CEO Compensation Design in a Multiplicative Model

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October 8, 2015

Abstract

We analyze the design of compensation contracts to motivate a risk neutral CEO's effort of developing risky project opportunity, then to induce his best project choice for the firm. Restricted stock induces the CEO to select the project that maximizes the firm's expected value while stock options are superior in incenting the managerial effort. When the risky project has sufficient "upside" value than the firm's existing safe project, it is optimal to pay the CEO solely in restricted stock. Otherwise, the firm faces a trade-off between motivating the CEO's effort and mitigating his excessive risk taking. The second best contract is a combination of restricted stock and stock options. We extend the model to consider a competitive CEO market and find out that there could be circumstances where larger firms hire lower ability CEOs in the market equilibrium. Our model also explores the optimal choice of exercise price of stock options and the relation between the firm size, risk and the optimal structure of compensation contract.

Keywords: Effort, Project Choice, Restricted Stock, Stock Options, CEO Ability, Firm Size, Positive Assortive Matching

JEL Classification: D86, G30

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1 Introduction

Both restricted stock and stock options are designed to link CEOs' future wealth to firms' stock price performance, therefore, to mitigate agency problems between the firms' shareholders and CEOs outlined in Jensen and Meckling (1976). But there is an open debate about whether stock options should be a part of the optimal CEO compensation contracts. Hall and Murphy (2002) and Jenter (2002) demonstrate that stock options are inefficient either because their economic value to risk averse CEOs or their created incentives are overstated. Yet, Lambert and Larcker (2004) shows that stock options dominate restricted stock in providing incentives in a model where effort affects the distribution of firm's stock price, i.e., more incentives are conveyed through stock options in the range where stock prices are higher. Dittmann and Maug (2007) calibrates the traditional principal-agent model in Hölmstrom (1979) with constant relative risk aversion and log-normal stock prices and find that the optimal contract should not include any stock options. Motivated by these conflicting findings, alternative models and theories to explain the observed CEO compensation contracts are developed.[1] In this paper, we rationalize the mix of restricted stock and stock options in the optimal CEO compensation contracts in a model where the CEO's preference and the firm's production functions are both multiplicative.

It is well recognized that non-linearities in the pay structure affect CEO's risk taking behavior. Guay (1999) suggests that boards add stock options to CEO compensation contracts to induce risk averse CEOs to adopt risky but value enhancing projects. However, in the aftermath of the 2008-2009 financial crisis, there emerges another concern that stock options provide CEOs with incentives to engage in excessive risk taking. [2] We show that stock options indeed induce the CEOs to follow a project choice rule that is both excessive risky and shareholders' value decreasing compared to the optimal one.

We present a model where the CEO first exerts effort to develop a new project for the firm. The new project is risky, yet increases the expected firm value compared to a safe project

[1] Papers such as Hemmer, Kim, and Verrecchia (1999), Dittmann, Maug, and Spalt (2010) and Chaigneau (2013) postulate the specific forms of CEO's preferences while papers such as Feltham and Wu (2001) and Dittmann and Yu (2011) justify the optimality of the observed mixed contracts with shareholders' risk-taking incentive. There is a paralleled inefficient contracting view as well. Yermack (1995) challenges the explanatory power of efficient contracting for the pattern of stock option awards and Bebchuk and Fried (2004) argues that CEOs of large corporations design their own pays to extract more rents.

[2] Regulatory actions have already been taken to limit the use of stock options. For instance, in the American Recovery and Reinvestment act of 2009, stock options are explicitly prohibited in the executive compensation plans for the TARP recipients. Only two types of compensation, base salaries and restricted stock limited no more than half of base salaries are allowed.

originally owned by the firm. However, the project implementation is not made based on the ex ante information. Instead, we allow the CEO to collect private information about the risky project after exerting effort and make a second choice about which project to implement based on that piece of information. The CEO's effort is assumed to have a multiplicative effect on the firm value. Thus, we consider that CEO's actions, such as exerting effort and making project choices, are "rolled out" across the entire firm and thus have a greater effect in a larger firm. Moreover, CEO's preference is also assumed to be multiplicative. Both of the assumptions are made because they are better fit to the empirical data compared to the ones in the traditional agency model, i.e., effort has an additive impact on firm value and CEO has an additive cost of effort that is independent of his wealth.[3]

In the paper, we first demonstrate that when stock options should be a part of the optimal compensation contract and derive the optimal mix of stock and options taking into account both effort and project choice problems. Different from the previous papers, risk taking in our model is excessive from the shareholders' perspective and is a side effect of the provision of effort incentives. The CEO, after privately observing the quality of new project generated from his effort, makes the project implementation choice between the new risky one and the old safe one to maximize his expected compensation. Paying the CEO in restricted stock perfectly aligns his interests with that of the firm's shareholders in selecting the project. On the other hand, the CEO makes excessively risky project choices from the shareholders' perspective when options are included in his pay contract. The reason is straightforward that options make the CEO's pay function convex in the firm's value. As a result of motivating effort and curbing excessive risk taking, stock should always be a part of the optimal contract. Moreover, options should not be included in the pay contract if stock perform perfectly the role of motivating effort. However, if the CEO's effort is not extremely productive, or the CEO's private benefit from shirking is quite high, restricted stock cannot motivate effort even if effort is still worth exerting. The described circumstances occur as a special feature of the model where CEO's utility is multiplicative between monetary compensation and private benefit. When the CEO shirks, the monetary compensation from restricted stock is lower as the firm value is lower, yet the CEO's total utility could be higher because the private benefit is higher. Thus, to motivate effort, the pay structure needs to be convex in the firm value, i.e., to be insensitive in the lower value to further reduce the monetary compensation when the CEO shirks. Under the circumstances

[3]For further justifications of the multiplicative specifications, readers can refer to Edmans and Gabaix (2009).

described above, stock options as a part of optimal compensation can be rationalized.[4] The shareholders face a tradeoff of motivating effort and mitigating excessively risky project choice induced by options when deciding the optimal mix of restricted stock and stock options in the compensation package.

Second, we extend the multiplicative model with CEO's effort issue in Edmans, Gabaix, and Landier (2009) to consider a second agency problem: CEO's project choice. In our model, the CEO's incentive compatibility constraint has the following form: The ratio of the CEO's expected pay in case of exerting effort to his expected pay in case of shirking must exceed the ratio of the CEO's private benefit in case of shirking to his private benefit in case of exerting effort. The optimal contract is no longer detail-neutral as in Edmans, Gabaix, and Landier (2009). It still predicts the amount of restricted stock or stock options to be proportional to the CEO's total pay, yet a proper measure of the firm risk is necessary to provide the right "amount" of risk taking incentives. This is a direct implication that the CEO makes project choice for the firm. When the CEO's action concerns firm's risk choice, the optimal mix of stock and options is a function of the optimal project choice.

The third objective of the paper is to analyze the general market equilibrium by embedding the above double agency problem into a labor market where firms with different asset sizes compete for CEOs with different ability. Our departure from the traditional work[5] is that the managerial contracting affects the general equilibrium matching between firms and CEOs. In our model, high ability CEOs are with high chances to discover risky project with better quality. We show that, in the circumstance where firms are able to contract for CEOs' effort and the optimal project choices, a firm's expected value will be higher if it is matched with a higher ability CEO. Thus, there is positive assortive matching in the market equilibrium: Larger firms hire higher ability CEOs. However, in the circumstance where the optimal project choice can not be attained, larger firms may tend to hire lower ability CEOs. In the second best contracting, CEOs make excessively risky project choices, in which case firms' expected value will be decreased. A higher ability CEO is with higher chances to generate the projects that are considered as excessively risky from the perspective of a firm's shareholders. To motivate effort

[4]Notice that in this paper, we focus on the choice between restricted stock and stock options. Including options is not the only means to increase the convexity of the pay function. Cash bonuses with a performance target, for instance, could have the similar effect. But we could show as in Innes (1990) and Inderst and Mueller (2010), options are optimal in providing convexity.

[5]In Edmans, Gabaix, and Landier (2009), managerial contracting is separated from the equilibrium assignment.

in the second best setting, a firm has to pay its CEO in stock options, thus makes concession in curbing the CEO's excessive risk taking. Consequently, when a higher ability CEO is hired, the likelihood that this CEO is able to implement the excessively risky project is higher. Once the value losses due to the excessive risk taking is large, a firm tends to hire a lower ability CEO to reduce the probability of excessive risk taking. Positive assortive matching between firm size and CEO ability may fail to hold because the friction generated from the managerial contracting.

Our paper is related to the traditional literature in the following aspects. First, our analysis of the optimal structure of compensation contract is obviously linked to the debate about the optimality of stock options. Our paper contributes to the debate by focusing on the different role of restricted stock and stock options in providing effort and risk taking incentives. Stock always provide CEOs with the "right" incentives in taking the project risk, while options dominate stock in providing the effort incentives. Thus, options and stock are not always perfect substitutes in the optimal compensation contract.

Our double agency framework is built on the earlier work of Lambert (1986) where firm contracts with CEO for both the managerial effort and best project choice. Hirshleifer and Suh (1992) further analyzes a model where the manager first selects the project risk then exerts effort and shows that the optimal curvature of the manager's compensation contract depends on the trade-off between controlling project risk and motivating effort. The focus of these models is to motivate the risk averse CEO to take value enhancing risk. Instead, our model analyzes the CEO's excessive risky project choice as a side effect of providing effort incentives and focus on the choice between restricted stock and stock options.

The agency issues are studied in a model of multiplicative specifications for CEO preference and production functions. The academic researches of CEO compensation have drawn attention to the multiplicative model primarily because its fit for the empirical data. Edmans, Gabaix, and Landier (2009) studies a multiplicative model in presence of CEO's moral hazard problem and obtains a parsimonious form of optimal incentive contract that rationalizes the CEO's low fractional ownership and its negative relationship with firm size. Edmans and Gabaix (2011) further introduces CEO's risk aversion into Edmans, Gabaix, and Landier (2009). Our model follows the multiplicative specifications but focuses on the structure of contract and the interaction between the double agency issues: effort and project choice. Thanassoulis (2013) applies a multiplicative model to analyze CEO's short termism behavior and its interaction with industry structure. Peng and Röell (2014) also analyzes a multiplicative model and finds convex contracts for some parameterizations but their main focus is on price manipulation.

The works as Edmans, Gabaix, and Landier (2009), Edmans and Gabaix (2011), Thanassoulis (2012) and Thanassoulis (2013) consider the optimal contracting problem with a labor market where firms act competitively in hiring CEOs to endogenize not only the pay structure and but also the pay level. In most papers, the general equilibrium prescribes a positive assorting matching assignment between firms with different sizes and CEOs with different ability. There is one distinction, Edmans and Gabaix (2011) shows taking in account the CEO's risk aversion, equilibrium assignment is distorted from the positive assortive matching. Our model also considers the equilibrium assignment in the CEO labor market and challenges the positive assortive matching from another angle. We allow CEOs to make project choice decision and show the circumstance where large firms prefer to hire CEOs with lower ability occurs when reducing the excess in the CEOs' risk taking choice is the firms' major concern.

Section 2 presents the setup of basic model. We analyze the optimal contracting where the board of a representative firm contracts with a representative CEO in fixed salary, restricted stock or stock options for effort and project choice in Section 3. Section 4 extends the baseline model to incorporate a competitive labor market where the competitive assignment between firms and CEOs is analyzed. Section 4 discusses the exercise price of options and the relationship between firm size and structure of optimal contract. Section 5 concludes.

2 The Basic Model

The basic model considers the case where a risk neutral female principal contracts with a risk neutral male agent to elect effort and induce best project choice in a firm. The principal can be the board whose objective is to maximize the expected net firm' value. The agent is the CEO hired by the firm whose objective is to maximize his expected utility. The model has one period and 3 dates $t = 0, 0.5, 1$. At $t = 0$, the board offers a contract to the CEO, the CEO chooses whether to exert effort to expand the firm's project opportunity. At $t = 0.5$, if effort is exerted, the CEO makes a second choice regarding the project implementation based on his private information. At $t = 1$, the return realizes.

2.1 Projects, effort and project choice

The initial ($t = 0$) value of the firm's asset is I. The firm has already owned a safe project. Investing asset I in the safe project, the firm's value at $t = 1$ is $\underline{y}I$, where the unit value of the safe project is certain $\underline{y} > 1$. Alternatively, the CEO can expend effort to innovate a new risky project for the firm. Conditional on the risky project being implemented, the firm's value at $t = 1$ is $\tilde{y}I$ if the project succeeds and zero if it fails. The success probability of the risky project is φ with the random unit value $\tilde{y}$ distributed on the interval $[\underline{y}, \overline{y}]$. If the CEO shirks, the only available project is the safe project. We assume that the risky project has higher expected value than the safe project at $t = 0$ and focus on the case that the firm always wants to induce the CEO's effort.

Denote the CEO's unobservable action to exert effort (shirk) as $e = 1$ ($e = 0$). Once effort is exerted, the CEO can privately observe a signal θ about the risky project at $t = 0.5$. The conditional distribution function of the unit value $\tilde{y}$ is $F(y|\theta)$ on $[\underline{y}, \overline{y}]$. We assume that the signal is informative in the sense that the project with higher θ has better distribution of value: $F(y|\theta_2)$ first order stochastically dominates $F(y|\theta_1)$ for any $\theta_2 > \theta_1$. The signal can be understood as the project quality, higher quality risky project has better value distribution. On the other hand, the board only knows that θ is distributed on the interval $[\underline{\theta}, \overline{\theta}]$ with distribution function $G(\theta)$.

After receiving the private signal about the quality of the new risky project, the CEO decides which project to implement at $t = 0.5$. We assume the project implementation is mutually exclusive between the safe and the risky project. The unit value of safe project is certain while the expected value of risky project increases in its realized quality θ[6], we assume the CEO's project choice follows a threshold rule, i.e., there exists a threshold $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$ such that he chooses the risky project once the quality is larger than the threshold $\theta > \hat{\theta}$ and the safe project otherwise.

2.2 CEO Compensation

The firm's value at $t = 1$ is observable and verifiable, yet, the action of exerting effort and project quality θ are the CEO's private information. The board faces a double agency problem. Judging from the realized firm value, she neither knows for sure whether a choice of risky

[6] We assumed that quality θ improves the distribution $F(y|\theta)$ in the sense of FOSD. A necessary condition of FOSD is that for any $\overline{\theta} \geq \theta_2 > \theta_1 \geq \underline{\theta}$, the expectation $E[y|\theta_2] > E[y|\theta_1]$.

project is because of its higher expected value nor a choice of safe project is not because the CEO shirks. To tackle with this problem, the board can design a CEO compensation contract with three components, fixed salary, restricted stock and stock options. Fixed salary α is the amount of cash granted to the CEO on a non-performance basis. Normalize the number of the bank's outstanding shares to be one, $\beta \in [0, 1]$ is the number of the shares granted to the CEO at $t = 0$. The CEO is allowed to sell these shares at $t = 1$. $\gamma \in [0, 1]$ is the number of the stock options granted to the CEO at $t = 0$ with an exercise price $\hat{y} \in (\underline{y}, \overline{y})$. To summarize, the CEO's total realized compensation at $t = 1$ is

$$c(yI) = \alpha + \beta yI + \gamma \max(y - \hat{y}, 0)I$$

Lastly, there is the normal limited liability constraint $\alpha \geq 0$.

2.3 Utility:

The board maximizes the expected net firm value (the total value net the compensation cost) from the investment. The CEO has a multiplicative utility function:

$$U = E[c\phi(e)].$$

c is denoted as the CEO's monetary compensation. We let $\phi(e)$ characterizes the cost of effort with $\phi(1) = 1 < \phi = \phi(0)$ and $\phi > 1$. Thus, $\phi(e)$ can be understood as the CEO's equivalent monetary utility from leisure, with more utility derived from leisure when the CEO shirks. The CEO's reservation utility is u. Notice that the CEO's utility is a multiplication of the monetary compensation and utility from leisure. This type of multiplicative preference is commonly used in Macroeconomics and Labor Economics. In Edmans, Gabaix, and Landier (2009), the authors find this type of multiplicative preference fits the empirical evidences regarding to the incentives and firm sizes quite well. Throughout the paper, we will assume the optimal level of effort is always to be $e = 1$.

3 Optimal Contracting in Partial Equilibrium

In this section, we discuss the contracting in the basic model treating the CEO's reservation utility u as exogenous. In the next section, we will extend the basic model to incorporate a competitive labor market. We first present the benchmark case where the contract could be directly written on the CEO's effort and best project choice maximizing the firm's value. We then move to the case where compensation contract needs to be designed to trade off the provision of effort and controlling excessive risk taking. We characterize the IC constraint in a multiplicative model, which is slightly different from the one in additive model. We show in the first best contracting, the optimal contract can use only stock to provide incentives. While the second best contract prescribes an optimal mix of shares and options trading off the needs to motivate effort and control excessive risk taking.

3.1 Benchmark:Contractible Effort and Project Choice

To start, suppose the contract can directly designate the CEO to exert effort and choose the project desired by the board. Then the board simply pays the CEO to participate $c = u$, demands the CEO's effort and chooses the project choice to maximize the expected net firm value

$$\Pi(\hat{\theta}) = \int_{\hat{\theta}}^{\bar{\theta}} E[y|\theta] I dG(\theta) + \int_{\underline{\theta}}^{\hat{\theta}} y_s I dG(\theta). \tag{1}$$

We assume this function is strictly concave to $\hat{\theta}$. Thus, the first order condition with respect to project choice $\hat{\theta}$ gives $E[y|\hat{\theta}] = \underline{y}$, where $E[y|\hat{\theta}] = \varphi \int_{\underline{y}}^{\bar{y}} y dF(y|\hat{\theta})$. The first best project choice is the solution of this first order condition. As we have already discussed, the expected value of risky project conditional on a project choice $\hat{\theta}$ is increasing in the project choice. We focus on the interior solution by assuming $E[y|\underline{\theta}] < E[y|\theta^{FB}] < E[y|\bar{\theta}]$. Then there exists a unique first best project choice $\theta^{FB} \in (\underline{\theta}, \bar{\theta})$ defined as the solution

$$E[y|\theta^{FB}] = \underline{y} \tag{2}$$

To maximize the expected firm value, The risky project is chosen if and only if its quality is higher than θ^{FB}. We then turn to the case that the compensation contract can only be contingent on the realization of the firm value.

3.2 Managerial Contracting in the Partial Equilibrium

In this subsection, we analyze the contracting problem between the board and the CEO. Under the equilibrium approach that effort is exerted, the CEO's expected compensation from a risky project with quality θ at $t = 0.5$ is

$$\alpha + \beta E[y|\theta]I + \gamma E[\max(y - \hat{y}, 0)|\theta]I$$

at $t = 0.5$. The component $E[\max(y - \hat{y}, 0)|\theta] = \varphi \int_{\underline{y}}^{\overline{y}} \max(y - \hat{y}, 0) dF(y|\theta)$ is the unit value of the option conditional on the project quality θ and exercise price $\hat{y}$. On the other hand, the CEO's compensation from the safe project is simply $\alpha + \beta \underline{y} I$. The risk neutral CEO chooses to implement the project with higher expected compensation, thus he chooses the risky project if and only if its quality is higher than the threshold, which is define as

$$\beta E[y|\hat{\theta}] + \gamma E[\max(y - \hat{y}, 0)|\hat{\theta}] = \beta \underline{y}. \tag{3}$$

For the similar reason, the unit option value increases in θ as well. We assume $\hat{\theta}$ defined in this equation is interior as well. Thus, $\hat{\theta}$ defined in (3) is existent and unqiue.

Anticipating the project choice rule $\hat{\theta}$, the expected compensation when the CEO exerts effort at $t = 0$ is

$$E[c\phi(e)|e = 1, \hat{\theta}] = \alpha + \int_{\hat{\theta}}^{\overline{\theta}} \{\beta E[y|\theta] + \gamma E[\max(y - \hat{y}, 0)|\theta]\} I dG(\theta) + \int_{\underline{\theta}}^{\hat{\theta}} \beta \underline{y} I dG(\theta).$$

This expression can be reformulated to

$$E[c\phi(e)|e = 1, \hat{\theta}] = \alpha + \beta[\underline{y} + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)]I + \gamma \int_{\hat{\theta}}^{\overline{\theta}} V_o(\hat{\theta}) dG(\theta))I$$

where the integrand $\Delta V_y(\theta) = E[y|\theta] - \underline{y}$ is the difference in expected value between implementing a risky project and a safe project conditional on the quality θ. Then, the term $\int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)$ is the unit "upside" value of implementing the risky project conditional on that the CEO follows project choice rule $\hat{\theta}$. The integrand $V_o(\theta) = E[\max(y - \hat{y}, 0)|\theta]$ is the expected unit value of option conditional on the quality θ. Similarly, the term $\int_{\hat{\theta}}^{\overline{\theta}} V_o(\hat{\theta}) dG(\theta))$ represents the unit option value conditional on that the CEO follows project choice rule $\hat{\theta}$. And the term in the parenthesis $\underline{y} + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y dG(\theta)$ is the expected unit firm value conditional on the project choice $\hat{\theta}$. On the other hand, the CEO's compensation is $E[c\phi(e)|e = 0] = \phi[\alpha + \beta \underline{y} I]$ when he shirks. The utility is a multiplication of the monetary compensation $\alpha + \beta \underline{y} I$ and ϕ the private benefit from leisure. Remember that the private benefit is 1 when the CEO works. Thus it is incentive compatible for the CEO to exert effort if $E[c\phi(e)|e = 1] \geq E[c\phi(e)|e = 0]$, that is

$$\frac{\alpha + \beta[\underline{y} + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta)dG(\theta)]I + \gamma \int_{\hat{\theta}}^{\overline{\theta}} V_o(\theta)dG(\theta))I}{\alpha + \beta \underline{y}I} \geq \phi \tag{4}$$

Notice that the IC constraint in a multiplicative preference model differs from the one in an additive model. The incentives to exert effort prescribes a ratio between the CEO's expected pay when he works to the expected pay when he shirks conditional on the CEO's ex post project choice. Motivating effort requires this ratio between the CEO's expected pays conditional on a certain project choice $\hat{\theta}$ outweighs the ratio between the CEO's private benefit from leisure when he shirks to the private benefit when he works $\frac{\phi}{1}$.

The CEO's participation constraint $E[c\phi(e)|e = 1] \geq u$ can be expresses as

$$\alpha + \beta[\underline{y} + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta)dG(\theta)]I + \gamma \int_{\hat{\theta}}^{\overline{\theta}} V_o(\theta)dG(\theta))I \geq u. \tag{5}$$

Then there is the limited liability constraints $\alpha \geq 0$. The expected end-of-period return of the firm can be expressed as:

$$\Pi = [\underline{y} + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta)dG(\theta)]I - \left\{\alpha + \beta[\underline{y} + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta)dG(\theta)]I + \gamma \int_{\hat{\theta}}^{\overline{\theta}} V_o(\theta)dG(\theta))I\right\}. \tag{6}$$

The firm maximizes the expected return (6) subjects to the project choice condition (3), the IC constraint for effort (4), the participation constraint (5) and the limited liability constraints.

We assume that firm's objective function Π is strictly concave to the project choice rule $\hat{\theta}$. To solve this program, we construct the Lagrange function, the firm's program can be write as:

$$\begin{aligned} \max_{\alpha,\beta,\gamma,\hat{\theta}} \quad & \mathbf{L} = \Pi + \lambda_1\{E[cg(e)|e = 1, \hat{\theta}] - E[cg(e)|e = 0]\} + \lambda_2\{E[cg(e)|e = 1, \hat{\theta}] - u\} \\ & + \eta\alpha + \mu\{\beta\underline{y} - \beta E[y|\hat{\theta}] - \gamma V_o(\hat{\theta})\}I. \end{aligned}$$

λ_1, λ_2 and η are non-negative lagrange multipliers associated with the IC constraint, IR constraint and the limited liability constraint. We solve this lagrange problem in the appendix. The solution is presented in the following Lemma.

Lemma 1. *The solution of the board's program satisfies*

$$(\lambda_1 + \lambda_2 - 1) + \eta = \phi\lambda_1 \tag{7}$$

$$(\lambda_1 + \lambda_2 - 1)[\underline{y} + \int_{\hat{\theta}}^{\bar{\theta}} \Delta V_y(\theta) dG(\theta)] = \phi\lambda_1 \underline{y} + \mu \Delta V_y(\hat{\theta}) \tag{8}$$

$$(\lambda_1 + \lambda_2 - 1) \int_{\hat{\theta}}^{\bar{\theta}} V_o(\theta) dG(\theta) = \mu V_o(\hat{\theta}) \tag{9}$$

$$-\Delta V_y(\hat{\theta}) + (\lambda_1 + \lambda_2 - 1)\left[-\beta \Delta V_y(\hat{\theta}) - \gamma V_o(\hat{\theta})\right] - \mu \left[\frac{dE[y|\hat{\theta}]}{d\hat{\theta}} + \frac{dV_o(\hat{\theta})}{d\hat{\theta}}\right] = 0 \tag{10}$$

There are two sets of solution depending on parameter value. (1) If the parameters satisfy the condition

$$1 + \int_{\theta^{FB}}^{\bar{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG(\theta) \geq \phi \tag{11}$$

the solution is $\mu = 0$, $\lambda_1 = 0$, $\lambda_2 > 0$ *and* $\eta = 0$. *(2) If the parameters are such that (11) does not hold, the solution is* $\mu > 0$, $\lambda_1 > 0$, $\lambda_2 > 0$ *and* $\eta > 0$. *There exists no solution for* $\mu < 0$.

Proof. See Appendix A □

The four first order conditions are derived with respect to the amount of fixed salary α, the number of shares β, the number of options γ and the project choice rule $\hat{\theta}$. We first consider the FOC of the board's program with respect to the project choice rule $\hat{\theta}$

$$-\Delta V_y(\hat{\theta}) + (\lambda_1 + \lambda_2 - 1)\left[-\beta \Delta V_y(\hat{\theta}) - \gamma V_o(\hat{\theta})\right] - \mu(\frac{dE[y|\hat{\theta}]}{d\hat{\theta}} + \frac{dV_o(\hat{\theta})}{d\hat{\theta}}) = 0$$

From the CEO's ex post project choice condition (3), the part

$$-\beta \Delta V_y(\hat{\theta}) - \gamma V_o(\hat{\theta}) = \beta \underline{y} - \beta E[y|\hat{\theta}] - \gamma E[\max(y - \hat{y}, 0)|\hat{\theta}] = 0.$$

Moreover, we know that $\frac{dE[y|\hat{\theta}]}{d\hat{\theta}} > 0$ and $\frac{dV_o(\hat{\theta})}{d\hat{\theta}} > 0$ because the distribution function $F(y|\theta)$ satisfies FOSD with respect to θ and both the integrands y and $\max(y - \hat{y}, 0)$ are increasing function of y. Thus, the sign of μ is determined by

$$sgn[\mu] = sgn[-\Delta V_y(\hat{\theta})] = sgn[\underline{y} - E[y|\hat{\theta}]].$$

When $\mu = 0$, the CEO maximizing the expected compensation based on the ex post information of quality follows the project choice rule $\underline{y} = E[y|\hat{\theta}]$. From the discussion in the last subsection, this equation defines the unique project choice rule that maximizes the expected value of the firm, i.e. $\hat{\theta} = \theta^{FB}$. In order this equation to be held, from (3), it must be $\gamma = 0$ and $\beta > 0$. The firm has to grant the CEO with positive amount of stock but no options. Intuitively, notice first the amount of fixed salary is irrelevant to the CEO's project choice decision as it is paid across all states. Second, options motivate the CEO to make more risky project choice. By applying implicit function theorem on (3), we obtain

$$\frac{\partial\hat{\theta}}{\partial\gamma} = -\frac{V_o(\hat{\theta})}{\frac{dE[y|\hat{\theta}]}{d\hat{\theta}} + \frac{dV_o(\hat{\theta})}{d\hat{\theta}}} < 0.$$

As to the stock, the impact on project choice decision depends on the range of project choice

$$\frac{\partial\hat{\theta}}{\partial\beta} = -\frac{\Delta V_y(\hat{\theta})}{\frac{dE[y|\hat{\theta}]}{d\hat{\theta}} + \frac{dV_o(\hat{\theta})}{d\hat{\theta}}} = \begin{cases} > 0 & \hat{\theta} < \theta^{FB} \\ = 0 & \hat{\theta} = \theta^{FB} \\ < 0 & \hat{\theta} > \theta^{FB} \end{cases}$$

Thus, at the first best project choice rule θ^{FB}, changing the amount of the stock has no marginal effect on the CEO's project choice decision. In order to induce the CEO to follow the first best choice rule, it is necessary that the firm uses solely restricted stock. But this is only possible only if (11) satisfies. To understand this condition, notice that it actually comes from (4)

$$\frac{\beta[\underline{y} + \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)]I}{\beta\underline{y}I} \geq \phi$$

with $\alpha = 0$ and $\gamma = 0$. Thus, this condition is, in fact, the necessary condition for the firm can use only stock to motivate the managerial effort. Notice that in the multiplicative model, the condition (11) prescribes a ratio between the expected compensation from stock when the CEO works and the expected compensation from stock when the CEO shirks. This ratio has to be higher than the private benefit from shirking ϕ. Actually, this condition is sufficient to induce the effort and first best project choice rule as well. This explains why in the Lemma 1, the IC constraint is slack $\lambda_1 = 0$. Notice that when (11) holds, the firm simply chooses the amount of fixed salary to make the CEO just participate to minimize the compensation cost, so $\lambda_2 > 0$ and $\eta = 0$, IR constraint is binding while LL constraint is slack. This leads to our Proposition 1 that characterizes the first best contract.

Proposition 1. *The first best CEO compensation contract is as follows. When condition (11) holds, effort and the project choice θ^* can be implemented through a contract with shares and fixed salary. It prescribes* $\left\{\alpha^{FB} = [1 - \frac{(\phi-1)\underline{y}}{\int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)}]\frac{u}{\phi}, \beta^{FB} I = \frac{\phi-1}{\int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)}\frac{u}{\phi}, \gamma^{FB} = 0\right\}$. *The expected firm value is*

$$\Pi(\theta^{FB}) - u = [\underline{y} + \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)]I - u \tag{12}$$

Proof. See Apendix B. □

To get a more intuitive understanding of this contracts, now (11) holds, the board use s-tock alone to motivate effort and implement the project choice θ^{FB}. The benefit of exerting effort for the firm is the "upside" value (expected increase in firm value) from the risky project $\int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta) I$. The cost of motivating effort for the board is the increase in the amount of total monetary compensation $\frac{\phi-1}{\phi}u$. Notice that when the CEO exerts no effort, the board offers a total pay $\frac{u}{\phi}$ to induce the CEO just to participate. To motivate effort, the compensation contract prescribes that the minimum amount of shares β^{FB} must be at least to maintain the equality between the benefit and cost $\beta^{FB} I \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta) = \frac{\phi-1}{\phi}u$. And remember that under the first solution the CEO's IC is slack.

Consider the second solution in Lemma 1. $\mu > 0$ corresponds to the case (11) does not hold. Effort and the first best project choice can not be induced together when the compensation contract is only consist of restricted stock and fixed salary. Under this case, the board can include options, which realize only if the high values occur $y > \hat{y}$ into the compensation contract to motivate the CEO's effort. In our model, the high firm values occur only if the CEO exerts effort to develop the risky project, but the argument is more general. According to Innes (1990), as long as effort increases the occurrence of the higher values in the sense that the distribution function of firm value satisfies the monotonic likelihood ratio property, option-like instrument is superior to motivate effort. In our current model, this advantage of options to motivate effort has to be balanced with the board's needs of controlling excessive risky project choice, which will be presented soon. Before presenting our main tradeoff regarding the restricted stock and stock options, we briefly analyze the case where only options are used to highlight the CEO's excessive risk taking induced by options.

Consider the case where the compensation contract is only consist of fixed salary α and γ units of options. Along the equilibrium approach that effort has already been exerted, the

CEO's expected compensation from a risky project with quality θ is $\alpha + \gamma V_o(\hat{\theta})$. On the other hand, the compensation from the safe project now results in only the fixed salary α. Options make the CEO's compensation non responsive to the project values that are below the exercise price $\hat{y}$. A direct implication is that the CEO always selects the risky project no matter its quality, as $\gamma V_o(\hat{\theta})I > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. In this case, we write the project choice rule $\hat{\theta} = \underline{\theta}$ denoting that any risky project is chosen by the CEO ex post.

Knowing the CEO always chooses the risky project, the option contract designed to motivate effort and ensure participation solves $\alpha + \gamma \int_{\underline{\theta}}^{\overline{\theta}} V_o(\theta)dG(\theta)I \geq \phi\alpha$ and $\alpha + \gamma \int_{\underline{\theta}}^{\overline{\theta}} V_o(\theta)dG(\theta)I \geq u$. Motivating managerial effort is not an issue when the compensation contract containing only fixed salary and options. There exists a continuum of contracts that results the same compensation cost for the firm. For instance, the board can simply choose the amount of fixed salary as $\alpha = 0$ and the unit of options as $\gamma I = \frac{u}{\int_{\underline{\theta}}^{\overline{\theta}} V_o(\theta)dG(\theta)}$. Under this contract, effort is induced while participation constraint is just binding.

The firm's expected value decreases as the project choice deviates from the first best rule θ^{FB}. The problem of options is that the CEO always makes the most dangerous project choice, i.e. $\hat{\theta} = \underline{\theta}$ under pure option contracts (the contracts without restricted stock). Thus, in the following, we discuss the tradeoff of granting options faced by the board.

Following the discussion of the case $\mu > 0$, we consider the compensation contract is consist of both stock and options. Stocks, even if they can not motivate effort when (11) does not hold, are granted to balance the excessive risk taking incentives of the CEO from the options. The board now faces a tradeoff in the designing of compensation contract: motivating effort against controlling the excessive risk taking. It can be seen from the derivatives $\frac{\partial\hat{\theta}}{\partial\beta} > 0$ and $\frac{\partial\hat{\theta}}{\partial\beta} > 0$ when $\hat{\theta} < \theta^{FB}$. Increasing marginally the amount of stock increases the project choice rule while increasing marginally the amount of options decreases the project choice conditional on the project choice rule lies in the range $[\underline{\theta}, \theta^{FB})$. The second best contract balancing this tradeoff prescribes the following: Given the amount of shares, the amount of options is chosen to be just enough to motivate the CEO's effort. This provides the CEO with the least incentives to make risky project choices that are considered as excessive risky according to the first best choice rule. Further reduces risk taking incentives by decreasing the amount of options is not feasible, as the CEO's IC constraint is binding, $\lambda_1 > 0$. Moreover, fixed salary is paid across all states, the CEO derives utility α when he works and $\phi\alpha > \alpha$ when he shirks. Thus, fixed salary actually disincentivizes the effort if the CEO has multiplicative preference. When the board needs to limit the amount of options to control excessive risk choice, there is no doubt she first chooses $\alpha = 0$. This is why the limited liability constraint is binding, $\eta > 0$ in the second solution.

Then the binding IC constraint can be expressed as

$$\frac{\beta[\underline{y} + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)]I + \gamma \int_{\hat{\theta}}^{\overline{\theta}} V_o(\theta) dG(\theta))I}{\beta \underline{y} I} = \phi$$

Combine with the project choice rule (3), this equation can be expressed as

$$1 + \int_{\hat{\theta}}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\hat{\theta})}{\underline{y}} \frac{V_o(\theta)}{V_o(\hat{\theta})} \right] dG(\theta) = \phi$$

Remember that $V_o(\theta) = E[\max(y - \hat{y}, 0)|\theta]$. We can decompose this equation into more intuitive form

$$\underbrace{1 + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)}_{\text{Incentives provided by 1 stock}} + \frac{-\Delta V_y(\hat{\theta})}{V_o(\hat{\theta})} \underbrace{\int_{\hat{\theta}}^{\overline{y}} V_o(\theta) dG(\theta)}_{\text{Incentives provided by 1 option}} = \phi \underline{y}.$$

Normalize the amount of restricted stock to be one, the term $\frac{-\Delta V_y(\hat{\theta})}{V_o(\hat{\theta})} > 0$ represents the amount of options required to maintain the project choice rule to be $\hat{\theta}$. The optimal contract prescribes that the incentives (expected compensation) provided by stock and options conditional on the project choice rule to be $\hat{\theta}$ just offset the CEO's private benefit of shirking, i.e., higher private benefit from leisure ϕ multiply the monetary compensation $\underline{y}$ from one stock.

Motivating the managerial effort in the second best setting in a multiplicative preference model is different from that in an additive model. In an additive model, if the limited liability constraint is binding, the CEO receives limited liability rent and the project choice is excessive risky compared to the first best level.[7] However, in a multiplicative model, when the limited liability constraint binds, the CEO receives no rent. The reason is that the board can scale up or down the level of stock and options while maintaining the optimal mix of the two to induce the CEO's effort. Then to minimize the compensation cost, the board chooses the level of stock and options to just bind the CEO's IR constraint, thus $\lambda_2 > 0$ again.

The following Proposition 2 characterizes the second best project choice θ^{**}, proves its existence and uniqueness and derives the optimal contract.

[7]For the moral hazard problem when effort and risk choice are both concerned in an additive model, readers can refer to the paper Inderst and Mueller (2010).

Proposition 2. *When the condition (11) does not hold, effort can not be motivated solely by stock and fixed salary. The board chooses the amount of restricted stock and stock options trading off the needs to motivate effort against the needs to curb the excessive risky project choice. The second best contract prescribes* $\{\alpha^* = 0, \beta^* I = \frac{u}{\phi \underline{y}}, \gamma^* I = \frac{-\Delta V_y(\theta^*)}{V_o(\theta^*)} \frac{u}{\phi \underline{y}}\}$*. The second best project choice* θ^* *solving*

$$1 + \int_{\theta^*}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\theta^*)}{\underline{y}} \frac{V_o(\theta)}{V_o(\theta^*)} \right] dG(\theta) = \phi \tag{13}$$

is existent and unique in $(\underline{\theta}, \theta^{FB})$*, thus it is more risky than the first best choice rule* θ^{FB}*. The expected firm value is*

$$\Pi(\theta^*) - u = [\underline{y} + \int_{\theta^*}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)] I - u \tag{14}$$

Proof. See Appendix C □

Now (11) does not hold, managerial effort can not be solely motivated by restricted stock. The total amount of options granted is $\gamma^* I = \frac{-\Delta V_y(\theta^*)}{V_o(\theta^*)} \frac{u}{\phi} > 0$ inducing the CEO to follow a more risky project choice rule than the first best one $\theta^* < \theta^{FB}$. Notice that $\Delta V_y(\theta^*) = E[y|\theta^*] - \underline{y} < 0$ under this case. This is due to the fact that $E[y|\theta]$ increases in θ. The optimal contract prescribes a mix of shares and options trading off the benefit of using options to motivate effort against its shortcoming of leading to excessive risky project choice.

Lastly, there exist no solution such that $\mu < 0$, i.e. $\hat{\theta} > \theta^{FB}$. This is due to our risk neutrality assumption of the CEO's preference. In our model, the distortion highlights the binding limited liability constraint and the private benefit is a proportion to the monetary compensation. As a result, there exists no risk and efficiency tradeoff as in the case where the CEO has a risk averse preference, he does not seek an insurance for bearing the risk from the uncertainty in the compensation. If $\mu < 0$ were a solution, it actually required that the board granted the CEO with negative amount of options $\gamma < 0$. This is because in the (3), $E[y|\hat{\theta}] > \underline{y}$ when $\hat{\theta} > \theta^{FB}$.

4 Optimal Contract in General Market Equilibrium

In this section, we incorporate a competitive labor market between firms and CEOs. In most models with the competitive labor market, the equilibrium between firms and CEOs results in a positive assortive matching assignment, i.e., larger firms are assigned to higher ability CEOs. Edmans, Gabaix, and Landier (2009) shows that this assignment maximizes the total surplus of production in the economy and can be separated from the provision of effort incentives. In our model, we present a different situation where provision of incentives matters for the equilibrium assignment. We show in this section, if the firms can always provide enough effort incentives while maintaining the first best project choice, then positive assortive matching holds. Otherwise, there could be circumstances where larger firms hire CEOs with lower ability to reduce the chance that CEOs make excessive project choices.

We first extend the basic model to consider a labor market that opens at $t = 0$. The market participants are N firms ranking by their asset sizes $I^1 < I^2 < ... < I^N$ and N CEOs ranking by their monitoring ability $G_1 < G_2 < ... < G_N$. The monitoring ability of a CEO n is characterized by the conditional distribution function $G_n(\theta)$ of the quality θ of the risky project generated by the CEO. We assume that the higher ability CEO has higher chances to generate the better quality projects. The ability improves the distribution of θ in the sense of first order stochastic dominance, i.e., for any two CEOs m, n with ranking $m < n$, $G_n(\theta)$ FOSD $G_m(\theta)$ [8]. Larger firms and higher ability CEOs are labeled in higher indices. We assume that CEOs' ability and firms' asset sizes are observable by all the market participants at $t = 0$. The firms make competitive bids in the form of compensation to hire the CEOs. One firm hires exactly one CEO to maximize expected firm value and each CEO accepts the contract with the highest expected total utility. Finally, we normalize the outside opportunity of the CEO with lowest ability to be $\underline{u}$.

We first consider the case where all CEOs can be hired in the first best contracts. Following the discussion in the last section, we can denote a firm's expected value in the partial equilibrium as a function of its asset size, the ability of CEO being hired and the optimal project choice. A firm indexed by i hires a CEO indexed by m in the first best contracting, its expected value is

$$\Pi(\theta^{FB}, i, m) - u_m = [\underline{y} + \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG_m(\theta)] I^i - u_m$$

Notice that in the first best contracting, the CEO's ability does not affect the optimal project choice θ^{FB}. The reason is that the first best project choice follows a "first order" rule, the CEO chooses the project to maximize compensation after the quality θ is realized. On the other hand, the ability of CEO m determines the distribution $G_m(\theta)$ of quality θ, thus affects the provision of the ex ante effort of CEO to develop risky project instead of the ex post project choice after

[8]Remember that the expected value of risky project $E[y|\theta]$ increases in the project quality θ because the conditional distribution function of project value $F(y|\theta)$ is assumed to be FOSD in the quality.

θ is realized. To attain the first best contracting requires a condition similar to (11) holds for all CEOs.

The following Lemma 2 establishes the complementary of CEO ability and firm asset in the first best contracting in order to generate the productive efficiency.

Lemma 2. *Once the sufficient and necessary condition for all N CEOs are hired in the first best contracts* $1 + \int_{\theta^{FB}}^{\overline{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG_1(\theta) \geq \phi$ *is satisfied, the total production* Π *of any firm-CEO pair exhibits complementarity between CEO's ability and firm's asset size. That is, for CEO* $m < n$ *and firm* $i < j$

$$\Pi(\theta^{FB}, j, n) + \Pi(\theta^{FB}, i, m) > \Pi(\theta^{FB}, j, m) + \Pi(\theta^{FB}, i, n)$$

Proof. See Appendix D □

If the firms can secure the first best contracting with the CEOs, there is complementarity between CEOs' ability and firms' asset in production. Consequently, firms with larger asset size hires higher ability CEOs produces larger economic surplus. The following Proposition 3 characterizes the general market equilibrium in the case of first best contracting.

Proposition 3. *In the circumstance where all CEOs can be induced to choose the optimal project choice* θ^{FB}*. The equilibrium in the competitive labor market results in positive assortive matching: a firm with kth largest asset size exactly hires a CEO with the same ranking in ability.*

Proof. See Appendix E. □

Remember that $G_i(\theta) < G_{i-1}(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. The complementarity property holds, a match between larger firm and higher ability CEO creates higher (expected) surplus. In the simplest case when two firms compete for two CEOs with different ability, the larger firm is able to offer the higher ability CEO with higher expected compensation. In equilibrium, the larger firm actually makes a bid equaling to the smaller firm's maximum willingness to pay for the higher ability CEO. Thus, the equilibrium results in a positive assortive matching. Apply this logic to the competitive bidding between N firms and N CEOs, we obtain the result in Proposition 3. In the equilibrium, the effort of CEOs can always be induced, a CEO's total utility equals to the expected pay. The total pay of CEO k matched with bank k is such that if this CEO were hired by firm $k-1$, firm $k-1$ would be indifference from hiring the lower ability CEO $k-1$. In a other words, the total pay u_k received by CEO k matched with firm k equals to bank $k-1$'s maximum willingness to pay for this CEO, that is

$$
\begin{aligned}
u_{k,k-1} &= \left[\int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG_k - \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG_{k-1} \right] I^{k-1} + u_{k-1} \\
&= \left[\int_{\theta^{FB}}^{\overline{\theta}} [G_{k-1}(\theta) - G_k(\theta)] \frac{d\Delta V_y(\theta)}{d\theta} d\theta \right] I^{k-1} + u_{k-1}.
\end{aligned}
$$

The second equation is the result of integrating by parts. As a result, the bank k simply offers $u_k = u_{k,k-1}$ to win the competition because it has more resources. In the proof of Proposition 4, we also derive each CEO's total pay in equilibrium by iteration.

We then turn to the case of second best contracting. We suppose now a firm indexed by i can only hire a CEO indexed m in the second best contracting. From Proposition 2, the firm i's expected value is

$$
\Pi(\theta_m^*, i, m) - u_m = [\underline{y} + \int_{\theta_m^*}^{\overline{\theta}} \Delta V_y(\theta) dG_m(\theta)] I^i - u_m
$$

Different from the case in the first best contracting, now the second best project choice θ^* is affected by the CEO's ability. θ^* is distorted downwards from θ^{FB} because options are included in the contract to motivate managerial effort. From the condition (13), we know that the distribution of θ affects the second best optimal threshold θ^*. As a result, we denote θ_m^* as the second best project choice rule made by the CEO m whose ability is characterized by $G_m(\theta)$.

The following Lemma 3 characterizes the relationship between the CEO ability and the second best project choice.

Lemma 3. *The sufficient and necessary condition for all N CEOs are hired in the second best contracts is* $1 + \int_{\theta^{FB}}^{\overline{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG_N(\theta) < \phi$. *Under this condition, the higher ability CEO makes safer second best project choice, i.e., for any two CEOs* m, n *with* G_n *FOSD* G_m, $\theta_m^* < \theta_n^* < \theta^{FB}$.

Proof. See Appendix F □

The intuition for this result has the root that higher ability CEO generates better distribution of risky project. Consider the equation determining the second best project choice (13). Normalize the number of restricted stock to be one, the higher ability CEO receives more compensation (in expectation) from restricted stock because the quality of risky project generated by his effort is better. On the other hand, ability does not affect the CEO's utility when he shirks. This means a firm's (unit) restricted stock deliver more incentives when higher ability CEO is hired. The firm thus use relatively less options when the higher ability CEO is hired. This leads to a less risky second best project choice rule made by the higher ability CEO.

Given the Lemma 4, the following Proposition 5 characterizes the hiring for CEOs in the second best contracting. Different from the first best case, we show that large firms can end up in hiring low ability CEOs.

Proposition 4. *Suppose N CEOs are hired in the second best contracting. There exists cases that the expected value of a firm is lower when a higher ability CEO is hired. The positive assortive matching does not always hold.*

Now we are in the second best world. Consider a firm's decision between hiring a two CEOs m and n with $G_n(\theta)$ FOSD $G_m(\theta)$, the CEO n has higher ability in generating new risky project. The firm is willing to pay the higher ability CEO with higher compensation only if the firm value increases under his management. Then we consider the difference in the expected unit value when the firm hires the higher ability CEO n other than the CEO m

$$\int_{\theta_n^*}^{\overline{\theta}} \Delta V_y(\theta) dG_n(\theta) - \int_{\theta_m^*}^{\overline{\theta}} \Delta V_y(\theta) dG_m(\theta).$$

It can be further expressed into the two terms

$$\int_{\theta_n^*}^{\overline{\theta}} \Delta V_y(\theta) d[G_n(\theta) - G_m(\theta)] - \int_{\theta_m^*}^{\theta_n^*} \Delta V_y(\theta) dG_m(\theta)$$

The second term in this expression $-\int_{\theta_m^*}^{\theta_n^*} \Delta V_y(\theta) dG_m(\theta)$ is positive. This is because $\theta_m^* < \theta_n^*$ and the risky project has a lower value than the safe project when its quality is lower than θ^{FB}. Thus the difference in expected value between the risky project and the safe project is negative $\Delta V_y(\theta) < 0$ under this case. This term represents a positive effect of the CEO ability on the firm value: when hiring a higher ability CEO, the firm is able to design contract to induce the CEO to take although still excessive but less risk. The firm's expected value increases accordingly due to this reason. We call this as the "risk reduction" effect.

Then we look at the difference $\int_{\theta_n^*}^{\overline{\theta}} \Delta V_y(\theta) d[G_n(\theta) - G_m(\theta)]$. We integrate it by parts and obtain

$$\int_{\theta_n^*}^{\overline{\theta}} \Delta V_y(\theta) d[G_n(\theta) - G_m(\theta)] = \Delta V_y(\theta)[G_n(\theta) - G_m(\theta)] \,|_{\theta_n^*}^{\overline{\theta}} - \int_{\theta_n^*}^{\overline{\theta}} [G_n(\theta) - G_m(\theta)] \frac{dV_y(\theta)}{d\theta} d\theta$$

The term $-\int_{\theta_n^*}^{\overline{\theta}} [G_n(\theta) - G_m(\theta)] \frac{dV_y(\theta)}{d\theta} d\theta$ is positive. This is because hiring higher ability CEO n than lower ability CEO m increases the distribution of quality of the risky project: the chance that θ is high increases. Notice that $G_n(\theta) - G_m(\theta)$ is negative for any $\theta \in [\underline{\theta}, \overline{\theta}]$. This effect is labeled as "distribution improving" effect, it increases the expected firm as well.

Lastly, the term

$$\Delta V_y(\theta)[G_n(\theta) - G_m(\theta)] \mid_{\theta_n^*}^{\overline{\theta}} = -\Delta V_y(\theta_n^*)[G_n(\theta_n^*) - G_m(\theta_n^*)]$$

is negative. Notice again, $\Delta V_y(\theta_n^*) < 0$ for $\theta_n^* < \theta^{FB}$ and $G_n(\theta_n^*) - G_m(\theta_n^*) < 0$. In the second best contracting, the CEO makes value reducing project choice for the quality lies in the interval $[\theta^*, \theta^{FB})$. Depending on the distribution function G, it could be possible that when a lower ability CEO m is hired, the chance that the quality of the risky project realizes in this interval is lower the one when a higher ability CEO n is hired. The reason is again due to the FOSD assumption. Then from ex ante ($t = 0$) point of view, the low ability CEO could have lower chance to choose the value reducing project even if he acts according to a more risky project choice rule $\theta_m^* < \theta_n^*$. This effect is the side effect of the fact that CEO ability improves distribution of quality of risky project. Due to this adverse effect, the firm value decreases when hiring higher ability CEO.

In sum, it is difficult to identify the final effect of CEO ability on firm's value in the second best world. It depends on the specific forms of the distribution functions G and F. When the adverse effect dominates the "risk reduction" and "distribution improving" effects, the firm value is actually lower when hiring a higher ability CEO. In case of first best contracting, there is only "distribution improving" effect. So in the first best contracting the unit firm value increases when hiring higher ability CEO.

5 Discussions

In this section, we discuss the issues that have not been covered so far. The first issue is the optimal exercise price of the options. The second issue is the relationship between firm size, risk and the structure of optimal contract.

5.1 Optimal Exercise Price y^*.

In this subsection, we analyze the role of exercise price $\hat{y}$ in the compensation design. In the previous analysis, $\hat{y} \in (\underline{y}, \overline{y})$ is treated as an exogenous variable. Now we discuss the optimal choice of exercise price $\hat{y}$ of the options.

We consider the optimal choice of exercise price as a two step program. First, the firm chooses the optimal mix of stock and options to maximize the expected firm value for any exercise price $\hat{y} \in (\underline{y}, \overline{y})$. Then firm chooses the optimal exercise price y^* from the feasible set $(\underline{y}, \overline{y})$. The first step has already been done in the section 3. We then consider the board's choice of exercise price y^*. To determine the optimal exercise price y^*, we impose another constraint on the total amount of shares and options:

$$\beta + \gamma < 1.$$

It means under any state of world, the CEO never owns the entire firm (remember that we normalize the firm's outstanding shares to be one).

The optimal choice of exercise price is irrelevant in the first best contracting as options are never used. We focus on the case of second best contracting. From Proposition 2, we know that the firm can always induce effort without giving any rent to the CEO. Thus, the exercise price only affects the optimal project choice defined as the solution of (13). We now write this condition as a function of exercise price as well

$$\underline{y} + \int_{\theta^*}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta) + \frac{-\Delta V_y(\theta^*)}{V_o(\theta^*, \hat{y})} \int_{\theta^*}^{\overline{y}} V_o(\theta, \hat{y}) dG(\theta) = \phi \underline{y}. \tag{15}$$

Notice that exercise price $\hat{y}$ affects the optimal project choice indirectly through its influence on the expected value of option $V_o(\theta, \hat{y}) = \varphi \int_{\underline{y}}^{\overline{y}} \max(y - \hat{y}, 0) dF(y|\theta)$.

An increasing in price $\hat{y}$ imposes an adverse "incentive" effect on project choice. It can be calculated that

$$\frac{\partial}{\partial \hat{y}} V_o(\theta, \hat{y}) = \frac{\partial}{\partial \hat{y}} \varphi \int_{\underline{y}}^{\overline{y}} \max(y - \hat{y}, 0) dF(y|\theta) = -\varphi \int_{\underline{y}}^{\overline{y}} dF(y|\theta) < 0$$

Increase $\hat{y}$ decreases the expected value of option $V_o(\theta)$ conditional on any quality θ of the risky project. The CEO's expected compensation from option $\int_{\theta^*}^{\overline{\theta}} V_o(\theta, \hat{y}) dG(\theta)$ conditional on the optimal project choice rule θ^* decreases as the expected value of option for all the risky project

$\theta \geq \theta^*$ decreases. The firm has to increase the amount of options ($\frac{-\Delta V_y(\theta^*)}{V_o(\theta^*,\hat{y})}$) to induce effort for each unit of stock granted. As a result, the CEO's optimal project choice becomes more risky, θ^* further decreases from θ^{FB}.

On the other hand, there is positive "project choice" effect. The project choice rule (3) prescribes for one unit stock granted, the amount of options required to maintain the project choice to be θ^* is $\frac{-\Delta V_y(\theta^*)}{V_o(\theta^*,\hat{y})}$. Exactly because the same reason, increase $\hat{y}$ decreases the expected value of option, the relative amount of options decreases for one unit of stock granted. Thus, the CEO is induced to make safer project choice ex post.

The two effects have opposite implications on the optimal project choice. The overall effect can be seen by taking partial derivative of equation (15) with respect to $\hat{y}$

$$\frac{-\Delta V_y(\theta^*)}{V_o(\theta^*, \hat{y})} \frac{\partial}{\partial \hat{y}} \left(\int_{\theta^*}^{\bar{y}} V_o(\theta, \hat{y}) dG(\theta) \right) + \frac{\partial}{\partial \hat{y}} \left(\frac{-\Delta V_y(\theta^*)}{V_o(\theta^*, \hat{y})} \right) \int_{\theta^*}^{\bar{y}} V_o(\theta, \hat{y}) dG(\theta)$$

The first term represents the negative "incentive" effect, while the second term represents the positive "project choice" effect. The overall effect is presented in the following lemma 2.

Lemma 4. *When $F(y|\theta)$ satisfies $F''_{y\theta} > 0$, the positive "project choice" effect dominates the negative "incentive" effect.*

Proof. See Appendix G □

Once the Lemma 2 holds, the following Proposition 3 summarizes the effect of exercise price on the optimal project choice.

Proposition 5. *When the condition in Lemma 4 holds, increase the exercise price of options decreases the CEO's incentives to make excessive projec choice, $\frac{\partial \theta^*}{\partial \hat{y}} > 0$.*

In the proof of Proposition 2, we have established the fact that the partial derivative of the equation (15) with respect to θ^* is negative. It ensures the uniqueness of optimal project choice. Under the condition presented in Lemma 2, the partial derivative of (15) with respect to $\hat{y}$ is positive. From the implicit function theorem, we have $\frac{\partial \theta^*}{\partial \hat{y}} > 0$. Increase the exercise price $\hat{y}$ actually induces the CEO to make the less risky optimal project choice.

Recall that in Proposition 2, the optimal amount of options is $\gamma^* I = \frac{-\Delta V_y(\theta^*)}{V_o(\theta^*,\hat{y})} \frac{u}{\phi \underline{y}}$, while the amount of stock is irrelevant to the exercise price. Due to the dominating "project choice" effect, the amount of options increases when exercise price $\hat{y}$ increase. To achieve the safest project choice, the firm optimally chooses the highest possible $\hat{y}$ such that the amount of option

γ^* is maximized, that is $1-\beta^*$. The optimal project choice and optimal exercise price are jointly determined by (15) and the binding constraint $\beta^* + \gamma^* = 1$.

Option induces the CEO to make excessive risky project choice compared the first best level. However, when the exercise price is endogenized, we obtain a result that the firm use the highest possible amount of options. This result is actually quite intuitive: as high exercise price reduces the chance that the option be finally exercised (the realized firm value is less likely above the exercise price), the CEO would rather select the safe project to enjoy the certain compensation from stock. Thus, a risky project is chosen only if its quality is high enough. A high exercise price of options decreases the CEO's incentive to make excessive project choice.

5.2 Firm size, risk and the structure of optimal contract

In this subsection, in order to analyze the relation between firm size and structure of optimal contract as well as risk, we further make two assumptions. First, we focus on an opposite situation to the one described in Proposition 4 by assuming away the ambiguity on the CEO ability. The benefit from "risk reduction" and "distribution improving" effects dominates the cost from the adverse effect. Second, we assume there will be different structure of optimal contract in the general equilibrium, thus the parameters are such that $1 + \int_{\theta^{FB}}^{\overline{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG_N(\theta) > \phi$ and $1 + \int_{\theta^{FB}}^{\overline{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG_1(\theta) < \phi$.

Under these assumptions, the partial equilibrium firm value could be either $\Pi(\hat{\theta}, j, n)$ or $\Pi(\theta_n^*, j, n)$ depending on whether the first best contracting is attained, that is (11) holds or not. The following lemma shows that under these assumptions, a firm always wants to hire higher ability CEO.

Lemma 5. *The expected (gross) firm value is higher when higher ability CEO is hired.*

Proof. See Appendix H □

Consider a firm j has the opportunity between hiring CEO m and n, with G_n FOSD G_m. Because ability improves the distribution of risky project in the sense of FOSD, the gross firm value is higher when hiring the higher ability CEO if first best contracting is attained when hiring both CEOs. Then by the assumption in this subsection that the expected firm value is also higher when hiring the higher ability CEO if only second best contracting is attained. The rest case is that the firm can attain first best contracting only when hiring CEO n, the higher ability one. Then it follows that θ^{FB} is first best project choice when hiring any CEOs, the firm still has higher expected value when hiring CEO n.

Follow the Lemma 5, we introduce the following Proposition 6. It characterizes a general market equilibrium where both contracting structures are presented.

Proposition 6. *There exists a critical CEO such that first best contracting can be attained for the CEOs with ability higher than (or equal to) him. The equilibrium results in PAM. There is seperation in structure of optimal contracts and project choice rule: firms hire CEOs with ability higher (lower) than the critical CEO in stock (stock and options) to implement the first (second) best project choice rule. CEOs in smaller firms follows more risky project choice rule.*

The intuition for this result is straightforward. The critical CEO is defined such that for CEOs who have higher ability than this CEO, (11) holds; while for CEOs who have lower ability than this CEO, (11) does not hold. Then there is a partition of the set of CEOs according to their ability. This partition is possible since the term $\int_{\theta^{FB}}^{\overline{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG(\theta)$ in (11) increases with the CEO's ability, i.e. for any G_n and G_m such that G_n FOSD G_m, the term is higher when the distribution is G_n. Follow the Lemma 5, we can establish the similar complementarity result when expected firm value increases when higher ability CEO is hired. Thus, the equilibrium assignment results in PAM. For the CEOs with ability higher than the critical CEO, the firms offer first best contracts to implement the same project choice rule θ^{FB}. While for the CEOs with ability lower than the critical CEO, the firms offer second best contracts to implement the project choice rule θ^*. Then from the Lemma 3, the CEOs with lower ability make more risky project choices under second best contracting. And firms with smaller asset sizes hire those CEOs with lower ability in positive assortive matching.

6 Conclusion

In this paper, we present a theoretical model where firms optimally design the compensation contracts to their CEOs. In the model, a CEO exerts privately observable effort as well as makes the project choice based on his own private information. Stocks perfectly incentivize the CEO to select the best project for the firm, while options are superior in motivating the CEO's effort. When effort is the prior concern of the firm, we find that both stock and options must be part of the second best optimal contract. The CEO and the firm in our model has multiplicative preference and production function. The incentive compatibility constraint prescribes a ratio between the expected pay in case of exerting effort and the pay in case of shirking must exceed the ratio of the CEO's private benefit in case of shirking and in case of exerting effort. We show that information about the project risk should be a part of the optimal compensation contract. We further analyze the productive efficiency in general equilibrium. We find that in the real world with distortion, there may be circumstance that larger firms would rather hire the CEOs with low ability to reduce the cost of excessive risky project choice.

Appendix A Proof of Lemma 1

Proof. Take the first order derivatives with respect to the four arguments α, β, γ and $\hat{\theta}$, we have the system of solution presented in the main text. We discuss different sets of solution according to the value of μ.

First, we look at the first order derivative with respective to $\hat{\theta}$ 10. From 6, we know that in 10, the term $-\beta\Delta V_y(\hat{\theta}) - \gamma V_o(\hat{\theta})$ equals to zero representing that the CEO makes the ex post project choice to maximize his expected compensation. Moreover, we know that from the FOSD assumption of distribution function $F(\theta)$, the two expectations $E[y|\hat{\theta}]$ and $V_o(\hat{\theta})$ are both increasing in $\hat{\theta}$. So the sign of the lagrange multiplier μ is determined by

$$sgn[\mu] = sgn[-\Delta V_y(\hat{\theta})].$$

Suppose $\mu = 0$, we have $E[y|\hat{\theta}] = \underline{y}$. It follows that γ must be zero from 3, as the expected option value $V_o(\hat{\theta})$ is strictly positive for all $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$. From our discussion in the last subsection, θ^{FB} is the unique value satisfing this equation, we have $\hat{\theta} = \theta^{FB}$. Then from 9, we have $\lambda_1 + \lambda_2 = 1$ when $\mu = 0$. Inserting $\lambda_1 + \lambda_2 = 1$ and $\mu = 0$ to 8, we have $\lambda_1 = 0$ and $\lambda_2 = 1$. Insert $\lambda_1 = 0$ and $\lambda_1 + \lambda_2 - 1 = 0$ into 7, we finally get $\eta = 0$. Thus, we find the first set of solution corresponding to the case that $\mu = 0$. Under this set of solution, IC and LL constraints are slack, IR constraint is binding. We insert $\hat{\theta} = \theta^{FB}$ and $\gamma = 0$ into the slack IC constraint 4 to get

$$\beta[\underline{y} + \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta) - \phi \underline{y}]I > (\phi - 1)\alpha$$

From the binding IR constraint, we solve $\alpha = u - \beta[\underline{y} + \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)]I$. Insert it into the first inequality, we get

$$\beta I > \frac{1}{\int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)} \frac{\phi - 1}{\phi} u.$$

Then from the binding IR constraint again, we have

$$0 < \alpha < \left[1 - \frac{(\phi - 1)\underline{y}}{\int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)}\right] \frac{u}{\phi}.$$

When $\mu = 0$, in order for the solution to be exist, a necessary condition is

$$1 + \int_{\theta^{FB}}^{\overline{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG(\theta) > \phi.$$

Otherwise, the LL constraint will be violated. On the other hand, once this condition satisfies, the board can always choose a combination of fixed salary and shares that satisfies the above two inequalities to implement θ^{FB} and induce effort while making the CEO break even. Thus, it is a sufficient condition as well.

Suppose $\mu > 0$, then $E[y|\hat{\theta}] < \underline{y}$. It follows that γ must be positive. We have $\hat{\theta} < \theta^{FB}$, the CEO makes more risky project choice than the one maximizing the expected firm value. From 9, we have $\lambda_1 + \lambda_2 > 1$. Then 8 becomes

$$\phi\lambda_1\underline{y} = (\lambda_1 + \lambda_2 - 1)[\underline{y} + \int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta)] - \mu[E[y|\hat{\theta}] - \underline{y}] > 0$$

This shows $\lambda_1 > 0$. Combining 7 and 8, we can solve

$$\eta = (\lambda_1 + \lambda_2 - 1) \int_{\hat{\theta}}^{\overline{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG(\theta) - \mu \frac{E[y|\hat{\theta}] - \underline{y}}{\underline{y}} > 0.$$

When $\mu > 0$, there exists a second set of solution. It entails both IC and LL is binding. Insert the project choice rule 3 and $\alpha = 0$ into the binding IC, we have the project choice $\hat{\theta}$ is the solution of

$$1 + \int_{\hat{\theta}}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\hat{\theta})}{\underline{y}} \frac{V_o(\theta)}{V_o(\hat{\theta})} \right] dG(\theta) = \phi \tag{A.16}$$

When $\mu < 0$, the CEO makes conservative project choice $\hat{\theta} > \theta^*$. From project choice rule 3, this requires $\gamma < 0$, which is impossible in our risk neutral setup.

To summarize, we found two sets of solutions corresponding to the different parameters value characterized in 11,

□

Appendix B Proof of Proposition 1

Proof. The proof of Proposition 1 follows directly the proof of Lemma 1. Notice that Lemma 1 predicts a continuum of contracts that induce effort and implement θ^{FB} when 11 holds. We choose the optimal contract as the one with maximum amount of fixed salary as in Edmans, Gabaix, and Landier (2009). □

Appendix C Proof of Proposition 2

Proof. When $\mu > 0$, the optimal project $\hat{\theta} < \theta^{FB}$ satisfies the equation A.16 from Lemma 1. Notice that now the limited liability constraint is binding, thus 11 does not hold.

We start by proving the existence and uniqueness of $\theta^* \in (\underline{\theta}, \theta^{FB})$ as a solution of A.16. For simplicity, we denote an auxiliary function as

$$\Psi(\hat{\theta}) = 1 + \int_{\hat{\theta}}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\hat{\theta})}{\underline{y}} \frac{V_o(\theta)}{V_o(\hat{\theta})} \right] dG(\theta) - \phi$$

We assume this function is continuous in $\hat{\theta}$. We first take the limit of $\hat{\theta}$ to θ^{FB}

$$\lim_{\hat{\theta} \to \theta^{FB}} \Psi(\hat{\theta}) = 1 + \int_{\theta^{FB}}^{\overline{\theta}} \frac{\Delta V_y(\theta)}{\underline{y}} dG(\theta) - \phi < 0$$

This is because now 11 does not hold. Then we assume that

$$\lim_{\hat{\theta} \to \underline{\theta}} \Psi(\hat{\theta}) = 1 + \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\underline{\theta})}{\underline{y}} \frac{V_o(\theta)}{V_o(\underline{\theta})} \right] dG(\theta) - \phi > 0$$

As the board can always choose the exercise price $\hat{y}$ of the option. Especially, the board can choose $\hat{y}$ high enough to make $-\frac{\Delta V_y(\hat{\theta})}{V_o(\hat{\theta})}$ sufficiently large when $\hat{\theta}$ is close to $\underline{\theta}$. By the continuity, there exists $\hat{\theta} \in (\underline{\theta}, \theta^{FB})$ such that $\Psi(\hat{\theta}) = 0$.

Then it can be calculated that

$$\frac{\partial \Psi(\hat{\theta})}{\partial \hat{\theta}} = \int_{\hat{\theta}}^{\overline{\theta}} \frac{V_o(\theta)}{\underline{y}} \frac{\partial}{\partial \hat{\theta}} \left(\frac{-\Delta V_y(\hat{\theta})}{V_o(\hat{\theta})} \right) dG(\theta) < 0$$

where

$$\frac{\partial}{\partial \hat{\theta}} \left(\frac{-\Delta V_y(\hat{\theta})}{V_o(\hat{\theta})} \right) = \frac{-\frac{\partial}{\partial \hat{\theta}} E[y|\hat{\theta}] V_o(\hat{\theta}) + \Delta V_y(\hat{\theta}) \frac{\partial}{\partial \hat{\theta}} V_o(\hat{\theta})}{[V_o(\hat{\theta})]^2} < 0$$

Remember that $\Delta V_y(\hat{\theta}) = E[y|\hat{\theta}] - \underline{y} < 0$ when $\hat{\theta} < \theta^{FB}$. And the two expectation $E[y|\hat{\theta}]$ and $V_o(\hat{\theta})$ increases in $\hat{\theta}$ because the FOSD assumption.

So the solution is existent and unique. We define it as θ^* such that $\Psi(\theta^*) = 0$.

Then knowing the value of θ^*, the board can always choose the amount of shares and options to make IR constraint binding. The reason is stated in the main text. With the binding LL constraint, the second best contract is consist of $\beta^* I$ units of shares and $\gamma^* I$ units of options.

Combining the binding IC and IR constraints, we have $\beta^* I = \frac{u}{\phi \underline{y}}$. Then use the project choice condition 3, $\gamma^* I = \frac{-\Delta V_y(\theta^*)}{V_o(\theta^*)}\beta^* I = \frac{-\Delta V_y(\theta^*)}{V_o(\theta^*)}\frac{u}{\phi \underline{y}}$. Notice that now $\Delta V_y(\theta^*) < 0$ for $\theta^* < \theta^{FB}$.

□

Appendix D Proof of Lemma 2

Proof. We first show when **??** satisfies, then a firm can secure first best contracting when hiring any CEOs in the labor market. For any CEO n other than the CEO with the lowest ability $G_1(\theta)$, we can show

$$\begin{aligned}
&\int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG_n(\theta) - \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG_1(\theta) \\
&= \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) d[G_n(\theta) - G_1(\theta)] \\
&= \Delta V_y(\theta)[G_n(\theta) - G_1(\theta)] \mid |_{\theta^{FB}}^{\overline{\theta}} - \int_{\theta^{FB}}^{\overline{\theta}} [G_n(\theta) - G_1(\theta)] \frac{dV_y(\theta)}{d\theta} d\theta \\
&= \int_{\theta^{FB}}^{\overline{\theta}} [G_m(\theta) - G_1(\theta)] \frac{dV_y(\theta)}{d\theta} d\theta > 0
\end{aligned}$$

Notice that $\Delta V_y(\theta^{FB}) = 0$ from the project choice rule 3. G_n FOSD G_1 for all higher ability CEOs than the CEO with the lowest ranking in ability. Then for $\theta \in [\theta^{FB}, \overline{\theta}]$, we have $G_m(\theta) > G_1(\theta)$. And $\frac{dV_y(\theta)}{d\theta} > 0$ showed in the last section. Then, **??** is the sufficient and necessary condition for firms to offer the first best contracts to CEO 1, it is the sufficient and necessary condition to attain first best contracting for any higher ability CEO.

By the definition of Π, we have

$$\begin{aligned}
&[\Pi(\theta^{FB}, j, n) - \Pi(\theta^{FB}, j, m)] - [\Pi(\theta^{FB}, i, n) - \Pi(\theta^{FB}, i, m)] \\
&= \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) d[G_n(\theta) - G_m(\theta)](I^j - I^i) \\
&= \left[\int_{\theta^{FB}}^{\overline{\theta}} [G_m(\theta) - G_n(\theta)] \frac{dV_y(\theta)}{d\theta} d\theta \right] (I^j - I^i) > 0
\end{aligned}$$

Again, G_n FOSD G_m, thus for all $\theta \in [\theta^{FB}, \overline{\theta}]$, we have $G_m(\theta) > G_n(\theta)$.

□

Appendix E Proof of Proposition 3

Proof. Now we are in the first best world. The proof follows Edmans, Gabaix, and Landier (2009) and Thanassoulis (2013). Consider the case that two firms i and j with $I^j > I^i$ compete for hiring two CEOs m and n with G_n FOSD G_m. Let's consider the bids offered by the firms are in the form of expected total utility in the eyes of CEOs. A CEO accepts the bid that delivers him the highest expected total utility. As as result, the firms choose the structure of the compensation contract to attain the optimal contracting described as in last section.

Let's analyze firm i's decision between hiring CEO m or CEO n. Notice that we assume the firm can induce the first best project choice. Fixed the outside utility of the CEO m as u_m, the firm's expected value of hiring this CEO is:

$$\Pi(\theta^{FB}, i, m) = [\underline{y} + \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta) dG_m(\theta)] I^i - u_m.$$

On the other hand, the firm i's highest possible bid $u_{i,n}$ for hiring higher ability CEO n is:

$$u_{i,n} = u_m + \left[\int_{\theta^{FB}}^{\overline{\theta}} [G_m(\theta) - G_n(\theta)] \frac{dV_y(\theta)}{d\theta} d\theta \right] I^i.$$

The term in the parenthesis is the expected value increasing from hiring the higher ability CEO n, which we have calculated in the proof of Lemma 3.

Similarly, we have the firm j's highest bid for the CEO n is:

$$u_{j,n} = u_m + \left[\int_{\theta^{FB}}^{\overline{\theta}} [G_m(\theta) - G_n(\theta)] \frac{dV_y(\theta)}{d\theta} d\theta \right] I^j.$$

Immediately we have the firm with larger asset size I^j can always bids higher for higher ability CEO

$$u_{j,n} - u_{i,n} = \left[\int_{\theta^{FB}}^{\overline{\theta}} [G_m(\theta) - G_n(\theta)] \frac{dV_y(\theta)}{d\theta} d\theta \right] (I^j - I^i).$$

Thus, in the equilibrium the larger firm matches the bids of the smaller firm $u_{j,n} = u_{i,n}$ to make the CEO just indifferent from accepting the two bids.

Iterate this argument for the hiring between the highest ability and second highest ability CEO N and $N-1$ and for the CEOs with lower ranks in their ability. We always have the firm with larger asset size bids higher for the more talented CEO. Because we assume the number of banks and CEOs are equal, the firm ranking in the k's position in asset size exactly hires the CEO ranking in the same position in ability.

Moreover, we can derive the total pay for CEO u_k by iteration

$$u_k = \sum_{i=2}^{k} \int_{\theta^{FB}}^{\overline{\theta}} [G_{i-1}(\theta) - G_i(\theta)] \frac{d\Delta V_y(\theta)}{d\theta} d\theta I^{i-1} + \underline{u}.$$

□

Appendix F Proof of Lemma 3

Proof. The condition that all CEOs are hired in the second best contracting is similar to the one in the first best contracting world. If the CEO with the highest ability can not be hired in first best contract then the result applies to all other CEOs with lower ability.

Define two auxiliary functions as

$$\Psi_m(\hat{\theta}) = 1 + \int_{\hat{\theta}}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\hat{\theta})}{\underline{y}} \frac{V_o(\theta)}{V_o(\hat{\theta})} \right] dG_m(\theta) - \phi$$

It is a function of the CEO's ability as well. Similarly, a CEO n's project choice θ_n^* is given by

$$\Psi_n(\hat{\theta}) = 1 + \int_{\hat{\theta}}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\hat{\theta})}{\underline{y}} \frac{V_o(\theta)}{V_o(\hat{\theta})} \right] dG_n(\theta) - \phi$$

Then the optimal project made by CEO m and n in the second best contracting are given by $\Psi_m(\theta_m^*) = 0$ and $\Psi_n(\theta_n^*) = 0$. We consider the following difference

$$\begin{aligned}
&\Psi_n(\theta_n^*) - \Psi_m(\theta_n^*) \\
&= \int_{\theta_n^*}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}} \frac{V_o(\theta)}{V_o(\theta_n^*)} \right] dG_n(\theta) - \int_{\theta_n^*}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}} \frac{V_o(\theta)}{V_o(\theta_n^*)} \right] dG_m(\theta) \\
&= \int_{\theta_n^*}^{\overline{\theta}} \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}} \frac{V_o(\theta)}{V_o(\theta_n^*)} \right] d[G_n(\theta) - G_m(\theta)] \\
&= \left\{ \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}} \frac{V_o(\theta)}{V_o(\theta_n^*)} \right] [G_n(\theta) - G_m(\theta)] \right\} \Big|_{\theta_n^*}^{\overline{\theta}} - \int_{\theta_n^*}^{\overline{\theta}} [G_n(\theta) - G_m(\theta)] d \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}} \frac{V_o(\theta)}{V_o(\theta_n^*)} \right] \\
&= - \int_{\theta_n^*}^{\overline{\theta}} [G_n(\theta) - G_m(\theta)] d \left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}} \frac{V_o(\theta)}{V_o(\theta_n^*)} \right] > 0.
\end{aligned}$$

The last term is larger than zero because first $G_n(\theta) < G_n(\theta)$ for any $\theta \in [\underline{\theta}, \overline{\theta}]$. Second, the derivative is

$$
\begin{aligned}
d\left[\frac{\Delta V_y(\theta)}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}}\frac{V_o(\theta)}{V_o(\theta_n^*)}\right] &= \frac{d\Delta V_y(\theta)/d\theta}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}}\frac{dV_o(\theta)/d\theta}{V_o(\theta_n^*)} \\
&= \frac{dE[y|\theta]/d\theta}{\underline{y}} - \frac{\Delta V_y(\theta_n^*)}{\underline{y}}\frac{dV_o(\theta)/d\theta}{V_o(\theta_n^*)} > 0.
\end{aligned}
$$

This is because $\frac{dE[y|\theta]}{d\theta} = \frac{d\int_{\underline{y}}^{\overline{y}} y dF(y|\theta)}{d\theta} > 0$ and $\frac{dV_o(\theta)}{d\theta} = \frac{d\int_{\underline{y}}^{\overline{y}} \max(y-\hat{y},0)dF(y|\theta)}{d\theta} > 0$ as $F(y|\theta)$ satisfies FOSD and the two integrands are increasing in y. And in the second best contracting $\theta_n^* < \theta^{FB}$, thus $\Delta V_y(\theta_n^*) = E[y|\theta_n^*] - \underline{y} < 0$. So we have $\Psi_m(\theta_n^*) < 0$.

From the Proof of Proposition 2, we know that $\frac{d\Psi_m(\hat{\theta})}{d\hat{\theta}} < 0$. Thus $\theta_n^* > \theta_m^*$ as θ_m^* is the value such that $\Psi_m(\theta_m^*) = 0$.

□

Appendix G Proof of Lemma 4

Proof. We use an auxiliary function similar to the one defined in the proof of Proposition 2 to take into account of the exercise price $\hat{y}$

$$
\Psi(\theta^*, \hat{y}) = \overline{y} + \int_{\theta^*}^{\overline{\theta}} \Delta V_y(\theta) dG(\theta) + \frac{-\Delta V_y(\theta^*)}{V_o(\theta^*, \hat{y})} \int_{\theta^*}^{\overline{y}} V_o(\theta, \hat{y}) dG(\theta) - \phi \underline{y}.
$$

From the proof of Proposition 2, we know that $\frac{\partial \Psi(\hat{\theta}, \hat{y})}{\partial \hat{\theta}} < 0$.

Then we consider exercise price $\hat{y}$. In the main text, we consider two effects of increasing the exercise price on the CEO's optimal project choice, a negative "incentive" effect and a positive "project choice" effect. The total effect of increasing $\hat{y}$ thus can be obtained as

$$
\begin{aligned}
&\frac{\partial}{\partial \hat{y}}\left(\frac{-\Delta V_y(\theta^*)}{V_o(\theta^*, \hat{y})}\int_{\theta^*}^{\overline{y}} V_o(\theta, \hat{y}) dG(\theta)\right) \\
&= \frac{-\Delta V_y(\theta^*)[1 - F(\hat{y}|\theta^*)]}{[V_o(\theta^*, \hat{y})]^2}\int_{\theta^*}^{\overline{\theta}} V_o(\theta, \hat{y}) dG(\theta) + \frac{\Delta V_y(\theta^*)}{V_o(\theta^*, \hat{y})}\int_{\theta^*}^{\overline{\theta}} [1 - F(\hat{y}|\theta)] dG(\theta) \\
&= \frac{-\Delta V_y(\theta^*)[1 - F(\hat{y}|\theta^*)]}{[V_o(\theta^*, \hat{y})]^2}\left(\int_{\theta^*}^{\overline{\theta}} V_o(\theta, \hat{y}) dG(\theta) - \int_{\theta^*}^{\overline{\theta}} \frac{1 - F(\hat{y}|\theta)}{1 - F(\hat{y}|\theta^*)} V_o(\theta^*, \hat{y}) dG(\theta)\right) \\
&= \frac{-\Delta V_y(\theta^*)[1 - F(\hat{y}|\theta^*)]}{[V_o(\theta^*, \hat{y})]^2}\int_{\theta^*}^{\overline{\theta}}\int_{\hat{y}}^{\overline{y}} [1 - F(\hat{y}|\theta)](y - \hat{y})\left(\frac{f(y|\theta)}{1 - F(\hat{y}|\theta)} - \frac{f(y|\theta^*)}{1 - F(\hat{y}|\theta^*)}\right) dy dG(\theta)
\end{aligned}
$$

Consider the following derivative

$$\frac{\partial}{\partial\theta}\frac{f(y|\theta)}{1-F(\hat{y}|\theta)} = \frac{f'_\theta(y|\theta)[1-F(\hat{y}|\theta)] + f(\hat{y}|\theta)F'_\theta(\hat{y}|\theta)}{[1-F(\hat{y}|\theta)]^2} > 0$$

Notice that we have assumed that the board's objective is strictly concave to $\hat{\theta}$. Thus $f'_\theta(y|\theta) = F''_{y\theta}(y|\theta) > 0$. The last term $F'_\theta(y|\theta)$ is positive as well under this assumption. So the term in the parenthesis is positive because the integration is on the interval $\theta \in [\theta^*, \overline{\theta}]$. Eventually, we have

$$\frac{\partial}{\partial\hat{y}}\left(\frac{-\Delta V_y(\theta^*)}{V_o(\theta^*, \hat{y})}\int_{\theta^*}^{\overline{y}} V_o(\theta, \hat{y})dG(\theta)\right) > 0.$$

Then $\frac{\partial\Psi(\theta^*,\hat{y})}{\partial\hat{y}} > 0$. And we have already proven in Proposition 2 that $\frac{\partial\Psi(\theta^*,\hat{y})}{\partial\theta^*} < 0$. By the implicit function theorem, $\frac{\partial\theta^*}{\partial\hat{y}} > 0$. The "project choice" effect dominates.

□

Appendix H Proof of Lemma 5

Proof. Suppose a firm j has the opportunity of hiring two CEOs m, n with G_n FOSD G_m. We have already proved in Lemma 2 that if first best contracting is attained when hiring both CEOs, $\Pi(\theta^{FB}, j, n) > \Pi(\theta^{FB}, j, m)$.

Then by the assumption in this subsection that if only second best contracting is attained when hiring both CEOs, we have $\Pi(\theta_n^*, j, n) > \Pi(\theta_m^*, j, m)$.

We now focus on the case that if first best contracting is attained for only one CEO. Recall in Lemma 2, first best contracting must be attained under the higher ability CEO G_n. Thus, we have the expected firm value when hiring these two CEOs are respectively

$$\Pi(\theta^{FB}, j, n) = [\underline{y} + \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta)dG_n(\theta)]I^j \quad \Pi(\theta^*, j, m) = [\underline{y} + \int_{\theta_m^*}^{\overline{\theta}} \Delta V_y(\theta)dG_m(\theta)]I^j.$$

Because θ^{FB} is the global maximizer of $\int_{\hat{\theta}}^{\overline{\theta}} \Delta V_y(\theta)dG_m(\theta)$. Then we have $\int_{\theta_m^*}^{\overline{\theta}} \Delta V_y(\theta)dG_m(\theta) < \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta)dG_m(\theta)$. The latter expression is the firm's expected unit value when hiring CEO m without any agency problem. We have it is in turn less than the expected unit value when hiring CEO n, $\int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta)dG_m(\theta) < \int_{\theta^{FB}}^{\overline{\theta}} \Delta V_y(\theta)dG_n(\theta)$, again from Lemma 2.

Thus, we conclude for a given firm it always wants to hire the higher ability CEO.

□

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