. Inequality (2) suggests that banks' retail debt exceeds their short-term wholesale debt, which represents a realistic case and helps simplify the analysis of bank run games.[10] Inequality (3) states that the penalty for early withdrawal is only moderate.[11] While we do not endogenize banks' capital structure (thus taking q, D_1, and D_2 as given), as long as the optimal capital structure satisfies the aforementioned conditions, all of our results apply.

2.2 The bank run game

A bank run game of complete information can have multiple equilibria. To refine the multiplicity, we take the global-games approach pioneered by Carlsson and Van Damme (1993) and assume that creditors observe noisy signals of banks' cash flows. We assume that a representative wholesale creditor j in a bank i observes a private signal x^i_j about his own bank's fundamentals. Specifically, $x^i_j = \theta^i + \epsilon^i_j$, with the noise ϵ^i_j drawn from a uniform distribution of support $[-\epsilon, \epsilon]$. In addition, all creditors in the bank i observe a signal y^{-i} about the other bank's fundamentals as outsiders. We assume $y^{-i} = \theta^{-i} + \eta^{-i}$, with the noise η^{-i} drawn from a uniform distribution of support $[-\eta, \eta]$. All noises are independent. We focus on the case $\epsilon < \eta$, so that creditors' private signals about their own bank's fundamentals are more accurate than their signal about the other bank's fundamentals.[12]

After receiving his signals (x^i_j, y^{-i}), the creditor j from the bank i takes one of two possible actions: to wait till $t = 2$, or to withdraw from his bank at $t = 1$. We focus on the following threshold strategy that is symmetric across all wholesale creditors

$$(x^i_j, y^{-i}) \longmapsto \begin{cases} \text{withdraw} & x^i_j < x(y^{-i}) \\ \text{wait} & x^i_j \geq x(y^{-i}). \end{cases} \tag{4}$$

[10] Despite the rapid growth of wholesale funding, most commercial banks and bank holding companies are still financed more by retail deposits than wholesale debt. For example, Cornett et al. (2011) document that the median core deposit to asset ratio for US commercial banks was 67.88% over the period from 2006 to 2009.

[11] For example, when $\underline{\theta}_G = \underline{\theta}_B = 0$, the condition states that $q > \frac{1}{2}$. That is, by withdrawing early, a wholesale creditor will not lose more than half of the face value of his claim. The moderate penalty for early withdrawal is in line with banks' role as liquidity providers as suggested by Diamond and Dybvig (1983).

[12] We show in the Online Appendix that assuming $\eta \geq \epsilon$ does not change our model's main results.

That is, a wholesale creditor withdraws from his bank if and only if his private signal is below a threshold $x(y^{-i})$.[13] Different from most global-games models, the threshold is not a constant but a function of the signal y^{-i} that the creditor receives as an outsider to the other bank. We assume the function $x(y^{-i})$ to be non-increasing so that the threshold strategy features two types of monotonicity: (1) the creditor's *action* is monotonic in his private signal, and (2) the *threshold* is monotonic in the signal that the creditor receives as an outsider. Focusing on $x(\cdot)$ that is non-increasing implies that a creditor's incentives to withdraw from his own bank should not be lower when the other bank's fundamentals become weaker.

A wholesale creditor's payoff depends both on his withdrawal decision and on the bank's solvency. The creditor will receive D_1, if he withdraws early and the bank does not fail at $t = 1$; he will receive $\frac{D_1}{q}$, if he waits and the bank stays solvent at $t = 2$. In the case of failure, a bank incurs a bankruptcy cost, such as the legal cost of bankruptcy. We assume the cost to be sufficiently high, so that if a wholesale creditor waits and the bank fails at either $t = 1$ or $t = 2$, the wholesale creditor will receive a zero payoff and a senior deposit insurance company obtains the residual value of the bank.[14] Finally, we assume that the creditor can obtain an arbitrarily small reputational benefit by running on a bank that fails at $t = 1$.[15]

A wholesale creditor forms rational beliefs about the fraction of withdrawals in his own bank and that in the other bank. We denote the two fractions by L^i and L^{-i}, respectively, and define the occurrence of a bank run as any positive mass of wholesale creditors withdrawing funds from their bank at $t = 1$.[16] By this definition, we have the number of bank runs $M = 1$ when $L^i > 0$ and $L^{-i} = 0$ or $L^i = 0$ and $L^{-i} > 0$. Similarly, $M = 2$ when $L^i > 0$ and $L^{-i} > 0$.

[13] In the finance application of global games, the threshold equilibrium is of primary interest. For example, see Morris and Shin (2004) and Liu (2016). Since creditors are ex-ante homogenous and banks are also assumed to have the same capital structure and *i.i.d.* cash flows, there is no loss of generality to focus on symmetric strategies.

[14] As it will be clear from the analysis, this case is off the equilibrium path when the noises of the private signals diminish.

[15] The reputational benefit may come from the fact that the creditor makes a 'right decision'. Rochet and Vives (2004) argue that most of wholesale deposits are held by investment funds whose managers are compensated if they build a good reputation. As we will show later, wholesale creditors receiving this small reputational payoff is also off the equilibrium path.

[16] Defining a bank run as a non-zero mass of withdrawals is an innocuous normalization. A run can be defined as the total withdrawals exceeding an alternative positive threshold. All results will qualitatively hold.

2.3 The secondary asset market

When facing withdrawals at $t = 1$, a bank has to liquidate its long-term assets in a secondary asset market. As early liquidation destroys a bank's value, we assume that a bank sells its assets if and only if it faces a bank run,[17] in which case, the bank sells its assets to the buyers who offer the highest price.

We assume that many identical, deep-pocketed buyers participate in the market and are called into action only when a run happens. When no bank run occurs, the asset buyers will not have the opportunity to move, and the game between wholesale creditors and asset buyers ends. The buyers observe neither the aggregate state nor any signals about the banks' cash flows. Thus, they cannot determine the exact quality of assets on sale. They can, nevertheless, observe the outcome of creditors' bank run game (i.e., the number of banks forced into liquidation) and infer the quality of assets on sale from the observation.[18]

An asset buyer bids according to the creditors' optimal strategy and her observation of the number of bank runs. When called upon to move as M bank runs happen, the buyer forms rational beliefs about the aggregate state s and the quality of assets on sale. Her strategy is a price schedule (P_1, P_2) with

$$M \longmapsto P_M, \;\; M \in \{1, 2\}. \tag{5}$$

We focus on symmetric strategies since the buyers are homogeneous and observe the same information. A strategy (P_1, P_2) can be viewed as an inverse demand function for banks' assets. When bidding competitively to purchase banks' assets in any contingency of M runs, the buyers break even in expectation.

2.4 The information structure and timing

The information structure of our model presumes players who are more closely linked to a bank receiving more precise signals on the bank's fundamentals,[19] which we consider as a realistic scenario

[17]Diamond and Rajan (2011) provide an exposition of why banks protected by the limited liability prefer not to sell their asset until runs happen, in which case the sale is too late and causes bank failures.

[18]For simplicity, we assume that asset buyers do not observe the precise size of runs if only a fraction of creditors withdraws early. Such a partial run becomes a zero-probability event when the noises of creditors' private signals approach zero.

[19]The information structure where different insiders and outsiders receive different signals is also prevalent in other finance literature, such as papers studying corporate disclosure, e.g., Goldstein and Yang (2017, 2019) and Xiong and Yang (2021). Semi-public signals — signals that are common knowledge only among a subset of players, such as y^{-i} in our model — are also featured in studies such as Morris and Shin (2007, 2018), in the context of central banks' forward guidance.

but is not analyzed in the global-games-based bank run literature. In particular, creditors who directly lend to a bank are assumed to receive the most accurate private signals about the bank's fundamentals, whereas creditors who do not lend to that particular bank still obtain relevant information about it when they lend to a bank of a similar business model.[20] The asset buyers, on the other hand, have not dealt with the banks until the occurrence of runs and are presumed to receive no informative signals in an equally timely manner.[21] Notably, a bank i's creditors are assumed to receive no common signal about their bank's fundamentals, which serves only as a modelling shortcut to highlight the coordination problem between creditors from different banks. Provided that the private information remains the most accurate, assuming the alternative will not qualitatively change the results of our paper.[22]

The timing of the game is summarized below, with events at $t = 1$ taking place sequentially.

t = 0	t = 1	t = 2
Banks are established, with their portfolios and liability structures as given.	1. s and (θ^1, θ^2) are realized sequentially. 2. Each creditor receives the noisy signals and decides whether to run on his own bank. 3. If any bank run occurs, buyers bid for and acquire assets on sale according to the number of runs.	1. Bank assets pay off. 2. Remaining obligations are settled.

3 Equilibrium bank runs and committed liquidity support

We solve the model using the concept of Perfect Bayesian Equilibrium.

Definition. *A PBE of the dynamic game consists of an equilibrium strategy profile and a system of beliefs. (i) Creditors play a symmetric threshold strategy: a representative creditor j from a bank i withdraws if and only if his private signal x^i_j falls below an equilibrium threshold $x^*(y^{-i})$. Asset buyers offer a price schedule (P^*_1, P^*_2) to purchase banks' assets on sale when observing M bank runs, $M \in \{1, 2\}$. (ii) Each creditor forms beliefs about the aggregate withdrawals in both banks based on his information (x^i_j, y^{-i}) and the equilibrium strategy profile described in (i). Asset buyers form beliefs about*

[20]Compared to an average market participant, financiers often learn better information about the firms that they fund. Botsch and Vanasco (2019) document evidence of 'learning by lending' in credit markets, showing that lenders are better able to gauge borrowers' creditworthiness as lending relationships progress.

[21]This assumption that the asset buyers receive no signals is equivalent to that they receive only very noisy signals, which are less informative for the banks' fundamentals as compared to the observed number of bank runs.

[22]In the Online Appendix, we analyze an alternative setting where all the bank i's creditors also observe the signal y^i in addition to their private signals. While this will change the analysis of the global games, the results of our paper will remain qualitatively the same as long as the private signal x^i_j is more accurate than y^i.

the qualities of banks' assets on sale and beliefs about the aggregate state based on their observation of M bank runs and the equilibrium strategy profile described in (i). (iii) The strategy profile described in (i) is sequentially rational, given the beliefs described in (ii).

For an equilibrium strategy and fundamentals (θ^1, θ^2), an equilibrium outcome in a laissez-faire market can either be no bank run and no asset liquidation, or be summarized by a duplex (M, P_M^*), $M \in \{1, 2\}$.

3.1 Competitive bidding in the secondary asset market

We first solve the asset buyers' bidding game. That is, given the creditors' strategy, what would be the secondary-market asset prices? Asset buyers observe neither fundamentals (θ^1, θ^2) nor the state s. Nevertheless, they form rational beliefs about the quality of assets on sale according to creditors' strategy and the observed number of runs $M \in \{1, 2\}$. In a subgame of competitive bidding, buyers who believe creditors using a threshold strategy understand that creditors withdraw from a bank if and only if their private signals are below a threshold signal x^*, and that given the creditors' threshold strategy, a bank run happens if and only if the bank's cash flow is below a threshold fundamental $\hat{\theta}$. Asset buyers also Bayesian update their beliefs about the aggregate state s. Since banks' cash flows are *i.i.d.* after the aggregate state is realized, more bank runs suggest State B being more likely. The buyers' posterior beliefs about the aggregate state s can be calculated as follows (details in the Online Appendix):

$$\omega_M^B\left(\hat{\theta}\right) \equiv Prob(s = B|\theta < \hat{\theta}, M) = \frac{\left(\hat{\theta} - \underline{\theta}_B\right)^M}{\left(\hat{\theta} - \underline{\theta}_B\right)^M + \kappa\left(\hat{\theta} - \underline{\theta}_G\right)^M}, \tag{6}$$

where $\kappa \equiv \frac{\alpha}{1-\alpha}\left(\frac{\bar{\theta}-\underline{\theta}_B}{\bar{\theta}-\underline{\theta}_G}\right)^2$ is a constant and $M \in \{1, 2\}$. Notably, buyers' beliefs about s are endogenous to their beliefs about the creditors' equilibrium strategy.

When buyers bid competitively for banks' assets on sale, the equilibrium of the secondary market requires buyers' bids to be equal to the expected asset quality. Specifically, when M bank runs occur, the homogeneous buyers will offer $P_M^* = \omega_M^B\left(\hat{\theta}\right) \cdot E\left(\theta \middle| \theta < \hat{\theta}, s = B\right) + \omega_M^G\left(\hat{\theta}\right) \cdot E\left(\theta \middle| \theta < \hat{\theta}, s = G\right)$. Since the buyers perceive the average quality of the assets on sale to be $E\left(\theta \middle| \theta < \hat{\theta}, s\right) = \frac{\underline{\theta}_s + \hat{\theta}}{2}$ for a given aggregate state $s \in \{B, G\}$, the competitive asset price can be written explicitly as:

$$P_M^* = \omega_M^B\left(\hat{\theta}\right) \cdot \frac{\underline{\theta}_B + \hat{\theta}}{2} + \omega_M^G\left(\hat{\theta}\right) \cdot \frac{\underline{\theta}_G + \hat{\theta}}{2} = \frac{E_s\left(\underline{\theta}_s|M\right) + \hat{\theta}}{2}. \tag{7}$$

Expression $E_s(\underline{\theta}_s|M) = \omega_M^B\left(\hat{\theta}\right) \cdot \underline{\theta}_B + \omega_M^G\left(\hat{\theta}\right) \cdot \underline{\theta}_G$ represents the expected lower bound of θ, based on M observed runs and the belief that a bank is forced into liquidation when its cash flow falls below $\hat{\theta}$.[23]

Creditors' strategy affects the secondary-market price in two ways. First, the critical signal x^* (and the resulting $\hat{\theta}$) directly determines the average quality of the assets on sale.[24] Second, the creditors' strategy affects buyers' perception of the aggregate state. For a given number of runs, a more pessimistic strategy on the creditors' side (i.e., a higher x^*) is associated with a more optimistic perception of the state s (i.e., a higher ω_M^G). Both channels suggest a higher x^* being associated with higher asset prices.

We show in Lemma 1 that any break-even price P offered by asset buyers must belong to interval $[\underline{P}, qD_2)$, with $\underline{P} \equiv \frac{\underline{\theta}_B + D_2}{2}$.[25] Intuitively, if the price $P \geq qD_2$, early liquidation will not hurt a bank's solvency so that its creditors would not run in the first place.[26] On the other hand, since all fundamentally insolvent banks (i.e., those with $\theta < D_2$) will be liquidated, the worst possible average asset quality is $\frac{\underline{\theta}_B + D_2}{2}$. It follows that the equilibrium asset prices $P_M^* \in [\underline{P}, qD_2)$. This restriction on the range of equilibrium asset prices will facilitate the solution of the bank run game in the next section.

Lemma 1. *When asset buyers believe that creditors follow a symmetric threshold strategy and that a bank fails if and only if its cash flow is lower than a threshold, the buyers' break-even price P cannot be greater than or equal to qD_2, nor can it be smaller than $\underline{P}$.*

Proof. See Appendix B.1. □

Furthermore, it holds that $P_M^* \geq \underline{P} > D_1$,[27] so that a bank can always repay its $t = 1$ liabilities and does not fail on the intermediate date. A run, however, can result in the bank's failure because liquidation losses lead to bankruptcy at $t = 2$. Specifically, while a partial liquidation can generate sufficient cash to

[23]Note that our model does not feature asset fire sales. Since the buyers pay the expected payoff of the asset given their information set, no welfare loss emerges due to the change of ownership of the asset. This differs from classic views of asset fire sales, such as in Shleifer and Vishny (1992).

[24]We derive explicitly the relationship between $\hat{\theta}$ and x^* in equation (16) when solving the creditors' bank run game.

[25]Parametric assumption (3) guarantees $qD_2 > \underline{P}$, so that the set $[\underline{P}, qD_2)$ is non-empty.

[26]For an asset price equal to qD_2, one can show that any run will reduce a bank's asset and liabilities by the same amount, resulting in a neutral impact of runs on the solvency of the bank.

[27]Note that for $q < 1$, parametric assumption (2) implies $D_2 > 2D_1$, because $D_2 = \frac{D_1}{q} + F > D_1 + F > 2D_1$.

pay early withdrawals at $t = 1$, the cash flow from the residual portfolio will be insufficient to cover the remaining liabilities at $t = 2$, making a bank that is otherwise solvent fail at $t = 2$.[28]

3.2 The bank run game

We solve the creditors' bank run game by examining the strategy of a representative creditor j from a bank $i \in \{1, 2\}$. We derive the creditor's threshold strategy as the best response to other players' equilibrium strategies $x^*(\cdot)$ and (P_1^*, P_2^*).

Facing a given asset price $P \in [\underline{P}, qD_2)$, the representative creditor will withdraw if and only if the aggregate withdrawal in his bank exceeds a critical level. To see this, note that when an L^i fraction of its creditors withdraw, the bank i faces a liquidity demand of $L^i D_1$ and needs to liquidate a $\lambda^i = \frac{L^i D_1}{P}$ fraction of its assets, where $\lambda^i \in (0, 1)$ because $P > D_1$. After the partial liquidation, the bank will fail at $t = 2$ if its remaining assets fall below its remaining liabilities, i.e., $(1-\lambda^i)\theta^i < F + (1-L^i)(1-E-F)r_D$. In other words, the bank fails at $t = 2$ if L^i exceeds a critical value L^c:

$$L^i > \frac{P \cdot (\theta^i - D_2)}{D_1 \cdot (\theta^i - P/q)} \equiv L^c(\theta^i, P). \tag{8}$$

If the representative creditor withdraws, his payoff will be $W_{run} = D_1$ since the bank does not fail at $t = 1$. If he waits instead, his payoff W_{wait} depends on L^i, θ^i and P. He would receive $\frac{D_1}{q}$ if the bank survives at $t = 2$, and 0 otherwise. Denote by $DW(L^i, \theta^i, P) \equiv W_{wait} - W_{run}$ the creditor's payoff difference from the two actions, we have

$$DW(L^i, \theta^i, P) = \begin{cases} (1-q)\frac{D_1}{q} & L^i \in [0, L^c(\theta^i, P)] \\ -D_1 & L^i \in (L^c(\theta^i, P), 1]. \end{cases} \tag{9}$$

The game features global strategic complementarities, with the creditor strictly preferring 'wait' ('withdraw') if L^i is lower (higher) than $L^c(\theta^i, P)$.

[28]Similar to Morris and Shin (2016), even if a bank survives $t = 1$ runs, it would be doomed to fail at $t = 2$. The funding liquidity risk is captured by a higher ex-ante probability of bank failure and the fact that the survival threshold is higher than the solvency threshold. This feature of no interim date failure also emerges in Ahnert et al. (2019).

With incomplete information, the creditor cannot directly observe his bank's cash flow θ^i or the aggregate withdrawal L^i. The asset price will also be endogenous to the number of runs. To determine his optimal action, the creditor has to form beliefs about θ^i, L^i, and P, given his information (x^i_j, y^{-i}) and the other players' equilibrium strategies. We take a step-by-step approach to derive the equilibrium of the incomplete information game: first analyzing a case without aggregate uncertainty for an illustrative purpose (Section 3.2.1) and then the fully-fledged model with the aggregate uncertainty (Section 3.2.2).

3.2.1 Equilibrium without the aggregate uncertainty

Suppose that there is no aggregate uncertainty and $\underline{\theta}_G = \underline{\theta}_B = \underline{\theta}$. We solve the game backward, starting with the asset market. As banks' cash flows are independently distributed, creditors' run in one bank provides no information about the other bank's fundamentals. The asset buyers thus offer a single price P independent of the number of runs observed. In other words, their strategy features a price schedule $(P_1, P_2) = (P, P)$, which suggests the demand for banks' assets to be perfectly elastic. A candidate equilibrium price P^* must satisfy the zero-profit condition (7). Without aggregate uncertainty, $E_s(\underline{\theta}_s|M)$ degenerates to $\underline{\theta}$, and the condition becomes

$$P^* = \frac{\underline{\theta} + \hat{\theta}}{2}. \tag{10}$$

To solve the bank run game, we first establish the existence of lower and upper dominance regions. When the representative creditor observes $x^i_j < x^L \equiv D_2 - \epsilon$ and knows his bank's fundamentals below $\theta^L \equiv D_2$, it is a dominant strategy for him to withdraw early, independent of his beliefs about L^i and for any asset price $P \in [\underline{P}, qD_2)$. Similarly, when the creditor observes $x^i_j > x^U \equiv \frac{F}{1-D_1/\underline{P}} + \epsilon$ and learns his bank's fundamentals above $\theta^U \equiv \frac{F}{1-D_1/\underline{P}}$, it is a dominant strategy for the creditor to wait, independent of his beliefs about L^i and for any asset price $P \in [\underline{P}, qD_2)$ (details in Appendix A.1).[29]

For an intermediate private signal $x^i_j \in [x^L, x^U]$, the representative creditor's optimal action depends on the asset price P and his beliefs about the aggregate withdrawal in his bank, L^i. When all other

[29]The upper dominance region is non-empty provided $\overline{\theta} > \frac{F}{1-D_1/\underline{P}}$. The sufficiently high upper bound is also in line with the global game literature with uniform and uninformative priors, e.g., Morris and Shin (1998) and Cong et al. (2020). We derive the precise conditions for the uninformative prior in the Online Appendix.

creditors in the bank i observe y^{-i} and take $x^*(y^{-i})$ as the critical signal, one can derive $L^i(\theta^i, x^*(y^{-i}))$ as the aggregate withdrawal faced by the bank i with a cash flow θ^i as follows (details in Appendix A.2):

$$L^i(\theta^i, x^*(y^{-i})) = \max\left\{\min\left\{\frac{x^*(y^{-i}) - \theta^i + \epsilon}{2\epsilon}, 1\right\}, 0\right\} = \begin{cases} 1 & \theta^i < x^*(y^{-i}) - \epsilon \\ \frac{x^*(y^{-i}) - \theta^i + \epsilon}{2\epsilon} & x^*(y^{-i}) - \epsilon \leq \theta^i \leq x^*(y^{-i}) + \epsilon \\ 0 & \theta^i > x^*(y^{-i}) + \epsilon. \end{cases} \tag{11}$$

$L^i(\theta^i, x^*(y^{-i}))$ increases in the critical signal $x^*(y^{-i})$ and decreases in the bank's cash flow θ^i. Based on his private signal, the representative creditor forms a posterior belief $\theta^i|_{x^i_j} \sim U(x^i_j - \epsilon, x^i_j + \epsilon)$ about his bank's fundamentals and expects the following aggregate withdrawal from his bank:

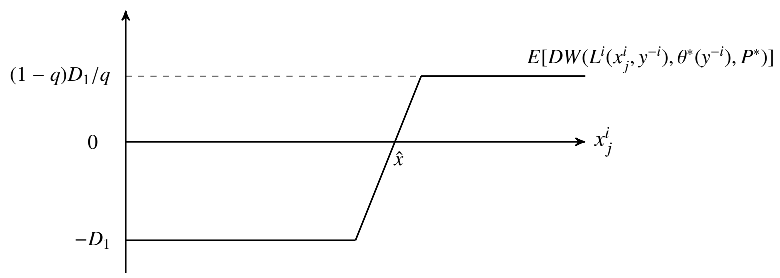
$$L^i(x^i_j, y^{-i}) = E\left[L^i(\theta^i, x^*(y^{-i}))\middle| x^i_j, y^{-i}\right] = \int_{x^i_j - \epsilon}^{x^i_j + \epsilon} L^i(\theta^i, x^*(y^{-i})) \cdot \frac{1}{2\epsilon} \cdot d\theta^i. \tag{12}$$

The creditor anticipates his bank to sell its assets at the price P^* if runs happen. Let $\theta^*(y^{-i})$ denote the critical fundamental below which the bank i fails. By condition (8), the creditor expects his bank with a fundamental $\theta^*(y^{-i})$ to fail if and only if

$$L^i > L^c(\theta^*(y^{-i}), P^*) = \frac{P^* \cdot (\theta^*(y^{-i}) - D_2)}{D_1 \cdot (\theta^*(y^{-i}) - P^*/q)}. \tag{13}$$

We now calculate the representative creditor's expected payoff difference conditional on his signals, $E\left[DW(L^i, \theta^i, P)\middle|(x^i_j, y^{-i})\right]$, which can be reformulated with (12) and (13) as $E\left[DW(L^i(x^i_j, y^{-i}), \theta^*(y^{-i}), P^*)\right]$. For a given y^{-i}, we illustrate in Figure 1 the expected payoff difference as a function of x^i_j. The representative creditor's best response to the other creditors' threshold strategy $x^*(y^{-i})$ is also a threshold strategy: to withdraw if and only if $x^i_j < \hat{x}(y^{-i}) = x^*(y^{-i}) - 2\epsilon \cdot [L^c(\theta^*(y^{-i}), P^*) - q]$ (details in Appendix B.2).

Figure 1: Payoff differences and the decision to withdraw



A symmetric threshold equilibrium requires $\hat{x}(y^{-i}) = x^*(y^{-i})$. So the equilibrium critical cash flow for the bank's failure, $\theta^*(y^{-i})$, must satisfy $L^c(\theta^*(y^{-i}), P^*) = q$, which implies

$$\theta^*(y^{-i}) = \frac{D_2 - D_1}{1 - qD_1/P^*}. \tag{14}$$

Furthermore, since the bank i with fundamentals $\theta^i = \theta^*(y^{-i})$ is on the verge of bankruptcy, the aggregate withdrawal $L^i(\theta^*(y^{-i}), x^*(y^{-i})) = \frac{x^*(y^{-i}) - \theta^*(y^{-i}) + \epsilon}{2\epsilon}$ must equal $L^c(\theta^*(y^{-i}), P^*)$, which further equals q in the equilibrium. This defines the equilibrium threshold signal:

$$x^*(y^{-i}) = \theta^*(y^{-i}) + (2q - 1)\epsilon. \tag{15}$$

Note that $\theta^*(y^{-i})$, in principle, differs from the critical fundamental $\hat{\theta}(y^{-i})$ that triggers runs, which is characterized by $L^i(\hat{\theta}(y^{-i}), x^*(y^{-i})) = 0$ and can be explicitly expressed as

$$\hat{\theta}(y^{-i}) \equiv x^*(y^{-i}) + \epsilon. \tag{16}$$

While a fundamental $\theta < \hat{\theta}(y^{-i})$ triggers runs and forces a bank to (partially) liquidate its asset, the bank will only fail when $\theta < \theta^*(y^{-i})$.[30] In line with the literature, we focus on $\theta^*(y^{-i})$ for the rest of the paper.

In a Perfect Bayesian equilibrium, asset buyers' belief about the critical signal must be consistent with the one associated with creditors' equilibrium strategy. The following condition must hold:

$$x^*(y^{-i}) = x^*. \tag{17}$$

Expressions (15), (16) and (17) imply that the equilibrium critical cash flows $\theta^*(y^{-i})$, $\hat{\theta}(y^{-i})$, and the equilibrium critical signal $x^*(y^{-i})$, if exist, are constants and do not depend on y^{-i}. Proposition 1 establishes the existence and the uniqueness of the equilibrium.

Proposition 1. *Without aggregate uncertainty, the game has a unique equilibrium: a wholesale creditor of a bank i's withdraws if and only if his private signal falls below x^*, which is unique and independent of y^{-i}. The asset buyers offer a price P^* to buy banks' assets, independent of the number of runs.*

Proof. See Appendix B.2 □

[30] It is straightforward to verify that $\hat{\theta}(y^{-i}) > \theta^*(y^{-i})$. When $\theta^i \in (\theta^*(y^{-i}), \hat{\theta}(y^{-i}))$, the bank survives at $t = 2$ despite the run and the partial asset liquidation at $t = 1$. $\hat{\theta}(y^{-i})$ converges to $\theta^*(y^{-i})$ when the noises of creditors' private signals approach zero.

Our model predicts two-way feedback between bank runs and distressed asset prices even in the absence of aggregate uncertainty. The equilibrium is unique and stable despite the two-way feedback. Intuitively, if creditors take a more optimistic strategy than the equilibrium one, they rationally anticipate the asset buyers to bid a lower price P^* according to (10). The lower P^*, however, implies an aggravated coordination problem by (14), which, in turn, restores the equilibrium threshold strategy.[31]

Our model differs from classic global-game-based bank run models such as Rochet and Vives (2004) and Vives (2014) because the liquidation value of banks' assets is endogenous: creditors in our model are forward-looking and understand the impact of their decisions to run on asset prices. The two-way feedback is related to Liu (2016) who studies the interaction between bank runs and rising interbank market rates when there is a limited supply of cash. We show, however, that given the lack of information on banks' solvency, runs depress asset prices even if the supply of cash is perfectly elastic.

3.2.2 Equilibrium with the aggregate uncertainty

We now characterize the equilibrium of the fully-fledged model with both idiosyncratic and aggregate risks. Different from the case without aggregate uncertainties, the equilibrium asset price P^*_M now depends on the number of runs M, which conveys information about the aggregate state s. In deciding whether to run, a creditor needs to anticipate the would-be asset price if his bank is forced into liquidation. To do that, the creditor needs to form rational expectations about the total withdrawals, not only in his own bank but also in the other bank.

For a sufficiently low or high y^{-i}, the representative creditor can tell whether the creditors in the other bank receive private signals that fall into the dominance regions. For example, when $y^{-i} < y^L \equiv x^L - \eta - \epsilon$, the creditor knows that all creditors in the bank $-i$ receive private signals lower than x^L and will withdraw. So the creditor expects $L^{-i}(x^i_j, y^{-i}) = 1$ independent of his private signal x^i_j. Similarly, for $y^{-i} > y^U \equiv x^U + \eta + \epsilon$, the representative creditor knows that all creditors in the bank $-i$ receive private signals in the upper dominance region, and expects $L^{-i}(x^i_j, y^{-i}) = 0$ independent of x^i_j.

[31]It is worth noticing that the information asymmetry in our model does not generate standard adverse selection problems where lower prices are associated with lower average qualities. Since banks in our model are forced into asset sales rather than strategically choose to do so, a lower asset price is associated with a *higher* average quality.

Lemma 2. *When observing $y^{-i} < y^L \equiv x^L - \eta - \epsilon$, the representative creditor expects $L^{-i}(x^i_j, y^{-i}) = 1$, independent of his private signal x^i_j. When observing $y^{-i} > y^U \equiv x^U + \eta + \epsilon$, the creditor holds a belief that $L^{-i}(x^i_j, y^{-i}) = 0$, independent of his private signal x^i_j.*

When observing $y^{-i} > y^U$, the representative creditor anticipates the asset price to be P^*_1 if his own bank is forced into liquidation because a run will not happen to the other bank. Similarly, upon observing $y^{-i} < y^L$, the representative creditor anticipates a run on the other bank, so that his own bank will have to sell assets for the price P^*_2 if a run happens. With such expectations of asset prices, we can derive the representative creditor's best response following the same procedure as in Section 3.2.1, except that the asset price is now given by equation (7) with $M = 1$ for $y^{-i} > y^U$, and with $M = 2$ for $y^{-i} < y^L$. The asset price (P^*_M), the bank i's creditors' threshold signal (x^*), and the bank's critical cash flows that trigger runs and failures ($\hat{\theta}_M$ and θ^*_M respectively), solve the following system of equations:

$$\begin{cases} P^*_M = \omega^B_M(\hat{\theta}_M) \cdot \frac{\theta_B + \hat{\theta}_M}{2} + \omega^G_M(\hat{\theta}_M) \cdot \frac{\theta_G + \hat{\theta}_M}{2} \\ x^*_M = \theta^*(y^{-i}) + (2q-1)\epsilon = x^*(y^{-i}) \\ \theta^*_M = \frac{D_2 - D_1}{1 - qD_1/P^*_M} = \theta^*(y^{-i}) \\ \hat{\theta}_M = x^*(y^{-i}) + \epsilon \equiv \hat{\theta}(y^{-i}) \end{cases} \quad \text{with} \quad M = \begin{cases} 1 & \text{if } y^{-i} > y^U \\ 2 & \text{if } y^{-i} < y^L. \end{cases} \tag{18}$$

We establish in Lemma 3 the existence and uniqueness of P^*_M, x^*_M, and θ^*_M, $M \in \{1, 2\}$, and show that $P^*_1 > P^*_2$, $x^*_1 < x^*_2$, and $\theta^*_1 < \theta^*_2$. Intuitively, the asset buyers form more pessimistic beliefs about the aggregate state s when observing more bank runs. As a result, they offer a lower price, which, in turn, increases the creditors' incentives to withdraw and the critical fundamental for banks to survive runs.

Lemma 3. *When observing $y^{-i} > y^U$, a wholesale creditor in a bank i withdraws if and only if his private signal falls below x^*_1 and expects his bank to liquidate its assets for a price P^*_1. The bank fails if and only if its cash flow is below θ^*_1, with P^*_1, x^*_1, and θ^*_1 being the unique solution to the system of equations (18) for $M = 1$. Similar, P^*_2, x^*_2, and θ^*_2 jointly solve the system of equations for $y^{-i} < y^L$ and $M = 2$. It holds that $P^*_1 > P^*_2$, $x^*_1 < x^*_2$, and $\theta^*_1 < \theta^*_2$.*

Proof. See Appendix B.3. ∎

Lemma 3 implies that the creditors' equilibrium threshold signal $x^*(y^{-i})$ must be a step function with only two possible values. Since the asset buyers can only observe the number of bank runs, $M \in \{1, 2\}$, and offer a corresponding price P_M^*, the creditors will choose their threshold strategy in expectation of one of the two equilibrium prices. We have established x_1^* and x_2^* as the two values of $x^*(y^{-i})$, for $y^{-i} > y^U$ and $y^{-i} < y^L$ respectively, so that the non-increasing function $x^*(y^{-i})$ must be bounded between x_1^* and x_2^*. Furthermore, there must exist a point of discontinuity $\hat{y} \in [y^L, y^U]$ such that $x^*(y^{-i}) = x_1^*$ for $y^{-i} \geq \hat{y}$, and $x^*(y^{-i}) = x_2^*$ for $y^{-i} < \hat{y}$. As y^{-i} rises above the cutoff value $\hat{y}$, the creditor's expected asset price changes from P_2^* to P_1^*.

To fully characterize the creditors' equilibrium strategy, we now derive $x^*(y^{-i})$ for $y^{-i} \in [y^L, y^U]$, in which case creditors in the bank $-i$ are no longer seen to have a dominant action. We establish in Proposition 2 that a unique equilibrium exists for the game with aggregate uncertainty and illustrate the creditors' equilibrium threshold signal in Figure 2.

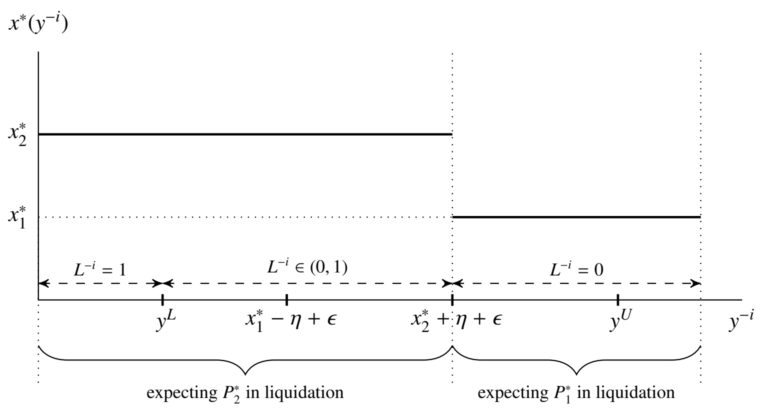
Proposition 2. *With aggregate uncertainty, the game has a unique equilibrium: a wholesale creditor of a bank i's withdraws if and only if his private signal falls below $x^*(y^{-i})$, with*

$$x^*(y^{-i}) = \begin{cases} x_2^* & y^{-i} < x_2^* + \eta + \epsilon \\ x_1^* & y^{-i} \geq x_2^* + \eta + \epsilon. \end{cases} \tag{19}$$

The asset buyers offer price P_M^* when observing M bank runs, $M = 1, 2$.

Proof. See Appendix B.4. □

Figure 2: The equilibrium threshold singal



To establish Proposition 2, we first derive the representative creditor's expectation of L^{-i} for those relatively high and low values of y^{-i} in the intermediate range. In particular, the creditor expects $L^{-i}(x_j^i, y^{-i}) = 0$ when observing $y^{-i} > x_2^* + \eta + \epsilon$, independent of his private signal. Indeed, such an observation reveals that all creditors in the bank $-i$ observe private signals no less than x_2^*. Therefore, none of those creditors will withdraw even if they follow the threshold signal x_2^*. The representative creditor then expects $L^{-i} = 0$ and sets his threshold signal to $x^*(y^{-i}) = x_1^*$. Similarly, when observing $y^{-i} < x_1^* - \eta + \epsilon$, the creditor knows that $\theta^{-i} < x_1^* + \epsilon$ must be true, and that a positive mass of creditors in the bank $-i$ will withdraw even if they follow the threshold signal x_1^*. As a result, the representative creditor expects $L^{-i} > 0$ and sets his threshold signal to x_2^*. Therefore, we know that the point of discontinuity must be in the range of $[x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon]$.

We sketch here the proof for why the point of discontinuity must be $x_2^* + \eta + \epsilon$, and provide the full proof by contradiction in Appendix B.4. Suppose that there is an alternative discontinuity point $\hat{y} < x_2^* + \eta + \epsilon$. When observing a $y^{-i} \in (\hat{y}, x_2^* + \eta + \epsilon)$, the bank i's creditors would follow a threshold signal x_1^*, which must be rationalized by the expectation that their bank — when forced into liquidation — will sell its assets for the price P_1^*. In other words, the creditors must not expect a run to occur to the bank $-i$. We can also make the following two observations. First, when a representative creditor from the bank i expects runs on his own bank (e.g., when observing $x_j^i = x_1^*$), the creditor cannot exclude the possibility that the bank $-i$'s creditors run according to the threshold x_2^*. This is because the representative creditor knows that $\theta^i < x_1^* + \epsilon$ as long as he does not receive the lowest private signal in his bank. Therefore, he cannot exclude the possibility that the bank $-i$'s creditors observe a signal $y^i < x_1^* - \eta + \epsilon$, which implies those creditors following the threshold signal x_2^*. Second, since the representative creditor also receives the signal $y^{-i} < x_2^* + \eta + \epsilon$, he perceives with a positive probability that $\theta^{-i} < x_2^* + \epsilon$, in which case those lowest private signals in the bank $-i$ would be lower than x_2^*. Combining these two observations, the representative creditor cannot exclude the possibility that a run also happens to the bank $-i$. Such a possibility, however, contradicts the expectation of the price P_1^*.

Our fully-fledged model with two groups of creditors and two-dimensional signals features a unique equilibrium despite the aggregate uncertainty. To gain some intuition for the uniqueness, note that the signal y^{-i} received by a bank i's creditors is about the other bank's fundamentals, which differs from the classic models where both private and public signals are about the same bank's cash flow.[32] Therefore, on top of the coordination game within each bank, the two groups of creditors in our model also play a cross-bank coordination game on the number of runs. To form rational beliefs about whether a run would happen to the bank $-i$, creditors in the bank i need to rely on both their private signals (to Bayesian update the threshold signal $x^*(y^i)$ adopted by the bank $-i$'s creditors) and the signal y^{-i} (to Bayesian update the range of private signals received by the bank $-i$'s creditors). Since each creditor has its own posterior about $x^*(y^i)$, there is a lack of common knowledge on whether a run will happen to the bank $-i$, which results in the uniqueness of the equilibrium.

Figure 3 gives a complete characterization of how the equilibrium outcome depends on the banks' fundamentals. Since the relative magnitude of η and ϵ does not affect any main results, we focus on a limiting case where $\epsilon \to 0$ and $\eta \to 0$.[33] As the uncertainties about both banks' fundamentals diminish, we have the critical cash flows $\theta_1^* = lim_{\epsilon\to 0} x_1^* = lim_{\epsilon\to 0}\hat{\theta}_1$ and $\theta_2^* = lim_{\epsilon\to 0} x_2^* = lim_{\epsilon\to 0}\hat{\theta}_2$, which make a unique partition of the set of bank fundamentals as shown in Figure 3.

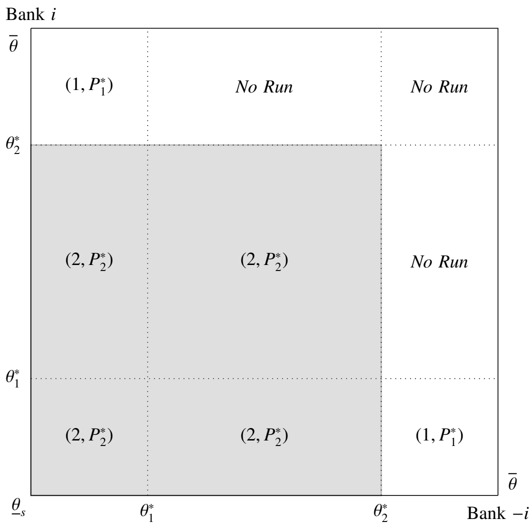
Assumption. *In the remaining of the paper, we focus on $\epsilon \to 0$ and $\eta \to 0$ unless otherwise stated.*

Our model features strategic complementarities between creditors from the two banks. Observing a low signal y^{-i} about the bank $-i$'s cash flow, creditors in the bank i know that the bank $-i$'s creditors are likely to run. Consequently, the bank i's creditors expect their bank to face the price P_2^* if it is forced into liquidation, which increases their incentives to withdraw. This creates financial contagion, since the bank $-i$'s weak fundamentals reduce the bank i's likelihood of survival. Note that a bank's liquidation depresses the asset price and generates contagion even though the cash is in perfectly elastic

[32]In models such as Morris and Shin (2001) and Hellwig (2002), the global-games refinement could fail to predict a unique equilibrium as the public signal becomes increasingly precise as compared to the private signals, and the players again coordinate on the public signal. In contrast, the uniqueness of the equilibrium of our model does not rely on the relative magnitude between ϵ and η because creditors do not observe a common signal about their own bank's fundamentals.

[33]For its tractability, it is common to study the limiting case in the literature: e.g., see Liu and Mello (2011).

Figure 3: Bank fundamentals and the equilibrium outcomes



supply. Rather, an informational channel is at play: when observing a bank run, asset buyers form more pessimistic beliefs about s and lower their bids, which in turn results in strategic complementarity between creditors across banks.[34] The bank i's exposure to the risk of contagion can be quantified accordingly as the increase in its probability of failure when the other bank's fundamentals weaken: $Prob(Bank\ \ i\ \ fails \mid \theta^{-i} < \theta_2^*) - Prob(Bank\ \ i\ \ fails \mid \theta^{-i} \geq \theta_2^*) = Prob(\theta_1^* < \theta^i < \theta_2^*)$.

When both banks' fundamentals fall below θ_2^*, both of them will fail because of equilibrium bank runs. In Figure 3, we depict the regions of fundamentals where both banks fail in grey, a scenario that we dub as '*systemic bank failures*' or '*a systemic crisis*'.

Corollary 1. *Systemic bank failures happen when both banks' fundamentals are below θ_2^*.*

Corollary 1 points out that a systemic crisis can emerge in a laissez-faire market, even if the fundamentals are strong, e.g., both banks' cash flows only marginally below θ_2^*. From an ex-ante perspective, the probability of such a crisis can be computed as $\text{SYS}(\theta_2^*) \equiv Prob(\theta^i \leq \theta_2^*, \theta^{-i} \leq \theta_2^*) = \alpha \cdot \left(\frac{\theta_2^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G}\right)^2 + (1-\alpha) \cdot \left(\frac{\theta_2^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B}\right)^2$. Since systemic bank failures are particularly detrimental for its costly

[34] This aspect of our model is in line with the literature of information contagion, e.g., Acharya and Thakor (2011) and Oh (2012). Also in a global-games framework, Ahnert and Bertsch (2020) model how a bank failure serves as a 'wake-up call' and triggers information acquisition about other banks' exposure to the aggregate risk. In contrast, we emphasize how the information constraint faced by asset buyers leads to liquidation losses, and how such losses trigger runs from forward-looking creditors who understand the price impact of bank asset liquidation.

resolution and negative impacts on the broader economy,[35] we consider liquidity interventions that would reduce the range of fundamentals where systemic bank failures can happen.

3.3 Policy Intervention

We now analyze a risk-neutral regulator's options for interventions when she strives to simultaneously achieve the following three desirable goals: (1) to mitigate systemic risks, (2) to make no loss in the intervention, and (3) to save no insolvent bank. The regulator tries to avoid making losses, as such losses would constitute a transfer to banks' claim holders and make the policy politically unpopular. It is also understood that the regulator should avoid using liquidity interventions to tackle insolvency problems since assisting insolvent banks would create zombie lending and weaken ex-ante discipline.

To highlight the importance of information friction, we first analyze a case where the regulator holds information on banks' solvency. With such information, the regulator can offer to purchase assets *only from solvent banks* for a price $P \geq qD_2$, which will costlessly reduce the critical cash flow for a bank's survival to D_2. In other words, runs are confined to the lower dominance region and occur only to the fundamentally insolvent banks.[36] Intuitively, since a solvent bank's assets are no longer pooled with those of insolvent banks', the increase in the price of the former will remove the first-mover advantage for creditors who run on such a bank. As a result, knowing the regulator's offer, no creditor would run a solvent bank in the first place. The policy simultaneously achieves all three desirable goals. This result echos how deposit insurance works in Diamond and Dybvig (1983): once a bank is known free of insolvency risks, liquidity support can costlessly eliminate bank runs.

Once the regulator is informationally constrained, the ideal allocation $\theta^* = D_2$ can no longer be achieved. In particular, an ex-post arrangement, in which the regulator chooses a supporting asset price $P_{A,M}$ after observing M bank runs, cannot improve on the market allocation.[37] Intuitively, when the

[35] A systemic banking crisis can threaten essential payment services and cause system-wide disintermediation, e.g., a credit crunch and the loss of soft information on borrowers. See Laeven et al. (2010) for the real and fiscal costs of systemic crises.

[36] To see this, note that even if all wholesale creditors withdraw from a solvent bank at $t = 1$, the bank only needs to sell a $\lambda = \frac{D_1}{P}$ fraction of its assets and can stay solvent at $t = 2$, because $(1 - \lambda)\,\theta \geq \left(1 - \frac{D_1}{qD_2}\right) \cdot D_2 = F$.

[37] In our view, the ex-post intervention is more in line with the traditional last resort policy. While the traditional policy also recommends the readiness of central bank liquidity support, the terms of lending are not specified beforehand. By contrast, with the committed liquidity support that we will propose, the terms of lending/the price for asset purchase should be specified before, and independent of, the occurrence of runs.

regulator intervenes by purchasing banks' assets after observing that bank runs have happened, she cannot offer better prices than what private market participants are willing to pay — at least not without incurring expected losses. The reason is that other than the coordination failure, the only friction in our model is the lack of information on banks' asset qualities, which applies to both the private asset buyers and the regulator. If the regulator only moves after observing the number of runs M, the efficiency of the policy intervention will be bounded by the market allocation.

By contrast, an ex-ante arrangement — with mutual commitments from the regulator and banks — can still reduce the systemic risk.[38] Formally, we assume that the regulator intervenes by committing to purchasing banks' assets for a price $P_A \geq P_2^*$ in case any bank is forced into liquidation at $t = 1$, and that the banks commit to raising liquidity by selling their assets to the regulator when experiencing runs.[39] The support price P_A is *pre-emptive*, in the sense that it is set up before the realization of the aggregate and idiosyncratic risks, and therefore before the observation of any actual bank run. In line with the three policy goals, we assume that the regulator solves the following program:

$$\min_{P_A \geq P_2^*} \quad \mathrm{SYS}(P_A)$$

$$\text{s.t.} \quad V_A(P_A) \geq 0 \tag{20}$$

$$\theta^*(P_A) \geq D_2. \tag{21}$$

The regulator's objective is to minimize the probability of a systemic crisis $\mathrm{SYS}(P_A)$ by setting the price support P_A. With $V_A(P_A)$ denoting the regulator's expected payoff from the intervention, constraint (20) requires the regulator to make no losses or net transfers to banks. With $\theta^*(P_A)$ denoting the equilibrium threshold cash flow under the regulator's intervention, constraint (21) states that the liquidity support should not save any insolvent bank.[40]

[38]As we will show in Section 4.3, an ex-ante liquidity arrangement with the regulator's unilateral commitment cannot achieve all three goals simultaneously, and in particular, requires the regulator to bear losses from the intervention.

[39]We analyze in Section 4.4 the case where banks can raise liquidity from a regulator by borrowing against their assets as collateral. We show that committed liquidity support in the form of purchasing banks' assets can still be preferred by the regulator to collateralized lending when she also pre-commits to the terms of lending in expectation of runs.

[40]Our modeling of the regulator's objective function is in line with the theory literature, e.g., Vives (2014) and Morris and Shin (2016). Both papers distinguish between a bank's liquidity and insolvency risks and assume the regulator's objective is to contain the risks. In our paper, the liquidity intervention aims to reduce the risk of systemic crises by preventing contagious bank runs while making no attempt to limit insolvency.

Observing a committed price $P_A \geq P_2^*$, creditors no longer need to consider the price impact of bank runs. The bank run game can be solved as in Rochet and Vives (2004). In the limiting case where $\epsilon \to 0$ and $\eta \to 0$, the creditors' equilibrium threshold signal and the banks' critical fundamentals that trigger runs and failures converge: $x^*(P_A) = \hat{\theta}(P_A) = \theta^*(P_A) = \frac{D_2 - D_1}{1 - qD_1/P_A}$. The risk of systemic bank failures under the intervention becomes $\text{SYS}(P_A) = \alpha \cdot \left(\frac{\theta^*(P_A) - \underline{\theta}_G}{\bar{\theta} - \underline{\theta}_G}\right)^2 + (1-\alpha) \cdot \left(\frac{\theta^*(P_A) - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right)^2$.

When choosing a P_A at $t = 0$, the regulator needs to calculate her expected payoff $V_A(P_A)$ by forming rational expectations about the possible number of runs and their associated probabilities.

$$V_A(P_A) = \sum_{s=G,B} Pr(s) \cdot \left(\sum_{M=1}^{2} Pr(\theta < \theta^*(P_A)|s)^M \cdot Pr(\theta > \theta^*(P_A)|s)^{2-M} \cdot C_2^M \cdot M \cdot \frac{D_1}{P_A} \cdot \pi(P_A|s) \right). \tag{22}$$

Here $\pi(P_A|s) = \frac{\underline{\theta}_s + \theta^*(P_A)}{2} - P_A$ denotes the regulator's expected payoff from purchasing a bank's one unit of assets for the price P_A in a given aggregate state s. In the equilibrium, a bank with a cash flow $\theta \in [\underline{\theta}_s, \theta^*(P_A))$ will experience runs and will have to sell a $\frac{D_1}{P_A}$ proportion of its portfolio to meet a D_1 amount of withdrawals. Therefore, $M \cdot \frac{D_1}{P_A} \cdot \pi(P_A|s)$ denotes the regulator's total payoff of purchasing assets from M banks, $M \in \{1, 2\}$. Since $Pr(\theta < \theta^*(P_A)|s)^M \cdot Pr(\theta > \theta^*(P_A)|s)^{2-M}$ is the probability that M and only M banks are forced into liquidation in a given aggregate state s, the term in the parentheses denotes the regulator's expected payoff in the state s. We characterize the solution of the regulator's program in Proposition 3.

Proposition 3. *There exists a unique $P_A^* \in (P_2^*, P_1^*)$ for $V_A(P_A^*) = 0$ such that the regulator can break even by offering the price. With both $SYS(P_A)$ and $V_A(P_A)$ monotonically decreasing in P_A, the regulator optimally commits to purchasing banks' assets for the break-even P_A^* and reduces the risk of systemic bank failures from $SYS(\theta_2^*)$ to $SYS(P_A^*)$.*

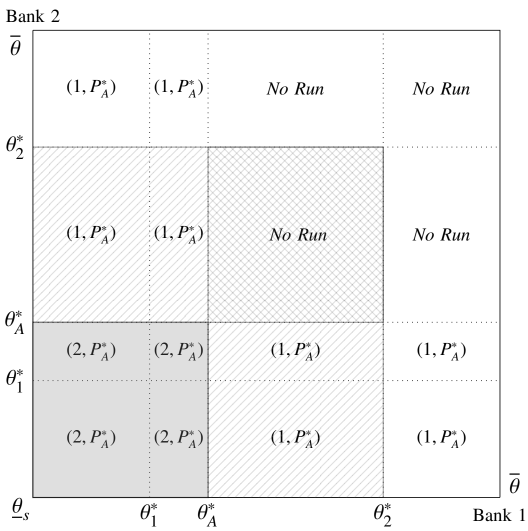
Proof. See Appendix B.5. □

Offering the price $P_A^* \in (P_2^*, P_1^*)$ at $t = 0$ allows the regulator to *break even across possible posterior beliefs of the state* s, whereas the private asset buyers who bid ex post at $t = 1$ have to *break even within a given belief of the state* s. To highlight this difference, we define $\Pi_M(P)$ as the expected payoff from purchasing one unit of bank assets for a *given* price $P \in (P_2^*, qD_2)$ in the contingency of M bank runs. Denoting the corresponding critical cash flow by $\theta^*(P)$, we have $\Pi_M(P) = \omega_M^B(\theta^*(P)) \cdot \pi(P|B) + \omega_M^G(\theta^*(P)) \cdot \pi(P|G)$. Here, $\omega_M^s(\theta^*(P))$ is the posterior belief about the state s upon the observation of M bank runs when the bank's assets are sold for a price P. One can verify that $\Pi_M(P)$ strictly decreases in P, and that the asset buyers' equilibrium bid P_M^* satisfies $\Pi_M(P_M^*) = 0$, for $M \in \{1, 2\}$. In other words, the buyers always break even for a *realized* M and the associated belief $\omega_M^s(P_M^*)$ about the state s. The regulator, on the other hand, can offer the price P_A^* to break even *across different numbers of runs* and thereby across possible posterior beliefs about the aggregate state s. To see this, we use the definition of $\Pi_M(P)$ to re-arrange $V_A(P_A)$ into the following form:

$$V_A(P_A) = \sum_{M=1}^{2} \left(\sum_{s=G,B} Pr(s) \cdot Pr(\theta < \theta^*(P_A)|s)^M \cdot Pr(\theta > \theta^*(P_A)|s)^{2-M} \right) \cdot C_2^M \cdot M \cdot \frac{D_1}{P_A} \cdot \Pi_M(P_A). \tag{23}$$

Importantly, the regulator does not require $\Pi_M(P_A^*) = 0$ but instead $V_A(P_A^*) = 0$. In fact, the regulator expects to make losses in the contingency of $M = 2$ and to profit in the contingency of $M = 1$. Since $\Pi_1(P_2^*) > 0$ and $\Pi_2(P_1^*) < 0$, we know that $V_A(P_A) > 0$ for $P_A = P_2^*$ and that $V_A(P_A) < 0$ for $P_A = P_1^*$. Since $V_A(P_A)$ monotonically and continuously decreases in P_A, there exists a unique $P_A^* \in (P_2^*, P_1^*)$ that allows the regulator to break even in expectation.

Figure 4: The equilibrium outcomes under the committed liquidity support



To appreciate the stability effect, suppose both banks' fundamentals are marginally below θ_2^*. We know that the equilibrium outcome in a laissez-faire market is that both banks fail and the prevailing asset price drops to P_2^*. Under the policy intervention, however, systemic bank failures will no longer happen. Knowing that their banks can sell assets to the regulator at a pre-specified price $P_A^* > P_2^*$, the creditors can no longer rationalize x_2^* as their threshold signal. In this case, promoting financial stability does not involve the regulator purchasing banks' assets. The regulator improves the allocation by offering a support price to break down the feedback loop between contagious bank runs and asset sales. The regulator can also reduce the risk of systemic bank failures when she actually purchases assets from the banks. In particular, the regulator will prevent systemic crisis when one bank's cash flow is below $\theta_A^* = \theta^*(P_A^*)$ while the other's belongs to interval $[\theta_A^*, \theta_2^*]$. The regulator saves the bank that has relatively strong fundamentals which would otherwise fail due to contagion. We illustrate the two cases by the cross-hatched and single-hatched areas in Figure 4, respectively.

4 Further Policy Discussion

We now extend the policy discussion by showing that the regulator can induce banks' voluntary participation in the upfront liquidity arrangement (Section 4.1), discussing the regulator's commitment power (Section 4.2), analyzing how the possibility for the regulator to bear losses from her intervention affects the effectiveness of the intervention (Section 4.3), and comparing the proposed intervention to collateralized lending schemes (Section 4.4). We collect the relevant proofs in the Online Appendix.

4.1 Banks' voluntary participation in the upfront liquidity arrangement

Under the mutually committed liquidity support, it is assumed that banks commit to raising liquidity by selling their assets to the regulator even if private asset buyers may offer more. Interestingly, the regulator does not have to command banks to participate in the upfront liquidity arrangement. We show that the regulator can induce banks' voluntary participation.

We analyze banks' incentives for voluntary participation with the following extension of our model. At $t = 0$, the regulator offers the two banks a price $P_A \in (P_2^*, P_1^*)$ at which it commits to purchasing banks' assets at $t = 1$ if runs happen. Banks' managements then decide whether to take the regulator's offer. Once a bank joins the arrangement, it is obliged to sell its assets only to the regulator (e.g., assets are encumbered for this purpose). Otherwise, the bank sells its assets to private asset buyers. The banks' managements simultaneously decide whether to accept the regulator's offer. At $t = 1$, both the private asset buyers and banks' creditors observe P_A and which bank has joined the arrangement. For simplicity, a bank's management is assumed to receive a constant compensation b conditional on his bank being afloat and decides whether to take the regulator's offer to maximize his expected compensation.

We establish in Proposition 4 a sufficient condition under which the regulator can induce banks' voluntary participation. Intuitively, when only one bank enters the agreement and receives the regulatory price support, observing a run on such a bank makes the private asset buyers particularly pessimistic about the aggregate state.[41] As a result, the bank that chooses not to participate in the program will face an even lower asset price when forced into liquidation. This leads to an increased critical cash flow for the latter bank to survive and gives its management incentives to join the program in the first place. In other words, the bank that participates in the program exerts negative externalities on the bank that chooses not to; and both banks participating in the program can emerge as a Nash equilibrium.

Proposition 4. *There exists a unique critical $P_A^C \in (P_2^*, P_1^*)$, such that, when the regulator offers a price $P_A > P_A^C$, it is a Nash equilibrium for both banks to accept the offer. When $V_A(P_A^C) > 0$, we have $P_A^* > P_A^C$, and the regulator can induce banks' voluntary participation in the upfront liquidity arrangement with the supporting price P_A^*.*

4.2 The regulator's commitment

While the regulator can induce voluntary participation from banks, the regulator's commitment is essential. In fact, in a crisis, it is usually harder for a regulator to refrain from intervening than to provide rescues. The regulator's tendency to provide rescues is the driving mechanism of the collective moral hazard problem highlighted by Farhi and Tirole (2012). The tendency is empirically documented by Brown and Dinç (2011): the regulators tend to be accommodative with distressed banks when the

[41] In terms of the policy intervention changing the informational environment, this result is related to Cong et al. (2020). In a dynamic global-games framework, the authors suggest that the initial policy intervention affects the informational environment of the subsequent interventions.

banking sector as a whole is undercapitalized.[42] To signal her commitment to providing liquidity support, the regulator can deploy commitment devices, such as creating financial stability funds.[43] The regulator can also achieve commitment with rule-based interventions by publishing the policy rules before any crisis.[44] Alternatively, the regulator can secure the commitment with legal obligations. For example, in providing its committed liquidity facility, the Reserve Bank of Australia enters binding agreements with participating banks that require the central bank to inject liquidity when fundings are needed.

Regulators can also be in positions to offer higher prices than private entities can do. To start with, regulators have different objective functions as compared to private entities. Negative externalities from systemic bank failures are not taken into account by private asset buyers but are major concerns to regulators.[45] As we will show in the next section, a regulator may well want to avoid a systemic crisis at some monetary losses. Moreover, regulators such as central banks are not subject to the same stark bankruptcy constraint of private institutions and can sit on temporary losses. Similarly, central banks do not face pressure to lower the bid for banks' assets to increase the financial returns from their interventions, which may not be the case if the committed liquidity support is privately provided.

The commitment pledged by the regulator can raise the concern of potential moral hazard from banks. In fact, such moral hazard problems and information constraints are intertwined: the regulator's inability to distinguish an illiquid bank from an insolvent one can lead to blind intervention that will benefit insolvent banks and compromise market discipline. Since any intervention that preserves some equity value in the rescue of a troubled bank would raise such a concern, a more pressing question is whether the proposed ex-ante intervention generates a less severe moral hazard problem than the traditional (ex-post) interventions do. This can be true because the committed liquidity support is limited to containing systemic crises and still allows banks with $\theta \in (D_2, \theta_A^*)$ to fail. Those banks are still

[42] In the context of interbank lending with insolvency risk spillover, Bernard et al. (2017) highlight that it can be hard for the regulator to commit to no intervention when the banking system features close interconnectedness.

[43] A similar observation can be made about deposit insurance. Most countries require banks to pay deposit insurance premium ex-ante into a deposit insurance fund, which adds to the credibility of deposit insurance schemes.

[44] The Bank of England's asset pre-positioning program provides a case in point. Bank of England (2019) publishes details of the haircuts for different assets when they are pre-positioned with the central bank as collateral for emergency funding.

[45] Also highlighting externalities, David and Lehar (2019) show that private institutions may not provide efficient resolution since they do not internalize positive externalities of the successful restructuring of a distressed bank's debt.

penalized for their mismanagement of risks, which would dampen risk-taking incentives. In contrast, the classic LoLR policy, as suggested by Bagehot (1873), aims to avoid inefficient liquidation of individual banks — aiming to save any bank with $\theta > D_2$, an ambitious goal that entails more bank rescues.

4.3 The possibility of public bail-outs

The policy intervention analyzed so far assumes no public bailouts, in the sense that the regulator incurs no expected loss and makes no net transfer to banks' claim holders. However, public authorities can make net losses in their interventions (e.g., see Laeven et al. (2010) for the fiscal costs of banking crises). So we relax this assumption to allow for (expected) losses from policy interventions.

We replace constraint (20) with $V_A(P_A) \geq -\overline{V}$, where $\overline{V} \geq 0$ indicates the regulator's loss-bearing capacity. When the regulator can bear a loss up to $\overline{V} > 0$, she can optimally commit to purchasing banks' assets at a price $P_A^{**} > P_A^*$ and reduce the risk of systemic failures from $\text{SYS}(P_A^*)$ to $\text{SYS}(P_A^{**})$. Since both the probability of systemic bank failures $\text{SYS}(P_A)$ and the regulator's expected payoff $V_A(P_A)$ decrease in P_A, the regulator can commit to a higher price P_A^{**} such that $V_A(P_A^{**}) = -\overline{V}$ once she is allowed to make a loss up to $\overline{V}$.[46]

The possibility for the regulator to take expected losses also broadens the set of policy tools that can be used to promote financial stability. In particular, it is now possible for the regulator to unilaterally commit to purchasing banks' assets for a price $P_U \geq P_2^*$ independently of the number of runs. When $P_U \geq P_1^*$, creditors expect banks to sell their assets only to the regulator in runs, and the analysis resembles that in Section 3.3. The regulator's expected payoff $V_U(P_U)$ is identical to expression (22) for $P_U \geq P_1^*$, i.e., $V_U(P_U) = V_A(P_U)$. Complications arise for $P_U \in [P_2^*, P_1^*)$. A creditor now expects his bank to sell its assets to the regulator at $P_U \geq P_2^*$ only when both banks experience runs. In contrast, the creditor expects his bank to sell its assets to the private asset buyers for $P_1^* > P_U$ when his bank is the only one that experiences a run. The unilaterally committed liquidity support, therefore, can co-exist with the private secondary market for banks' assets. When offering a $P_U \in [P_2^*, P_1^*)$, the regulator's

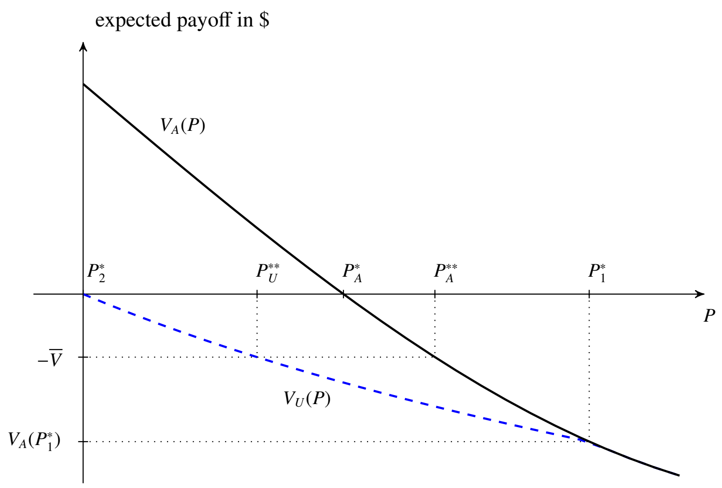
[46]In fact, when the loss-bear capacity is sufficiently large ($\overline{V} \geq -V_A(qD_2)$), constraint (21) becomes binding. The regulator can offer a price of qD_2 to completely eliminate funding liquidity risks. Since this corner case trivializes the analysis, we focus on the interior case $0 < \overline{V} < -V_A(qD_2)$.

expected payoff becomes $V_U(P_U) = \sum_{s=G,B} Pr(s) \cdot \left(Pr(\theta < \theta^*(P_U)|s)^2 \cdot 2 \cdot \frac{D_1}{P_U} \cdot \pi(P_U|s)\right)$, and any price higher than P_2^* implies expected losses.[47] The regulator, again, aims to minimize the risk of systemic bank failures $\text{SYS}(P_U) = \alpha \cdot \left(\frac{\theta^*(P_U)-\underline{\theta}_G}{\bar{\theta}-\underline{\theta}_G}\right)^2 + (1-\alpha) \cdot \left(\frac{\theta^*(P_U)-\underline{\theta}_B}{\bar{\theta}-\underline{\theta}_B}\right)^2$, subject to the constraint $V_U(P_U) \geq -\overline{V}$. For $V_U(P_U)$ decreases in P_U, the regulator optimally commits to purchasing banks' assets for a price P_U^{**} that satisfies $V_U(P_U^{**}) = -\overline{V}$.

Given a loss-bearing capacity $\overline{V} \geq 0$, we compare the stability effects of the arrangement with the regulator's unilateral commitment and that with the mutual commitments from the regulator and banks. An arrangement achieves greater stability if it allows the regulator to offer a higher price for banks' assets or, equivalently, if it generates a higher regulator's payoff for the same price that she offers. We establish in Proposition 5 that the ex-ante liquidity support with the mutual commitments dominates that with the regulator's unilateral commitment. Figure 5 illustrates the comparison.

Proposition 5. *When the regulator's loss-bearing capacity $\overline{V} < -V_A(P_1^*) = -V_U(P_1^*)$, the upfront liquidity arrangement with the mutual commitments outperforms that with the regulator's unilateral commitment in terms of containing the systemic risk. The two arrangements are equally effective otherwise.*

Figure 5: The regulator's payoffs under mutual vs. unilateral commitments



When the regulator cannot make any expected loss, i.e., $\overline{V} = 0$, the ex-ante liquidity support with the mutual commitments strictly dominates that with the regulator's unilateral commitment, since the

[47]In the equilibrium, creditors' critical signal is as follows: $x^*(y^{-i}, P_U) = x^*(P_U)$ when $y^{-i} < x^*(P_U)+\eta+\epsilon$, and $x^*(y^{-i}, P_U) = x_1^*$ otherwise. A bank survives if its cash flow exceeds $\theta^*(P_U) = \frac{D_2-D_1}{1-qD_1/P_U}$. The proof is provided in the Online Appendix.

regulator can only offer P_2^* under the latter arrangement. The former arrangement also outperforms the latter when the regulator has a loss-bearing capacity $\overline{V} \in (0, -V_A(P_1^*))$. For such a moderate loss-bearing capacity, the regulator can offer a price $P_A^{**} \in (P_A^*, P_1^*)$ under the mutual commitments. In contrast, she can only offer a lower price $P_U^{**} \in (P_2^*, P_A^{**})$ with her unilateral commitment. Intuitively, the regulator expects that she will purchase banks' assets with a lower quality under the unilateral commitment, which in turn, suggesting that she will offer a lower price P_U^{**}. Lastly, when the regulator can make large losses, i.e., $\overline{V} > -V_A(P_1^*)$, she would commit to a price greater than P_1^* and receive the same expected payoff under both arrangements. Consequently, the two arrangements become equally effective.

We believe it is both reasonable and realistic to demand commitments from banks. From a normative point of view, banks' access to public liquidity support should be commensurate with their regulatory obligations. Banks should take regulatory obligations (e.g., entering a binding agreement with the regulator) in exchange for public liquidity support during a crisis. Furthermore, it is feasible for banks to make such commitments. For instance, banks can put assets in encumbrance and reserve these assets exclusively for raising liquidity from central banks. Alternatively, banks can pay for public liquidity support upfront. For example, the committed liquidity facility of the Reserve Bank of Australia requires banks participating in the program to make ex-ante payments for the central bank's liquidity insurance.

4.4 Committed liquidity support: asset purchase vs. collateralized lending

Having focused on an ex-ante asset purchase arrangement to demonstrate the benefits of committed liquidity support, we now compare such an intervention to collateralized lending. In particular, we consider an ex-ante liquidity intervention where the regulator commits to lending to banks at a pre-specified interest rate. When lending happens at $t = 1$, the information-constrained regulator sets a unified interest rate r and requires a borrowing bank to post up to $\beta \in (0, 1)$ fraction of its assets as collateral. The bank repays the regulator $1 + r$ on the amount borrowed if it is solvent at $t = 2$. If the bank fails, the regulator seizes the collateral. For a fair comparison, we further assume that $\beta = \frac{D_1}{P_A^*}$, so that the bank has at its disposal the same amount of assets either for sale or for use as collateral.

In the limiting case where $\epsilon \to 0$ and $\eta \to 0$, the two types of interventions have the same impact on financial stability. Indeed, any bank that requires liquidity assistance from the regulator at $t = 1$ would fail at $t = 2$ because the critical cash flow $\hat{\theta}$ that triggers a run (i.e., $L(\hat{\theta}, x^*) = 0$) converges to the critical cash flow θ^* that triggers a bank failure (i.e., $L(\theta^*, x^*) = q$). The regulator would seize the β units of collateral at $t = 2$ if she lends to the bank and would acquire the same amount of assets if she purchases assets from the bank. The level of systemic risk achievable under the two types of interventions will be the same because the regulator expects the same payoff.

The two types of securities make a difference when we *move away from the limiting case*. We now derive the interest rate r^* that minimizes the risk of systemic bank failures when the regulator pre-commits to lending to banks, and compare its stability effects with that when the regulator pre-commits to purchasing assets at a price P_A^* in Proposition 6.

Proposition 6. *There exists a critical $\hat{q} \in (\frac{1}{2}, 1)$, such that for $q > \hat{q}$, committing to purchasing banks' assets on sale at a price P_A^* achieves a strictly lower risk of systemic bank failures than committing to lending to banks at an interest rate r^* when $\epsilon > 0$.*

Provided that the committed lending and the committed asset purchase achieve the same critical cash flow, lending is preferable to the regulator if $\theta \in (\hat{\theta} - 2\epsilon, \theta^*)$, whereas purchasing bank assets is preferred when $\theta \in [\theta^*, \hat{\theta})$. To gain the intuition of the result, note that a partial run happens for those fundamentals, i.e., $L \in (0, 1)$. In the former case, the bank fails despite the intervention since $\theta < \theta^*$. The regulator would seize a $\frac{D_1}{P_A^*}$ amount of the bank's assets at $t = 2$ due to the collateralization of her lending. Instead, she would only acquire a $\frac{L \cdot D_1}{P_A^*}$ amount of the bank's assets at $t = 1$ when committing to purchasing the bank's assets. In the latter case, both the asset purchase and the lending save the bank since $\theta \geq \theta^*$, but purchasing the bank's assets allows the regulator to access the upside of the cash flow, while her return is bounded by the pre-specified interest rate when she lends to the bank. For q increases to 1, the length of interval $(\hat{\theta} - 2\epsilon, \theta^*)$ diminishes to zero while the length of interval $[\theta^*, \hat{\theta})$ expands to 2ϵ. Thus, for a sufficiently large q, the regulator's expected payoff is higher with the committed asset purchase, which in turn allows her to make better offers and to reduce the critical cash flow θ_A^*.

While the proposed asset purchase arrangement illustrates the benefits of ex-ante intervention, it should not be understood as the only way to provide upfront liquidity support. As we just demonstrated, whether the regulator can achieve greater financial stability with an ex-ante agreement on asset purchase or collateralized lending depends on the level of q and the possibility of over-collateralization. Regulators also have policy tools beyond asset purchases or lending at their disposal. For example, it is possible to provide liquidity via guarantees or even indirectly via recapitalization. While it is beyond the scope of the current paper to make exhaustive comparisons between all possible interventions and their combinations, we shall emphasize that — independent of the particular instrument that the regulator deploys — an ex-ante intervention should be more effective than ex-post interventions in the presence of the information constraint. The reason is that an ex-ante intervention allows the regulator to break the vicious cycle with a break-even constraint across different aggregate states.[48]

5 Concluding remarks

In this paper, we revisit a classic issue of providing liquidity support for troubled banks. We emphasize that both private investors and central banks can face the information constraint that it can be difficult — if not impossible — to distinguish illiquid banks from insolvent ones in crisis times. We introduce such an information constraint into a global-games framework where the solvent-but-illiquid banks are endogenously defined. We endogenize the liquidation value of banks' assets under the information constraint and show how a bank's funding illiquidity interacts with its asset illiquidity. In a two-bank setting with aggregate uncertainties, a vicious cycle emerges between contagious bank runs and falling asset prices. We analyze a global games model with multiple groups of players and multi-dimensional signals to obtain a unique equilibrium for clear-cut policy analysis despite the two-way feedback between distressed asset prices and contagious bank runs.

Our model illustrates how the lack of information on banks' asset quality creates financial fragility and simultaneously restricts the set of feasible policy tools: without granular information on individual

[48]We illustrate in the Online Appendix that ex-ante intervention dominates ex-post intervention for $\epsilon > 0$, whether the liquidity support is via asset purchase or collateralized lending. We also derive conditions under which the ex-ante asset purchase arrangement dominates the ex-post collateralized lending.

banks' solvency, it is infeasible for central banks to target only solvent-but-illiquid banks as suggested by Bagehot's principles. Instead, we show that a regulator with information on neither the individual banks' solvency nor the aggregate risk factor can still break down the two-way feedback between failing asset prices and contagious bank runs with upfront liquidity support. In particular, we recommend an arrangement where a regulator and banks mutually commit to an agreement for the regulator to purchase a bank's assets for a pre-specified price to contain contagious bank runs. Our theory on committed liquidity support helps rationalize some recent policy practices, such as the asset prepositioning program of the Bank of England and the committed liquidity facility of the Reserve Bank of Australia.

Appendix A Preliminaries of the bank run game

Appendix A.1 Upper and Lower dominance regions

We show that $\theta^L \equiv D_2$ defines a lower dominance region $[\underline{\theta}_s, \theta^L)$, where it is a creditor's dominant strategy to withdraw early — independent of the other creditors' actions and for any asset price $P \in [\underline{P}, qD_2)$. Indeed, when the bank is fundamentally insolvent (i.e., $\theta^i < D_2$), the inequality $(1-\lambda^i)\theta^i < F + (1-L^i)(1-E-F)r_D$ will hold for $L^i = 0$, and the creditor will always be better off to withdraw than to wait. He will receive 0 by choosing to wait because of the bank failure but will receive D_1 if he withdraws early.[49] Moreover, if the creditor's private signal falls below $D_2 - \epsilon$, he is sure that $\theta^i < D_2$.

Similarly, $\theta^U \equiv \frac{F}{1-D_1/\underline{P}}$ defines an upper dominance region $(\theta^U, \overline{\theta}]$, where it is a creditor's dominant strategy to wait – independent of other creditors' actions and for any asset price $P \in [\underline{P}, qD_2)$. Suppose that all other creditors withdraw early (i.e., $L^i = 1$) and that the asset price is the least favorable one $\underline{P}$. The bank can still repay its liabilities in full if its fundamentals exceed $\theta^U = \theta^U(\underline{P})$. Therefore, the creditor will receive D_1 if he withdraws early but will receive $\frac{D_1}{q}$ if he waits. Provided that $\overline{\theta} > \frac{F}{1-D_1/\underline{P}}$ and that $\theta^U(P)$ decreases in P, the upper dominance region exists for any $P > \underline{P}$. Additionally, when a creditor's private signal exceeds $\theta^U + \epsilon$, he will be sure that $\theta^i > \theta^U$.

[49]Recall from Lemma 1 that banks will not fail at $t = 1$, since an equilibrium asset price must be higher than $\underline{P}$, which is further higher than D_1.

Appendix A.2 The fraction of withdrawals in the bank i and the bank $-i$

We derive a bank i's representative creditor j's rational expectations on L^i and L^{-i} — given that all other creditors take the equilibrium threshold strategy and that the creditor observes signals (x^i_j, y^{-i}).

The fraction of early withdrawals in the bank i. Observing a signal y^{-i} about θ^{-i}, the representative creditor knows that all other creditors in the bank i withdraw if and only if their private signals are below $x^*(y^{-i})$. So L^i is a function of the bank's fundamentals θ^i and the critical signal $x^*(y^{-i})$, i.e., $L^i(\theta^i, x^*(y^{-i}))$. We derive the function form of $L^i(\theta^i, x^*(y^{-i}))$. For a realized θ^i, there can be three cases. (i) When $\theta^i < x^*(y^{-i}) - \epsilon$, all creditors withdraw early in the bank i and $L^i(\theta^i, x^*(y^{-i})) = 1$. (ii) When $\theta^i > x^*(y^{-i}) + \epsilon$, all creditors wait in the bank i and $L(\theta^i, x^*(y^{-i})) = 0$. (iii) For $\theta^i \in [x^*(y^{-i}) - \epsilon, x^*(y^{-i}) + \epsilon]$, we denote by x^i_k the private signal of another creditor k of the bank i. We have: $L^i(\theta^i, x^*(y^{-i})) = Prob\left(\tilde{x}^i_k < x^*(y^{-i}) \middle| \theta^i, y^{-i}\right) = Prob\left(\tilde{\epsilon}^i_k < x^*(y^{-i}) - \theta^i\right) = \frac{x^*(y^{-i}) - \theta^i + \epsilon}{2\epsilon}$. Consequently, $L^i(\theta^i, x^*(y^{-i}))$ can be expressed as (11). The creditor j perceives $L^i(\theta^i, x^*(y^{-i}))$ uncertain since he only receives a noisy signal x^i_j about θ^i. In particular, the representative creditor will form a posterior belief $\tilde{\theta}^i|_{x^i_j} \sim U(x^i_j - \epsilon, x^i_j + \epsilon)$ and rationally expect the total withdrawals $L^i(x^i_j, y^{-i})$ in the bank i to be given by equation (12). It can be verified that $L^i(x^i_j, y^{-i})$ has the following functional form: $L^i(x^i_j, y^{-i}) = 0$ when $x^i_j > x^*(y^{-i}) + 2\epsilon$; $L^i(x^i_j, y^{-i}) = \frac{(x^*(y^{-i}) - x^i_j + 2\epsilon)^2}{8\epsilon^2}$ when $x^i_j \in (x^*(y^{-i}), x^*(y^{-i}) + 2\epsilon]$; $L^i(x^i_j, y^{-i}) = \frac{1}{2}$ when $x^i_j = x^*(y^{-i})$; $L^i(x^i_j, y^{-i}) = \frac{1}{2} + \frac{x^*(y^{-i}) - x^i_j}{2\epsilon} - \frac{(x^*(y^{-i}) - x^i_j)^2}{8\epsilon^2}$ when $x^i_j \in [x^*(y^{-i}) - 2\epsilon, x^*(y^{-i}))$; and $L^i(x^i_j, y^{-i}) = 1$ when $x_j < x^*(y^{-i}) - 2\epsilon$.

The fraction of early withdrawals in the bank $-i$. L^{-i} is a function of θ^{-i} and the bank $-i$'s creditors' critical signal $x^*(y^i)$, i.e., $L^{-i}(\theta^{-i}, x^*(y^i)) = \max\left\{\min\left\{\frac{x^*(y^i) - \theta^{-i} + \epsilon}{2\epsilon}, 1\right\}, 0\right\}$. Note that y^i is the bank $-i$'s creditors' signal about the bank i's cash flow. The creditor j perceives the critical signal $x^*(y^i)$ uncertain: he considers y^i a random variable $\tilde{y}^i$. To derive the creditor j's rational expectations on L^{-i}, we first calculate his posterior belief about y^i conditional on the private signal x^i_j, i.e., the conditional density $f(y^i|x^i_j)$. Since $\tilde{\eta}^i \sim U(-\eta, \eta)$, we have: $Prob(\tilde{y}^i \leq y^i|\theta^i) = Prob(\tilde{\eta}^i \leq y^i - \theta^i) = \max\left\{\min\left\{\frac{y^i - \theta^i + \eta}{2\eta}, 1\right\}, 0\right\}$. The creditor also perceives $Prob(\tilde{y}^i \leq y^i|\theta^i)$ uncertain since he only receives a noisy signal x^i_j about θ^i. We further have: $Prob\left(\tilde{y}^i \leq y^i \middle| x^i_j\right) = \int_{x^i_j - \epsilon}^{x^i_j + \epsilon} \max\left\{\min\left\{\frac{y^i - \theta^i + \eta}{2\eta}, 1\right\}, 0\right\} \cdot \frac{1}{2\epsilon} \cdot d\theta^i$. Take the first order derivative of $Prob\left(\tilde{y}^i \leq y^i \middle| x^i_j\right)$ with respective to y^i, we obtain the conditional density of $\tilde{y}^i$ as follows:

$$f(y^i|x^i_j) = \begin{cases} \frac{y^i-(x^i_j-\eta-\epsilon)}{4\eta\epsilon} & x^i_j-\eta-\epsilon < y^i \le x^i_j-\eta+\epsilon \\ \frac{1}{2\eta} & x^i_j-\eta+\epsilon < y^i \le x^i_j+\eta-\epsilon \\ \frac{(x^i_j+\eta+\epsilon)-y^i}{4\eta\epsilon} & x^i_j+\eta-\epsilon < y^i \le x^i_j+\eta+\epsilon \\ 0 & \text{otherwise.} \end{cases} \tag{A.24}$$

$f(y^i|x^i_j)$ is everywhere non-negative and strictly positive when $y^i \in (x^i_j - \eta - \epsilon, x^i_j + \eta + \epsilon]$. Additionally, based on the signal y^{-i}, the creditor j will form a posterior belief $\tilde{\theta}^{-i}|_{y^{-i}} \sim U(y^{-i} - \eta, y^{-i} + \eta)$ about θ^{-i}. Lastly, $\tilde{\theta}^{-i}|_{y^{-i}}$ and $\tilde{y}^i|_{x^i_j}$ are independently distributed because $\tilde{\epsilon}^i_j$, $\tilde{\eta}^i$ and $\tilde{\eta}^{-i}$ are independently distributed. The joint density function of $\tilde{\theta}^{-i}|_{y^{-i}}$ and $\tilde{y}^i|_{x^i_j}$ is $\frac{1}{2\eta} \cdot f(y^i|x^i_j)$. Apply Fubini's theorem, the representative creditor j's expectations on the total withdrawals $L^{-i}(x^i_j, y^{-i})$ in the bank $-i$ is as follows:

$$L^{-i}(x^i_j, y^{-i}) = E\left\{E\left[L^{-i}(\theta^{-i}, x^*(y^i))\middle|y^{-i}\right]\middle|x^i_j\right\} = \int_{x^i_j-\eta-\epsilon}^{x^i_j+\eta+\epsilon}\int_{y^{-i}-\eta}^{y^{-i}+\eta} L^{-i}(\theta^{-i}, x^*(y^i)) \cdot \frac{1}{2\eta} \cdot d\theta^{-i} \cdot f(y^i|x^i_j) \cdot dy^i. \tag{A.25}$$

Monotonicity of $L^i(x^i_j, y^{-i})$ and $L^{-i}(x^i_j, y^{-i})$. The monotonicity of $L^i(x^i_j, y^{-i})$ and $L^{-i}(x^i_j, y^{-i})$ can be calculated as follows: $\frac{\partial L^i(x^i_j,y^{-i})}{\partial x^i_j} \le 0$, $\frac{\partial L^i(x^i_j,y^{-i})}{\partial y^{-i}} \le 0$, $\frac{\partial L^{-i}(x^i_j,y^{-i})}{\partial x^i_j} \le 0$ and $\frac{\partial L^{-i}(x^i_j,y^{-i})}{\partial y^{-i}} \le 0$. The details of the derivations can be found in the Online Appendix.

Appendix B Proofs to lemmas and propositions

Appendix B.1 Proof of Lemma 1

Proof. Since buyers' bid cannot be negative, an ex-post break-even price P after observing M runs, $M \in \{1, 2\}$, if exits, must be in one of the three regions: $[0, \underline{P})$, $[\underline{P}, qD_2)$, or $[qD_2, +\infty)$. We show that it cannot be greater than or equal to qD_2, nor can it be lower than $\underline{P} = \frac{\theta_B+D_2}{2}$.

Suppose $P \ge qD_2$, then it is not sequentially rational for the wholesale creditors to withdraw from a solvent bank, i.e., $\theta^i \ge D_2$. To see this, one can take the perspective of a representative creditor j of a bank i. Even when all other creditors withdraw, the bank needs to liquidate no more than $\frac{D_1}{qD_2}$ fraction of its asset, for $P \ge qD_2$. While the bank's $t = 2$ liability drops to F, its residual cash flow is $(1 - \frac{D_1}{P}) \cdot \theta^i \ge \left(1 - \frac{D_1}{qD_2}\right) \cdot D_2 = F$ as $\theta^i \ge D_2$. As a result, by running on the bank, creditor j will only incur a penalty for early withdrawal. This implies that whenever a run happens when $P \ge qD_2$, the bank must be fundamentally insolvent with $\theta^i < D_2$. Therefore, buyers must expect asset quality to be lower than $\frac{D_2+\theta_G}{2}$, which is in turn lower than qD_2 given our parametric assumption (3). Buyers would make a loss by offering $P \geq qD_2$, a contradiction.

A break-even price P cannot be smaller than $\underline{P}$ either. Note that when a bank is insolvent with cash flow $\theta^i < D_2$, it is a dominant strategy for its wholesale creditors to run independently of the asset price. To see this, notice that if $P \geq D_1$ and the bank does not fail at $t = 1$, a creditor is better off to run and receive D_1 than to wait and receive 0.[50] On the other hand, if $P < D_1$, a creditor will receive a zero payoff for his claim whether he runs or not, but can still obtain an arbitrarily small reputational benefit by running on a bank that is doomed to fail. This implies that runs must happen to those banks with $\theta^i < D_2$, and the expected quality of assets on sale is at least $\frac{\theta_B+D_2}{2} = \underline{P}$. As asset buyers break even with their competitive bidding, the price they offer must be greater than or equal to $\underline{P}$. □

Appendix B.2 Proof of Proposition 1

Proof. To start with, we present a bank i's representative creditor j's payoff difference function and derive his best response to other players' equilibrium strategy, i.e., $x^*(\cdot)$ and P^*. Rationally expecting a price P^* independent of the runs, the representative creditor's payoff difference $E\left[DW(L^i(x^i_j, y^{-i}), \theta^*(y^{-i}), P^*)\right]$ can be expressed as follows, with the expression of $L^c(\theta^*(y^{-i}), P^*)$ given by (13):[51]

$$E\left[DW(L^i(x^i_j, y^{-i}), \theta^*(y^{-i}), P^*)\right] = \begin{cases} \frac{1-q}{q} D_1 & x^i_j > \overline{x} \\ \frac{D_1}{q}[L^c(\theta^*(y^{-i}), P^*) - q] + \frac{D_1}{q}\frac{x^i_j - x^*(y^{-i})}{2\epsilon} & x^i_j \in (x^*(y^{-i}), \overline{x}] \\ \frac{D_1}{q}[L^c(\theta^*(y^{-i}), P^*) - q] & x^i_j = x^*(y^{-i}) \\ \frac{D_1}{q}[L^c(\theta^*(y^{-i}), P^*) - q] - \frac{D_1}{q}\frac{x^*(y^{-i}) - x^i_j}{2\epsilon} & x^i_j \in [\underline{x}, x^*(y^{-i})) \\ -D_1 & x^i_j < \underline{x}. \end{cases} \tag{B.26}$$

Observe the following results. First, the payoff difference function is linear in x^i_j with a slope $\frac{D_1}{2q\epsilon} > 0$ when $x^i_j \in [\underline{x}, \overline{x}] \subset [x^*(y^{-i}) - 2\epsilon, x^*(y^{-i}) + 2\epsilon]$. Second, the payoff difference function equals a constant $(1-q)\frac{D_1}{q} > 0$ when $x^i_j \geq \overline{x}$ and another constant $-D_1 < 0$ when $x^i_j \leq \underline{x}$. Therefore, there

[50] Note that the ex-post asset sale will never revive an insolvent bank as we prove that $P \geq qD_2$ could never happen.

[51] In (B.26), $\overline{x} \in (x^*(y^{-i}), x^*(y^{-i}) + 2\epsilon]$ and $\underline{x} \in [x^*(y^{-i}) - 2\epsilon, x^*(y^{-i}))$ are two cutoffs of x^i_j, which solve $L^c(\theta^*(y^{-i}), P^*) = 1 - \frac{\overline{x} - x^*(y^{-i})}{2\epsilon}$ and $L^c(\theta^*(y^{-i}), P^*) = \frac{x^*(y^{-i}) - \underline{x}}{2\epsilon}$ respectively. The derivation follows the standard global games approach. Details are included in the Online Appendix.

must exist a unique $\hat{x} \in [\underline{x}, \overline{x}]$ such that $E\left[DW(L^i(\hat{x}, y^{-i}), \theta^*(y^{-i}), P^*)\right] = 0$, $\forall y^{-i} \in [\underline{\theta}_s - \eta, \overline{\theta} + \eta]$. This establishes a function $\hat{x}(y^{-i})$, $\forall y^{-i} \in [\underline{\theta}_s - \eta, \overline{\theta} + \eta]$. The creditor j's best response to the other creditors' threshold strategy (i.e., $x^*(\cdot)$) is a threshold strategy: to withdraw if $x^i_j < \hat{x}(y^{-i})$ and to wait if $x^i_j > \hat{x}(y^{-i})$, $\forall y^{-i} \in [\underline{\theta}_s - \eta, \overline{\theta} + \eta]$. We have $\hat{x}(y^{-i}) = x^*(y^{-i}) - 2\epsilon\left[L^c(\theta^*(y^{-i}), P^*) - q\right]$ by solving $E\left[DW(L^i(\hat{x}, y^{-i}), \theta^*(y^{-i}), P^*)\right] = 0$. In a symmetric equilibrium, $\hat{x}(y^{-i}) = x^*(y^{-i})$ must be true. Therefore, the critical cash flow must satisfy $L^c(\theta^*(y^{-i}), P^*) = q$, which gives the equilibrium condition (14).

As established, $\theta^*(y^{-i})$, $\hat{\theta}(y^{-i})$ and $x^*(y^{-i})$, if exist, are constants and do not depend on y^{-i}. Let us denote them as θ^*, $\hat{\theta}$ and x^*. We then show that there exists a unique combination (θ^*, x^*, P^*) jointly solving the system of equations: $\theta^* = \frac{D_2 - D_1}{1 - qD_1/P^*}$, $x^* = \theta^* + (2q-1)\epsilon$, $P^* = \frac{\hat{\theta} + \underline{\theta}}{2}$, and $\hat{\theta} \equiv x^* + \epsilon$.

Let P^* be the argument and express θ^*, x^* and $\hat{\theta}$ as: $\theta^* = \theta^*(P^*) = \frac{D_2 - D_1}{1 - qD_1/P^*}$, $x^* = x^*(P^*) = \theta^*(P^*) + (2q-1)\epsilon$ and $\hat{\theta} = \hat{\theta}(P^*) = \theta^*(P^*) + 2q\epsilon$. Define a function $\Pi(P) = \frac{\theta^*(P) + 2q\epsilon + \underline{\theta}}{2} - P = \frac{1}{2}\left(\frac{D_2 - D_1}{1 - qD_1/P} + 2q\epsilon + \underline{\theta}\right) - P$ as the asset buyers' expected profit from purchasing banks' assets at a price P upon observing runs. The equilibrium asset price P^*, if exists, must satisfy the zero-profit condition $\Pi(P^*) = 0$. Indeed, one can verify that $\Pi(P)$ monotonically decreases in P, $\Pi(\underline{P}) > 0$, and $\Pi(qD_2) < 0$ for small ϵ. Therefore, the equilibrium asset price $P^* \in [\underline{P}, qD_2)$ exists and is unique. With the unique P^*, it is then straightforward to verify that the associated $\theta^* \in [\theta^L, \theta^U]$ and $x^* \in [x^L, x^U]$ exist and are unique. □

Appendix B.3 Proof of Lemma 3

Proof. **Step 1:** We prove the existence of $(\theta^*_M, x^*_M, P^*_M)$ as a unique solution to the system of equations (18), $\forall M \in \{1, 2\}$. Let P^*_M be the argument and express θ^*_M, x^*_M and $\hat{\theta}_M$ as: $\theta^*_M = \theta^*(P^*_M) = \frac{D_2 - D_1}{1 - qD_1/P^*_M}$, $x^*_M = x^*(P^*_M) = \theta^*(P^*_M) + (2q-1)\epsilon$ and $\hat{\theta}_M = \hat{\theta}(P^*_M) = \theta^*(P^*_M) + 2q\epsilon$. Let $\Pi_M(P)$ be the asset buyers' expected profit function from purchasing banks' assets at a price P when observing M runs. We have: $\Pi_M(P) = \omega^B_M\left(\hat{\theta}(P)\right) \cdot \left(\frac{\underline{\theta}_B + \hat{\theta}(P)}{2} - P\right) + \omega^G_M\left(\hat{\theta}(P)\right) \cdot \left(\frac{\underline{\theta}_G + \hat{\theta}(P)}{2} - P\right)$. Also define $\pi(P|s) = \frac{\underline{\theta}_s + \hat{\theta}(P)}{2} - P$ as the buyers' payoff from purchasing one unit of assets in a given state s. Then, P^*_M, if exists, must satisfy the asset buyers' break-even condition: $\Pi_M(P^*_M) = \omega^B_M\left(\hat{\theta}(P^*_M)\right) \cdot \pi(P^*_M|B) + \omega^G_M\left(\hat{\theta}(P^*_M)\right) \cdot \pi(P^*_M|G) = 0$. One can check that $\frac{d\Pi_M(P)}{dP} < 0$, $\Pi_M(\underline{P}) > 0$ and $\Pi_M(qD_2) < 0$. Hence, there exists a unique $P^*_M \in [\underline{P}, qD_2)$

such that $\Pi_M(P_M^*) = 0$, $\forall M \in \{1,2\}$. The associated $\theta_M^* \in [\theta^L, \theta^U]$ and $x_M^* \in [x^L, x^U]$ exist and are unique.

Step 2: We prove $\theta_2^* > \theta_1^*$, $x_2^* > x_1^*$ and $P_2^* < P_1^*$. For simplicity, let θ be the argument. Denote $P^*(\theta) = \frac{qD_1\theta}{\theta-(D_2-D_1)}$ as the inverse of $\theta^*(P)$. We reformulate the asset buyers' zero-profit condition as: $\Pi_M(\hat{\theta}_M) = \omega_M^B(\hat{\theta}_M)\cdot\pi(\hat{\theta}_M|B) + \omega_2^G(\hat{\theta}_M)\cdot\pi(\hat{\theta}_M|G) = 0$. Here, $\pi(\theta|s) = \frac{\underline{\theta}_s+\theta}{2} - P^*(\theta)$. The proof then hinges on the monotonicity of two ratios: $\frac{\omega_M^B(\theta)}{\omega_M^G(\theta)} = \frac{(\theta-\underline{\theta}_B)^M}{\kappa(\theta-\underline{\theta}_G)^M}$ and $\frac{\pi(\theta|G)}{\pi(\theta|B)} = \frac{(\underline{\theta}_G+\theta)/2-P^*(\theta)}{(\underline{\theta}_B+\theta)/2-P^*(\theta)}$ for $\theta \in [\theta^L, \theta^U]$. Observe that $\frac{dP^*(\theta)}{d\theta} < 0$. Then, one can verify that both ratios strictly *decrease* in θ when $\theta > D_2 > \underline{\theta}_s$. Moreover, $\frac{\omega_1^B(\theta)}{\omega_1^G(\theta)} < \frac{\omega_2^B(\theta)}{\omega_2^G(\theta)}$ must hold $\forall\theta > D_2$. Indeed, $\kappa\cdot\frac{\omega_1^B(\theta)}{\omega_1^G(\theta)} = \left(\frac{\theta-\underline{\theta}_B}{\theta-\underline{\theta}_G}\right) < \left(\frac{\theta-\underline{\theta}_B}{\theta-\underline{\theta}_G}\right)^2 = \kappa\cdot\frac{\omega_2^B(\theta)}{\omega_2^G(\theta)}$ because $\frac{\theta-\underline{\theta}_B}{\theta-\underline{\theta}_G} > 1$.

We then prove the result by contradiction. Suppose $\hat{\theta}_1 \geq \hat{\theta}_2$. By the monotonicity of $\frac{\pi(\theta|G)}{\pi(\theta|B)}$, we have: $\frac{\pi(\hat{\theta}_1|G)}{\pi(\hat{\theta}_1|B)} \leq \frac{\pi(\hat{\theta}_2|G)}{\pi(\hat{\theta}_2|B)}$. By the equilibrium condition $\Pi_M(\hat{\theta}_M) = 0$ for $M \in \{1,2\}$, we have: $\frac{\pi(\hat{\theta}_1|G)}{\pi(\hat{\theta}_1|B)} = -\frac{\omega_1^B(\hat{\theta}_1)}{\omega_1^G(\hat{\theta}_1)}$ and $\frac{\pi(\hat{\theta}_2|G)}{\pi(\hat{\theta}_2|B)} = -\frac{\omega_2^B(\hat{\theta}_2)}{\omega_2^G(\hat{\theta}_2)}$. Therefore, we obtain: $\frac{\omega_2^B(\hat{\theta}_2)}{\omega_2^G(\hat{\theta}_2)} \leq \frac{\omega_1^B(\hat{\theta}_1)}{\omega_1^G(\hat{\theta}_1)}$. By $\frac{\omega_1^B(\theta)}{\omega_1^G(\theta)} < \frac{\omega_2^B(\theta)}{\omega_2^G(\theta)}$ $\forall\theta > D_2$, we further have: $\frac{\omega_2^B(\hat{\theta}_2)}{\omega_2^G(\hat{\theta}_2)} \leq \frac{\omega_1^B(\hat{\theta}_1)}{\omega_1^G(\hat{\theta}_1)} < \frac{\omega_2^B(\hat{\theta}_1)}{\omega_2^G(\hat{\theta}_1)}$. Apply the monotonicity of $\frac{\omega_2^B(\theta)}{\omega_2^G(\theta)}$, we obtain $\hat{\theta}_2 > \hat{\theta}_1$, a contraction. Therefore, $\hat{\theta}_2 > \hat{\theta}_1$ must hold, which implies $x_2^* > x_1^*$. Then, $\theta_2^* = x_2^* - (2q-1)\epsilon > x_1^* - (2q-1)\epsilon = \theta_1^*$, and $P_2^* < P_1^*$ follows directly from the monotonicity of $P^*(\theta)$. □

Appendix B.4 Proof of Proposition 2

Proof. Having established in the text that $x^*(y^{-i}) = x_1^*$ when $y^{-i} > \hat{y}$ and $x^*(y^{-i}) = x_2^*$ when $y^{-i} < \hat{y}$, where $\hat{y} \in [y^L, y^U]$, we show here there exists a unique $\hat{y} = x_2^* + \eta + \epsilon$. The proof hinges on deriving a bank i's representative creditor j's rational expectations on the fractions of withdrawals (i.e., L^i and L^{-i}) based on his signals (x_j^i, y^{-i}). We analyze whether $L^{-i}(x_j^i, y^{-i})$ is non-zero to determine the creditor's expectations on the number of runs M and the equilibrium asset price.[52] Note $L^{-i}(x_j^i, y^{-i}) = E\left\{E\left[L^{-i}(\theta^{-i}, x^*(y^i))\middle|y^{-i}\right]\middle|x_j^i\right\}$ is given by (A.25) in Appendix A.2.

Step 1: We prove that upon observing $y^{-i} \in [y^L, x_1^* - \eta + \epsilon]$ and $y^{-i} \in [x_2^* + \eta + \epsilon, y^U]$, the creditor j expects $L^{-i}(x_j^i, y^{-i}) > 0$ and $L^{-i}(x_j^i, y^{-i}) = 0$, respectively, both independent of his private signal x_j^i. We derive the creditor j's expectation of $L^{-i}(x_j^i, y^{-i})$ when he observes $y^{-i} = x_1^* - \eta + \epsilon$ and has a

[52]To determine whether $L^i(x_j^i, y^{-i})$ is non-zero is less involving. In particular, the representative creditor knows that $L^i(x_j^i, y^{-i}) > 0$ when $x_j^i \leq x^*(y^{-i}) + 2\epsilon$ and $L^i(x_j^i, y^{-i}) = 0$ when $x_j^i > x^*(y^{-i}) + 2\epsilon$ from Appendix A.2.

posterior $\theta^{-i}\big|_{y^{-i}} \sim U(x_1^* - 2\eta + \epsilon, x_1^* + \epsilon)$. Note that the creditors in the bank $-i$ can follow either x_2^* or x_1^* as the critical signal depending on their signal y^i. If those creditors withdraw according to x_1^* (i.e., $x^*(y^i) = x_1^*$), we can compute the expectation $E\left[L^{-i}(\theta^{-i}, x^*(y^i))\middle|y^{-i}\right]$ in the expression of $L^{-i}(x_j^i, y^{-i})$ as:[53] $E\left[L^{-i}(\theta^{-i}, x_1^*)\middle|x_1^* - \eta + \epsilon\right] = \int_{x_1^*-2\eta+\epsilon}^{x_1^*-\epsilon} 1 \cdot \frac{1}{2\eta} d\theta^{-i} + \int_{x_1^*-\epsilon}^{x_1^*+\epsilon} \frac{x_1^*-\theta^{-i}+\epsilon}{2\epsilon} \cdot \frac{1}{2\eta} d\theta^{-i} = \frac{2\eta-\epsilon}{2\eta} > 0$. Instead, if they withdraw according to x_2^*, one can verify easily that $L^{-i}(\theta^{-i}, x_2^*) = 1$ and $E\left[L^{-i}(\theta^{-i}, x_2^*)\middle|x_1^* - \eta + \epsilon\right] = \int_{x_1^*-2\eta+\epsilon}^{x_1^*+\epsilon} L^{-i}(\theta^{-i}, x_2^*) \cdot \frac{1}{2\eta} \cdot d\theta^{-i} = 1$. Either way, the creditor j expects $L^{-i}(x_j^i, y^{-i}) > 0$, because $f(y^i|x_j^i)$ is everywhere non-negative, and it is strictly positive when $y^i \in [x_j^i - \eta - \epsilon, x_j^i + \eta + \epsilon]$. By the monotonicity of $L^{-i}(x_j^i, y^{-i})$ with respect to y^{-i}, we establish that $L^{-i}(x_j^i, y^{-i}) > 0$ when $y^{-i} \in [y^L, x_1^* - \eta + \epsilon]$.

Consider the representative creditor j's expectation of $L^{-i}(x_j^i, y^{-i})$ when observing $y^{-i} \geq x_2^* + \eta + \epsilon$. He knows with certainty that the lowest possible private signal received by the bank $-i$'s creditors is higher than x_2^*. The representative creditor then expects $L^{-i}(x_j^i, y^{-i}) = 0$. Follow the same argument in Lemma 3, we can obtain $x^*(y^{-i}) = x_2^*$ when $y^{-i} \in [y^L, x_1^* - \eta + \epsilon]$ and $x^*(y^{-i}) = x_1^*$ when $y^{-i} \in [x_2^* + \eta + \epsilon, y^U]$.

Step 2: Given that all other creditors follow strategy (19), we prove that the creditor j expects $L^{-i}(x_j^i, y^{-i}) > 0$ when observing $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$ and $x_j^i \leq x_2^* + 2\epsilon$. We prove this result by first establishing that it holds for $x_j^i = x_2^* + 2\epsilon$. Then by the monotonicity of $L^{-i}(x_j^i, y^{-i})$ in x_j^i established in Appendix A.2, we have $L^{-i}(x_j^i, y^{-i}) > 0, \forall x_j^i \leq x_2^* + 2\epsilon$. Upon observing $x_j^i = x_2^* + 2\epsilon$, the representative creditor forms beliefs about the signal y^i received by the other bank's creditors. He can calculate, by (A.24), that y^i has a positive conditional density $f(y^i|x_2^* + 2\epsilon)$ on the interval $[x_2^* - \eta + \epsilon, x_2^* + \eta + 3\epsilon]$.

$$f(y^i|x_2^* + 2\epsilon) = \begin{cases} \frac{y^i - (x_2^* - \eta + \epsilon)}{4\eta\epsilon} & x_2^* - \eta + \epsilon < y^i \leq x_2^* - \eta + 3\epsilon \\ \frac{1}{2\eta} & x_2^* - \eta + 3\epsilon < y^i \leq x_2^* + \eta + \epsilon \\ \frac{(x_2^* + \eta + 3\epsilon) - y^i}{4\eta\epsilon} & x_2^* + \eta + \epsilon < y^i \leq x_2^* + \eta + 3\epsilon \\ 0 & \text{otherwise.} \end{cases}$$

Following strategy (19), the bank $-i$'s creditors withdraw according to the critical signal $x^*(y^i) = x_2^*$ when they observe $y^i < x_2^* + \eta + \epsilon$ and $x^*(y^i) = x_1^*$ when they observe $y^i \geq x_2^* + \eta + \epsilon$. In particular, the representative creditor expects that the creditors in the bank $-i$ will follow x_2^* with a probability $Prob\left(x^*(y^i) = x_2^* | x_j^i = x_2^* + 2\epsilon\right) = \int_{x_2^*-\eta+\epsilon}^{x_2^*-\eta+3\epsilon} \frac{y^i - (x_2^* - \eta + \epsilon)}{4\eta\epsilon} dy^i + \int_{x_2^*-\eta+3\epsilon}^{x_2^*+\eta+\epsilon} \frac{1}{2\eta} dy^i = \frac{2\eta - \epsilon}{2\eta} > 0$.

[53] Note that $L^{-i}(\theta^{-i}, x_1^*) = 1$ if $x_1^* - 2\eta + \epsilon < \theta^{-i} < x_1^* - \epsilon$ and $L^{-i}(\theta^{-i}, x_1^*) = \frac{x_1^* - \theta^{-i} + \epsilon}{2\epsilon}$ if $x_1^* - \epsilon < \theta^{-i} < x_1^* + \epsilon$.

Conditional on that creditors from the bank $-i$ actually follow the critical signal x_2^*, the representative creditor can calculate the aggregate withdrawal in the bank $-i$ as $E[L^{-i}(\theta^{-i}, x_2^*)|y^{-i}] = \int_{y^{-i}-\eta}^{y^{-i}+\eta} L^{-i}(\theta^{-i}, x_2^*) \cdot \frac{1}{2\eta} \cdot d\theta^{-i}$ since $\tilde{\theta}^{-i}|_{y^{-i}} \sim U(y^{-i}-\eta, y^{-i}+\eta)$. When the representative creditor observes a signal $x_1^* - \eta + \epsilon < y^{-i} < x_2^* + \eta + \epsilon$, he knows that the expected aggregate withdrawal has a lower bound

$$E\left[L^{-i}(\theta^{-i}, x_2^*)\middle|y^{-i}\right] \geq \int_{y^{-i}-\eta}^{x_2^*+\epsilon} \frac{x_2^* - \theta^{-i} + \epsilon}{2\epsilon} \cdot \frac{1}{2\eta} \cdot d\theta^{-i} = \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} > 0. \tag{B.27}$$

The first inequality in (B.27) is true because $E\left[L^{-i}(\theta^{-i}, x_2^*)\middle|y^{-i}\right]$ decreases in y^{-i} and the functional form of $L^{-i}(\theta^{-i}, x_2^*)$ is $\frac{x_2^*-\theta^{-i}+\epsilon}{2\epsilon}$ if $y^{-i} - \eta \leq \theta^{-i} < x_2^* + \epsilon$ and 0 if $x_2^* + \epsilon \leq \theta^{-i} \leq y^{-i} + \eta$, conditional on y^{-i} slightly lower than $x_2^* + \eta + \epsilon$ (i.e., $y^{-i} \in (x_2^* + \eta - \epsilon, x_2^* + \eta + \epsilon)$).

Upon observing $x_j^i = x_2^* + 2\epsilon$ and $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$, the creditor j rationally expects:

$$\begin{aligned}
L^{-i}(x_2^* + 2\epsilon, y^{-i}) &= \int_{x_2^*-\eta+\epsilon}^{x_2^*+\eta+\epsilon} E\left[L^{-i}(\theta^{-i}, x_2^*)\middle|y^{-i}\right] \cdot f(y^i|x_2^* + 2\epsilon) \cdot dy^i + \int_{x_2^*+\eta+\epsilon}^{x_2^*+\eta+3\epsilon} E\left[L^{-i}(\theta^{-i}, x_1^*)\middle|y^{-i}\right] \cdot f(y^i|x_2^* + 2\epsilon) \cdot dy^i \\
&\geq \int_{x_2^*-\eta+\epsilon}^{x_2^*+\eta+\epsilon} \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} \cdot f(y^i|x_2^* + 2\epsilon) \cdot dy^i = \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} \cdot Prob\left(x^*(y^i) = x_2^*|x_j^i = x_2^* + 2\epsilon\right) \\
&= \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} \cdot \frac{2\eta - \epsilon}{2\eta} > 0
\end{aligned}$$

from (A.25). The inequality in the second line follows (B.27), the fact that $E\left[L^{-i}(\theta^{-i}, x_1^*)\middle|y^{-i}\right] \geq 0$[54], and the density of y^i being non-negative everywhere.

Step 3: We establish the existence of the equilibrium strategy (19) by analyzing the representative creditor's best response. We have already proved that $x^*(y^{-i}) = x_2^*$ for $y^{-i} \leq x_1^* - \eta + \epsilon$ and $x^*(y^{-i}) = x_1^*$ for $y^{-i} \geq x_2^* + \eta + \epsilon$. Moreover, for $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$, given that all other creditors follow strategy (19), the representative creditor expects $L^i(x_j^i, y^{-i}) > 0$ when $x_j^i \leq x_2^* + 2\epsilon$, i.e., a positive mass of withdrawals in his own bank. Therefore, for $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$ and $x_j^i \leq x_2^* + 2\epsilon$, the creditor expects the number of runs to be $M = 2$ and the asset price to be P_2^*. Instead, when observing $x_j^i > x_2^* + 2\epsilon$, he expects $L^i(x_j^i, y^{-i}) = 0$, i.e., no run in his own bank. We can follow the same procedure in the proof of Proposition 1 and Lemma 3 to show that the creditor j optimally withdraws if and only if $x_j^i < \hat{x}(y^{-i})$, where $\hat{x}(y^{-i}) = x_2^* - 2\epsilon[L^c(\theta^*(y^{-i}), P_2^*) - q]$. A symmetric equilibrium, if exists, features

[54]The creditor j also expects that the creditors in the bank $-i$ will follow x_1^* with a probability $\int_{x_2^*+\eta+\epsilon}^{x_2^*+\eta+3\epsilon} f(y^i|x_2^* + 2\epsilon) \cdot dy^i$ and calculates the expected aggregate withdrawal in the bank $-i$ as $E\left[L^{-i}(\theta^{-i}, x_1^*)\middle|y^{-i}\right]$ in that case.

$L^c(\theta^*(y^{-i}), P_2^*) = q$. The bank's critical cash flow, the bank i's creditors' threshold signal, and the asset price, jointly solve the system of equations (18) for $M = 2$, with the unique solution being $\left(\theta_2^*, x_2^*, P_2^*\right)$ by Lemma 3. Therefore, we obtain $x^*(y^{-i}) = x_2^*$ for $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$. When all other creditors in the bank i and $-i$ take strategy (19), the creditor j's best response is to follow the same strategy.

Step 4: We prove the uniqueness by contradiction. Suppose that all other creditors follow an alternative symmetric threshold strategy with a discontinuity point $x_1^* - \eta + \epsilon < \hat{y} < x_2^* + \eta + \epsilon$, so that $x^*(y^{-i}) = x_1^*$ when $y^{-i} \in (\hat{y}, x_2^* + \eta + \epsilon)$. Consider that the representative creditor j observes a private signal $x_j^i = x_1^*$. Following the same procedure in Step 2, the probability that creditors in the bank $-i$ following the critical signal x_2^* is positive: $Prob\left(x^*(y^i) = x_2^* | x_j^i = x_1^*\right) = \int_{x_1^* - \eta - \epsilon}^{x_1^* - \eta + \epsilon} \frac{y^i - (x_1^* - \eta - \epsilon)}{4\eta\epsilon} \cdot dy^i = \frac{\epsilon}{2\eta} > 0$. Conditional on that creditors from the bank $-i$ follow the threshold signal x_2^* and that the representative creditor observes $y^{-i} \in (\hat{y}, x_2^* + \eta + \epsilon)$, the aggregate withdrawals again satisfy the inequality (B.27). The representative creditor j rationally expects $L^{-i}(x_1^*, y^{-i})$ to be strictly positive. That is, $L^{-i}(x_1^*, y^{-i}) \geq \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} \cdot \frac{\epsilon}{2\eta} > 0$. By monotonicity, the representative creditor expects $M = 2$ and the secondary market asset price to be P_2^* when observing $x_j^i \leq x_1^*$ and $y^{-i} \in (\hat{y}, x_2^* + \eta + \epsilon)$. Consequently, the creditor j expects his own bank (i.e., the bank i) to sell its assets for the price P_2^* when runs happen. The price P_2^*, however, contradicts the threshold signal x_1^* as dictated by the alternative threshold strategy. In particular, the creditor j is not indifferent between waiting and withdrawing when observing $x_j^i = x_1^*$. As $\hat{y}$ is arbitrary in interval $(x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$, we establish the result. □

Appendix B.5 Proof of Proposition 3

Proof. The proof for the existence of $P_A^* \in (P_2^*, P_1^*)$ is provided in the text, we focus on the uniqueness.

One can directly calculate $V_A(P_A)$ in (22) as: $V_A(P_A) = \sum_{s=G,B} Pr(s) \cdot Pr(\theta < \theta^*(P_A)|s) \cdot \left(2 \cdot \frac{D_1}{P_A} \cdot \pi(P_A|s)\right)$. Combined with $\pi(P_A|s) = \frac{\underline{\theta}_s + \theta^*(P_A)}{2} - P_A$, one can obtain the first order derivative of $V_A(P_A)$ with respect to P_A as: $\frac{dV_A(P_A)}{dP_A} = \sum_{s=G,B} \frac{Pr(s)}{(\bar{\theta} - \underline{\theta}_s)} \cdot 2 \cdot \left\{\frac{d\theta^*(P_A)}{dP_A} \cdot (\theta^*(P_A) - P_A) - \frac{(\theta^*(P_A) + \underline{\theta}_s)(\theta^*(P_A) - \underline{\theta}_s)}{2P_A}\right\} \cdot \frac{D_1}{P_A} < 0$. Notice that the term inside the brace is negative because $\frac{d\theta^*(P_A)}{dP_A} = \frac{d}{dP_A}\left(\frac{D_2 - D_1}{1 - qD_1/P_A}\right) < 0$ and $\theta^*(P_A) > D_2 > P_A > \underline{\theta}_s$ for $P_A \in [\underline{P}, qD_2)$. Therefore, there exists a unique $P_A^* \in (P_2^*, P_1^*)$ such that $V_A(P_A^*) = 0$.

One can also verify the following: $\frac{d\text{SYS}(P_A)}{dP_A} = \sum_{s=G,B} \frac{Pr(s)}{(\bar{\theta}-\underline{\theta}_s)^2} \cdot 2 \cdot \left(\theta^*(P_A) - \underline{\theta}_s\right) \cdot \frac{d\theta^*(P_A)}{dP_A} < 0$. So the regulator indeed chooses the unique P_A^* such that $V_A(P_A^*) = 0$ to minimize the risk of systemic crises, i.e., constraint (20) is binding. Also, note that constraint (21) is slack at optimum because $P_A^* < P_1^* < qD_2$.

Lastly, it is easy to verify that $\text{SYS}(P_A^*) < \text{SYS}(\theta_2^*)$ as $P_A^* > P_2^*$ and $\theta_A^* = \theta^*(P_A^*) < \theta^*(P_2^*) = \theta_2^*$. □

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