Contagious Bank Runs and the Net Stable Funding Ratio Requirement

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Abstract

In a dynamic global-games model with asset sales, bank runs, and aggregate uncertainties, we analyze whether an increase in banks' net stable funding can contain contagious bank runs. Conventional wisdom suggests that banks with a high net stable funding can better fend off runs. An increase in the net stable funding can also depress banks' asset prices due to asset buyers' pessimistic inferencing of the aggregate risk. Therefore, given that a bank faces a run, buyers' valuation of its assets decreases in its level of net stable funding. Asset buyers' low willingness to pay, in turn, can precipitate runs in all other banks in the first place. We show that an increase in the net stable funding can lead to an unambiguous increase in the risk of contagion.

Keywords: Bank Runs, Asset Sales, Global Games, the NSFR

JEL Classification: G01, G11, G21

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1 Introduction

The 2007-2009 recent financial crisis witnessed, yet again, banking industry is fragile, and the fragility was no longer limited to banks' funding side as described by Diamond and Dybvig (1983). At the same time, as documented by Brunnermeier and Pedersen (2008), banks' market liquidity also evaporated. In addition, the modern financial system is more interconnected than ever before, the failure of one bank could trigger a systemic meltdown of the industry, and eventually endanger the whole system. As a natural response, the Basel Committee of Banking Supervision (BCBS) has introduced the Net Stable Funding Ratio (NSFR) requirement as part of its new liquidity regulation in the Basel III Accord. By limiting banks' over-reliance on short-term wholesale funding, BCBS intends to use the requirement to reduce banks' *risk of failure* and prevent *potential systemic stress* due to the failure.[1] A bank's NSFR is equal to its Available Stable Funding (ASF) divided by the Required Stable Funding (RSF), i.e., its potential funding needs, and is required to be larger than some predetermined ratio. A bank's capital and liabilities are divided into 5 categories according to their funding stability, with respective ASF factors (100%, 95%, 90%, 50%, and 0%). The liabilities receiving a higher factor are considered less likely to be withdrawn over the considered time horizon (1 year). The amount of ASF is then an weighted average of $\sum_i \text{ASF factor}_i \times \text{Liabilities}_i$, i indicates a particular category. Similarly, RSF is measured as bank's assets according to their exposures to liquidity risk, with 8 categories and respective RSF factors (0%, 5%, 10%, 15%, 50% 65%, 85% and 100%). The assets receiving a lower RSF factor are considered to be more liquid. The amount of RSF can be calculated as the same fashion as ASF.[2]

Due to the importance, there are many research attempts on the newly implemented liquidity requirements. Most of them take a positive approach by empirically evaluating the effect of the regulation on banks' risks.[3] There are several papers, such as Vives (2014), König et al. (2015) and Morris and Shin (2016), trying to provide normative analyses with theoretical models. Our paper also intends to take a normative view on the NSFR requirement, and contributes

[1] According to the Consultative Document 'Basel III, the NSFR requirement is "A sustainable funding structure is intended to reduce the likelihood that disruptions to a bank's regular source of funding will erode its liquidity position in a way that would increase the *the risk of its failure* and potentially lead to a broader *systemic stress*. The NSFR limits overreliance on short-term wholesale funding..."

[2] The NSFR requirement was initially proposed in 2010 and re-proposed in January 2014. On October 31, 2014, the Basel Committee on Banking Supervision (BCBS) issued its final Net Stable Funding Ratio. According to BCBS, all banks worldwide have until 2018 to meet the NSFR standard if they are engaged in international banking.

[3] For instance, King (2013) shows an adverse effect of liquidity regulation on bank profitability and eventually bank risk.

to the existing literatures in three aspects. First, we take fully consideration of rational economic agents' optimal response to the NSFR requirement policy when evaluating the effect of the policy itself. Second, we build a general equilibrium model incorporating both banks' credit market and asset market. We use the model to provide a general equilibrium analysis of the NSFR requirement, which is in contrast to the most previous partial equilibrium approach papers. Our general equilibrium approach allows us to deliver new insight about the effect of the NSFR requirement on bank's risk of failure. Lastly, we extend our basic model to a multi-period setting to analyze the policy implication on the risk of contagion and spill-over among banks, while most previous models pay their attention solely to the risks at the individual bank level.

To deliver unique policy prediction, we analyze a bank run model refined by the global games technic based on Li and Ma (2018). In the basic model with a single bank, creditors receive noise signals about their bank's asset return and make a choice between withdraw their claims immediately and wait until maturity. Once there are withdrawals, the (passive) bank has to raise liquidity by liquidating its portfolio in a secondary asset market. Buyers in that market bid competitively to purchase bank's asset on sale. When making the binary choice, creditors form posterior belief about the fraction of total withdrawals not only based on their signals, but also depend on their rational expectations about the equilibrium price of bank's asset. On the other hand, asset buyers make their bids by rationally anticipating the quality of asset on sale, which relates to the creditors' equilibrium bank run strategy and an aggregate risk factor that determines the distribution of asset. The global games refinement pins down a unique equilibrium consists of a triplet of a threshold signal, a threshold asset return and an equilibrium asset price: A bank run occurs when the realized asset return is lower than the threshold value.

Equipped with the theoretical model, we conduct policy evaluation of the NSFR requirement. While our paper confirms that a NSFR requirement does force a bank to contain it risk of failure, our analysis also reveals that once asset buyers' response to the regulation is considered, an increase in retail funding can also have unintended consequences for bank's market liquidity via buyers' beliefs. An increase in retail funding makes the bank more resilient to runs. In particular, buyers' posterior beliefs about a bank's asset quality will deteriorate when a run happens to a bank that maintains a high stable funding ratio. So they will bid less to purchase bank's asset on sale. This will, however, intensify the coordination failure among bank's creditors, offset some stable effect of the NSFR requirement. Accordingly, we show a NSFR requirement aiming to limit the bank's risk of failure to a certain level may not fulfill its goal once the regulator neglect to take in account players' rational response of the requirement. In other words, such a NSFR requirement may be subject to Lucas Critique.

Furthermore, we extend our one-period model to a multi-period framework to analyze the financial contagion between banks. The global games refinement also predicts a uniqueness result. We show that in equilibrium the second bank's risk of failure increases follows a run/failure in the first bank. This is due the the fact that the failure of the first bank exerts negative externalities on the second bank. Indeed, creditors and asset buyers of the second bank will revise downwards their beliefs of the aggregate risk factor after observing a failure of the first bank.

Then our parsimonious dynamic model can be used to analyze the effect of NSFR requirement on the risk of contagion. It is interesting to point out an increase in retail funding by the first bank to satisfy the NSFR requirement will increase the second bank's risk of contagion. The intuition hinges on players' inferencing about the aggregate risk factor. A bank failure is more likely to be triggered either because of a bad aggregate state or creditors take aggressive bank run strategy. Then a run on a bank with high stable funding ratio actually suggests the realization of the aggregate risk is more likely to be bad. So creditors take more aggressive run strategy and buyers bid less in the second period game, which results in a higher risk of contagion follow an increase in stable funding ratio.

1.1 Related literature

There are unintended consequences of the current liquidity regulation, such as the NSFR requirement policy when the regulators do not take fully into account rational market participants' optimal responses to the policy. This main argument of our paper most obviously relates to the vast literature on macro policy discussion initiated by Robert Lucas in his 1976 paper (later edited in Brunner et al. (1983)), and Goodhart (1984) with an application in banking. Compared to those macroeconomic analyses, our paper is among the first to apply Lucas' insight to analyze the effects of the new macro-prudential regulations in banking industry after the recent financial crisis.

In their two recent papers, Horváth and Wagner (2016) and Horváth and Wagner (2017) also take a Lucas Critique view on macro-prudential policies, and point out the potential unintended consequences. Horváth and Wagner (2017) focuses on the counter-cyclical capital regulation, and points out such capital requirements increase banks' systemic risk taking exactly because they insure banks against aggregate shocks. Horváth and Wagner (2016) pointed out three areas in which countercyclical policies are likely to be subject to the Lucas Critique. Our paper differs from theirs in three aspects. First, we focus on the recent Net Stable Funding Ratio liquidity requirement, and explore its unintended consequences on bank's risk of failure and financial contagion, that are, the primary targets of the policy. Second, we take a general equilibrium approach by modeling both bank's asset market and credit market differing with their partial equilibrium analysis. Lastly, our paper focus on the policy implication on informational contagion channelled through players' belief updating about the aggregate risk, while they look for how banks' risk taking incentive will be distorted after the implementation of capital requirement.

To predict fundamentally stronger institutions impose greater financial spillovers, Choi (2014) has a similar result as ours. Our paper mainly differs from his in three aspects. First, our paper takes an informational contagion channel by considering sequential bank run decisions where the previous outcomes are observed by the later players. In contrast, Choi (2014) assumes players in different banks move simultaneously and region switching occurs when the total withdrawals of all banks surpass a certain fundamental value of economy. Lastly, his paper endogenize the liquidation price (collateral value) of banks' assets following the limited participation logic a la Allen and Gale (1994). In contrast, we take an asymmetric information approach and analyze a competitive bidding game among asset buyers. Lastly, in terms of policy, we analyze the policy implications of the ex ante liquidity requirement while his main focus is on the ex post bail out policies.

The main purpose of this paper is to provide positive view on the recent liquidity requirement in Basel III. With the similar purpose, Vives (2014), König et al. (2015) and Morris and Shin (2016) are the most related papers. All three papers build on a global games setup and analyze the implication of liquidity regulation on bank risks. However, the emphasis of all those papers is tbe comparative static analyses. For instance, Vives (2014) shows how the degree of strategic complementarity of investors' actions affect the policy analysis, while Morris and Shin (2016) digs how the variation of liquidity ratio, outside option ratio and fundamental risk ratio affects bank's illiquidity and insolvency risk. König et al. (2015) disentangles a countervailing solvency effect of liquidity regulation. Holding all else equal, investing more liquid assets reduces both bank's liquidity mismatch and its profitability. Bank's creditors could have higher incentive to withdraw early because they worry about bank's insolvency due to this low profitability. None of these papers, like ours, builds their arguments by taking into account agents' rational response to the policy when conducting their policy analysis.

In terms of predicting feedback between market liquidity and funding liquidity, our model is most related to Li and Ma (2018). That paper studies how central banks' Dealer of Last Resort policies can trim the belief-driven multiple equilibria and restore financial stability, while our main focus is to analyze the unintended consequences of the recent NSFR requirement in Basel III Accord. Additionally, our paper uses a dynamic global games setup and predicts a unique equilibrium, while there are multiple equilibria in their paper. Liu (2016) and Brunnermeier and Pedersen (2008) also analyze the two-way feedback mechanism between financial intermediation's asset side and funding side, but none of these papers conducts a Lucas Critique type of policy analysis as ours. Moreover, they do not consider bank's risk of contagion as well.

From a technical point of view, the paper relates to the broader literature that use global-games refinement to study bank runs, e.g., Morris and Shin (2000), Rochet and Vives (2004), Goldstein and Pauzner (2005), and Morris and Shin (2016),[4] but we relax the common simplifying assumption of exogenous liquidation losses.[5] From a modeling point of view, the simplifying assumption implicitly excludes the possibility for bank runs to affect asset prices, despite that bank failures often put downward pressure on asset prices in reality and that the mechanism is central to theories such as Allen and Gale (1998) and Gromb and Vayanos (2002). We introduce the impact of runs on asset prices in the framework of global games. We emphasize that buyers' lack of knowledge about the aggregate state and the inability to distinguish illiquid banks from insolvent ones result in a downward spiral between runs and declining asset prices. In the broader literature of global games, Ozdenoren and Yuan (2008) and Angeletos and Werning (2006) also introduce endogenous asset prices and predict the co-existence of price volatility and multiple equilibria. In generating multiple equilibria, both papers emphasize the impact of endogenous market price on the precision of public signals. In contrast,

[4]The literature refines the multiple equilibria in Diamond and Dybvig (1983) and emphasizes the role of liquidation loss in causing bank runs. That is, to prevent runs, an extra buffer of cash flow is needed against the liquidation loss. A bank that fails to provide the extra buffer will become "solvent but illiquid"—being able to repay its debt in full if no run happens, but will fail in equilibrium as its creditors do not roll over their debt. This bridges the panic view and the fundamental view of bank runs.

[5]For example, Rochet and Vives (2004) assume an exogenous fire-sale discount; **?** assumes unit liquidation value; and Morris and Shin (2016) assume an exogenous haircut of 100%.

we study a case where asset prices directly affect players' payoffs in coordination games. In this context, we show that even if the price is endogenous, one can still have a unique equilibrium, which disappears only upon the introduction of aggregate uncertainty. Also, our focus is not the multiplicity *per se*, but the policy intervention that reduces financial fragility and price volatility.

Lastly, our paper also studies information contagion, and the policy implication of the NSFR requirement on it. In line with the literature of information contagion, e.g., Acharya and Thakor (2016) and Oh (2013), contagious bank runs in our model are caused not only by the actual realization of the common risk but also by its perception: An extra bank failure casts shadow on the perception of the common risk factor, and the negative informational externalities affect all other banks. Such informational contagion is introduced into global-games setups in Ahnert and Bertsch (2015) and in Chen and Suen (2016). Yet, neither paper allows the outcome of later stages to affect coordination games in earlier stages. As a result, the papers predict sequences of unique equilibrium that are path-dependent. In contrast, we show that once creditors in the bank run game are *forward-looking*, the later stage will feedback to the bank run decision, and multiple equilibria will emerge.

Our analyses unfold as follows. Section 2 lays out the basic model with only one bank, bank's creditors and the secondary asset market for bank's asset. We derive the unique equilibrium of this general equilibrium model. In Section 3, we use our basic model to analyze the Net Stable Funding Ratio requirement proposed in Basel III Accord. Our main focus is to compare a mis-conducted policy evaluation of the NSFR requirement on the bank's risk of failure with a properly performed one. We extend the basic model to a dynamic setting with multiple banks, and analyze the unintened consequence of the NSFR requirement on the risk of contagion is Section 4. In Section 5, we conduct three robustness checks. Section 6 concludes the paper. All proofs are provided in the Appendix.

2 Basic Model

We first consider a economy with 1 banks, a unit measure of wholesale creditors. There are one period and 3 dates $t = 0, 0.5, 1$. We assume there is universal risk neutrality along our whole analysis.

2.1 Bank's Assets and Liabilities

The bank, thereafter Bank 1, holds a unit portfolio of long-term assets, and finances the portfolio with F proportion of retail deposits and $1 - F$ proportion of short-term wholesale debts.[6] We consider bank as passive contractual arrangement among claim holders, designed to fulfill the function of liquidity transformation and retail deposits are fully insured. So the active players in our basic model is bank's wholesale creditors.

Investing at $t = 0$, Bank 1's assets mature one period after at $t = 1$ and generate a cash flow $\tilde{\theta}_1 \sim U\left(\underline{\theta}_s, \overline{\theta}\right)$. The realization of the cash flow is not only affected by the idiosyncratic risk of Bank 1, but also by a systematic risk factor s. The systematic risk, as indicated by the subscript of the lower bound, determines the distribution of bank's cash flow.[7] There are two possible aggregate states, $s = G$ and $s = B$, with $\underline{\theta}_G > \underline{\theta}_B$. State G is assumed to be more favorable. All players hold a prior belief that State G and B occur with probabilities α and $1 - \alpha$ respectively.[8] Note that the upper bound of each bank's cash flow is assumed to be the same across the two aggregate states. This reflects the fact that banks hold mostly debt claims whose highest payoffs are capped by their face values. Once the systematic risk factor s realizes, bank's cash flow is determined by its idiosyncratic risk.

On the liability side, we assume that retail deposits are fully protected by deposit insurance and this financial safety net is provided to banks free of charge.[9] Therefore, retail depositors will passively hold their claims to maturity and demand only a gross risk-free rate which we normalize to 1. On the other hand, each bank's wholesale debts are risky, demandable at the mid-term of the period, and raised from a continuum of creditors of mass 1. Following most global games bank run papers, we assume each bank issues its wholesale debts to separate measures of creditors.[10] Provided that a bank does not fail, a wholesale debt contract promises a gross interest rate $r_D > 1$ if a wholesale creditor waits until project matures at the end of the period, and qr_D if the wholesale creditor withdraws early at the mid-term of the period. Here $q < 1$ reflects the penalty for the early withdrawal and captures qualitatively the feature of optimal demandable debt as in Diamond and Dybvig (1983). A bank run occurs if a positive

[6] We consider banks' balance sheets as exogenous, but private optimally chosen in this model. We also assume away equity capital as all our qualitative results will not be affected.

[7] The terminology 'systematic risk' will be evident when we introduce a second bank, Bank 2.

[8] Probabilities α and $1 - \alpha$ have a frequentist interpretation. One can consider them derived from historical observations and corresponding to the long-run frequencies of economic booms and recessions respectively.

[9] An unfairly priced deposit insurance premia will not affect our results.

[10] For instance, Goldstein and Pauzner (2004) and Chen and Suen (2016) have the same assumption as ours, while Li and Ma (2018) assumes banks issue wholesale debts to the same measure of creditors.

mass of wholesale creditors withdraw funds from their bank at the mid-term before assets mature. For the ease of future presentation, we denote by D_1 the total amount of debts bank 1 needs to repay at the mid-term if *all* wholesale creditors withdraw early, and by D_2 the total amount of debt bank 1 needs to repay at the end of the period if *no* wholesale creditor withdraws early. Accordingly, we have $D_1 \equiv (1-F)qr_D$ and $D_2 \equiv (1-F)r_D + F$. We further make the following three parametric assumptions.

$$D_2 > \underline{\theta}_s \tag{1}$$

$$F > D_1 \tag{2}$$

$$q > \frac{1}{2} + \frac{\underline{\theta}_G}{2D_2} \tag{3}$$

Inequality (1) states that banks are not risk free, and there is a positive probability of bankruptcy for both banks even in State G. Inequality (2) suggests that both banks' retail deposits exceed their short-term wholesale debts,[11] which is a realistic scenario and helps to simplify the analysis of bank run games.[12] Finally, inequality (3) states that the penalty on early withdrawal is only moderate,[13] which is in line with banks' role as liquidity providers (**?**). While we do not endogenize the bank's capital structure (therefore taking q, D_1, D_2 as given), as long as the optimal capital structure satisfies the aforementioned conditions, all of our results will apply.

We assume that banks' long-term assets cannot be physically liquidated at the mid-term. If a wholesale run happens, a bank has to financially liquidate its assets in a secondary market and sell them to outside asset buyers. Because early liquidation is costly in this model, a bank will sell its assets if and only if it faces a bank run. In the next two subsections, we describe the behavior of wholesale creditors' and that of outside asset buyers'.

2.2 Bank Run Game

At the beginning of $t = 0.5$, both systematic risk (State s) and bank 1's idiosyncratic risk (cash flow θ_1) have realized, but the information is not fully revealed to players. We take the global-games approach pioneered by Carlsson and Van Damme (1993) and assume that the

[11]Note that for $q < 1$, inequality (2) also implies $D_2 > 2D_1$ because $D_2 = D_1/q + F > D_1 + F > 2D_1$.

[12]The condition is more than a technical assumption. It is realistic in the sense that despite the rapid growth of wholesale funding, most commercial banks and bank holding companies are still financed more by retail deposits than wholesale debt.

[13]For example, when $\underline{\theta}_G = \underline{\theta}_B = 0$, the condition states that $q > 1/2$. That is, by withdrawing early, a wholesale creditors will not lose more than a half of the face value of his claim.

wholesale creditors observe independent noisy signals for the bank's cash flow. Specifically, a representative Creditor j of bank 1 privately observes a noisy signal $x_1^j = \theta_1 + \epsilon_1^j$ about bank 1's realized cash flow θ_1, where noise ϵ^j is drawn from a uniform distribution with a support $[-\epsilon, \epsilon]$. We will focus on a limiting case where ϵ approaches zero. After receiving his signals, wholesale creditor j has two possible actions at bank 1: to wait until assets mature at $t = 1$ or to withdraw early at $t = 0.5$. We assume that creditors play a bank run game with each other in the bank simultaneously, and focus on threshold strategy as most global games models do. That is, any creditor j withdraws from bank 1 if and only if $x_1^j < x_1^*$. In the limiting case where $\epsilon \to 0$, the critical cash flow θ_1^* converges to x_1^*, then a bank run will happen if and only if bank 1's cash flow $\theta_1 < \theta_1^*$ in equilibrium.[14] A creditor's payoff from bank 1 depends both on the timing of his withdrawal and on bank 1's solvency. Provided that bank 1 does not fail, the creditor's payoff will be $(1 - F)qr_D$ if he withdraws at $t = 0.5$, and $(1 - F)r_D$ if he waits until $t = 1$. We assume that bankruptcy costs are sufficiently high such that once bank 1 fails, the wholesale creditors receive zero payoffs and only a senior deposit insurance company obtains the bank's residual value. Therefore, when bank 1 fails, either at $t = 0.5$ or $t = 1$, a wholesale creditor will receive a zero payoff for his claim whether he runs or not. Finally, we assume that a creditor can obtain an arbitrarily small reputational benefit by running on a bank that is doomed to fail.[15,16] As in standard global-games models, creditors formulate posterior beliefs about the bank's realized fundamental θ_1 and the fraction of creditors who will withdraw early in each bank.

2.3 Asset Sale

Facing withdrawals, bank 1 has to liquidate its long-term assets in a secondary asset market. We assume that a large number of identical, deep-pocketed buyers participate in the market and that they are called into action only when a bank run happens. The buyers observe neither the aggregate state s nor any signals about bank 1's cash flow. Thus, they cannot determine the

[14] Due to its tractability, it is common to study the limiting in the literature. For example, see **?**. In our model, the limiting case also allows for a clear-cut definition for a bank run. In equilibrium, either all creditors withdraw or no one withdraws from the bank. That a fraction of creditors withdraw becomes a zero probability event.

[15] The reputational benefit may come from the fact that the creditor makes a "right decision". More detailed discussion on this assumption can be found in Rochet and Vives (2004). The authors argue that the vast majority of wholesale deposits are held by collective investment funds, whose managers are compensated if they build a good reputation, and penalized otherwise.

[16] As we will show later, wholesale creditors receiving this small positive payoff is also off equilibrium path in this model.

exact quality of bank 1's assets on sale. They can, however, observe the outcome of creditors' bank run game, i.e., whether bank 1 is forced to liquidation and can infer the quality of assets on sale from the observation. When buyers observe a run on bank 1, they compete in prices to purchase the bank's assets on sale. A strategy for an asset buyer is a price offer P_1. The price offered by an asset buyer will aggregate all information available to her. First, the buyer understands the creditors' bank run game and knows that the quality of assets on sale must be below an equilibrium threshold θ_1^*. Second, after the aggregate state realizes, the distribution of the bank's cash flow is determined, so that a bank run suggests State G less likely. On the other hand, when no run happens in bank 1, the asset buyers will not have the opportunity to move, and the period one game between wholesale creditors and asset buyers ends. However, they still update their beliefs that State G being more likely based on the observation of no run. Note that buyers' belief about s is endogenous to creditors' strategy, which we will discuss in the analysis in detail. Finally, we assume asset buyers make competitive price offers. Therefore, the equilibrium price should leave the buyers break even in expectation upon the run has occurred. In fact, the equilibrium strategy can be viewed as the market demand for bank assets.

2.4 Timing

The timing of the basic model is summarized in Figure 2. Events at $t = 0.5$ take place sequentially.

Figure 1: Timing of the game

t = 0	t = 0.5	t = 1
Banks are established, with their portfolios and liability structures as given.	1. s and θ_1 realize. 2. Creditors of bank 1 receive private noisy signals about θ_1, and decide to run or not. 3. After observing the outcome of bank run, buyers purchase assets on sale.	1. Returns become public. 2. Remaining obligations are settled.

2.5 Equilibrium in the Basic Model

In this section, we solve the market equilibrium of our basic model. The basic model resembles to the one in Li and Ma (2018). To start, we use the Rational Expectations Equilibrium (REE) as our equilibrium concept.

Definition. *A symmetric REE of our basic model is characterized by: (i) An equilibrium strategy profile* $\left(x_1^*, \theta_1^*, P_1^*\right)$*. Each creditor* j *in bank* 1 *plays a threshold strategy*

$$a(x_1^j) = \begin{cases} \textit{withdraw} & x_1^j \leq x_1^* \\ \textit{wait} & x_1^j > x_1^* \end{cases}$$

Bank 1 *fails because of run when its realized cash flow* $\theta_1 \leq \theta_1^*$*. On the other hand, asset buyers bid a price* P_1^* *to purchase bank* 1*'s assets on sale. (ii) Creditors form the 'correct' belief about banks' secondary market asset prices, and the distribution of early withdrawals following the realized cash flows. On the other hand, each asset buyer forms the 'correct' beliefs about the qualities of banks' asset on sale and the realized aggregate state* s *following the observed bank runs.*

We take the normal approach of backward induction to solve the basic model. The detailed solution resembles to that of Li and Ma (2018), we only provide the critical equations and steps here.

We start with the asset buyers' bidding game. The buyers are called upon to move only if a bank run occurs. In a Rational Expectations Equilibrium, buyers who believe that creditors using a symmetric switching threshold x_1^* understand a run happens in Bank 1 if and only if its cash flow is lower than θ_1^*. They also Bayesian update about State s as follows.

$$\omega_R^G\left(\theta_1^*\right) = Pr(s = G|O_1 = R) = \frac{1}{1 + \left(\frac{\theta_1^* - \underline{\theta}_B}{\theta_1^* - \underline{\theta}_G}\right)\kappa_0} = 1 - \omega_R^B\left(\theta_1^*\right), \tag{4}$$

where κ_0 is defined as $\frac{1-\alpha}{\alpha}\left(\frac{\overline{\theta} - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_B}\right)$. The subscript F denotes that an outcome of run in Bank 1, i.e., $O_1 = R$,[17]while the superscript G denotes buyers' posterior belief that $s = G$.

When asset buyers bid competitively for the bank's assets on sale, the equilibrium of the secondary market competition entails buyers' bid to be equal to the expected asset quality.

[17]Note that we are in the limiting case, thus Bank 1 is doomed to fail when there is run. In fact, O_1 could take two values R or S, where $O_1 = S$ denotes there is no run on the bank. But recall that the asset buyers are called upon to move only if the bank faces a run and sells its assets to meet the withdrawals.

Otherwise, undercutting will happen among the buyers. The competitive price offered by the buyers can be written as follows.

$$P_1^* = P(\theta_1^*) = \omega_R^G\left(\theta_1^*\right)\frac{\underline{\theta}_G + \theta_1^*}{2} + \omega_R^B\left(\theta_1^*\right)\frac{\underline{\theta}_B + \theta_1^*}{2} = \frac{E(\underline{\theta}_s|O_1 = R) + \theta_1^*}{2} \tag{5}$$

Note that for a given aggregate state s, the buyers perceive the average quality of Bank 1's asset on sale to be $E_s\left[\tilde{\theta}_1\middle|\tilde{\theta}_1 < \theta_1^*\right] = \int_{\underline{\theta}_s}^{\theta_1^*}\theta\cdot\frac{1}{\theta_1^*-\underline{\theta}_s}d\theta = \frac{\underline{\theta}_s+\theta_1^*}{2}$.

Then it can be showed that a candidate equilibrium price of bank 1's assets on sale must belongs to the region $\left[\overline{P}, qD_2\right)$, where $\overline{P} = (\underline{\theta}_B + D_2)/2$.[18] A direct implication of this result is that Bank 1 never fails at $t = 0.5$ because $P_1^* \geq \overline{P} > D_1$. So bank 1 can always repay its $t = 0.5$ liabilities. However, runs on the intermediate date *do* lead to bank failure in equilibrium. It is also worth noticing that parametric assumption (3) guarantees $qD_2 > \overline{P}$, so that the set of candidate equilibrium prices is non-empty.

We then move backwards to solve Bank 1 creditors' coordination game when they anticipate an asset price $P_1^* \in [\overline{P}, qD_2)$ and derive an equilibrium condition of θ_1^*.

We first establish two dominance regions for bank 1's cash flow, defined as $\theta^L = D_2$ and $\theta^U = F/(1 - D_1/P_1^*\overline{P})$. It is a dominant strategy for a representative creditor j to withdraw when $\theta_1 \in \left[\underline{\theta}_s, \theta^L\right)$, and to wait when $\theta_1 \in \left(\theta^U(P_1^*), \overline{\theta}\right]$. Second, we characterize a representative creditor j's posterior belief about the fraction of withdrawals when Bank 1's fundamental is out of the dominance regions and derive her best response to the other creditors' threshold strategy. It can be shown that in a symmetric equilibrium the strategy profile is characterized by $a(x_1^j)$ stated in the definition. On the other hand, the critical cash flow θ_1^* must satisfy the following condition.

$$\theta_1^* = \frac{D_2 - D_1}{1 - qD_1/P_1^*} \tag{6}$$

It can be shown that the critical signal $x_1^* \to \theta^*$ when $\epsilon \to 0$.

Then a REE of the game is a solution of the system of two equations (18) and (6). So the creditors hold the 'correct' belief about the equilibrium asset price P_1^*, and the same for asset buyers. We summarize the result in the following Lemma and provide the entire proof in the Appendix.

[18] We provide the proof in the Appendix.

Lemma 1. *When $\epsilon \to 0$, there exists a unique REE of the game characterized by $\left(x_1^*, \theta_1^*, P_1^*\right)$. θ_1^* and P_1^* jointly solving the system of equations* (5) *and* (6) *such that $\theta_1^* \in [\theta^L, \theta^U(P_1^*)]$ and $P_1^* \in [\overline{P}, D_2)$. And in the limiting case of our model, we have $x_1^* \to \theta_1^*$.*

Proof. See Appendix A.2 □

It should be noticed that there is the bank run and asset sale mutually feed back each other and determines the (general) equilibrium. First consider the partial equilibrium condition (5) in the secondary market. The price is not sticky, an increase in the critical cash flow θ_1^* leads to an increase in buyers' bid P_1^*.

$$\frac{\partial P_1^*}{\partial \theta_1^*} = \frac{1}{2} + \frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G - \underline{\theta}_B) = \frac{1}{2} + \frac{\kappa_0(\underline{\theta}_G - \underline{\theta}_B)^2}{[(\theta_1^* - \underline{\theta}_G) + \kappa_0(\theta_1^* - \underline{\theta}_B)]^2} > 0$$

A marginal increase in θ_1^* increases the quality of asset on sale in two channels. The maximum return of the asset on sale increases as in the first term of the above equation. There is also an indirect effect through the buyers' inferencing about state, i.e., a bank run is more easily triggered when either coordination problem among creditors aggravates or a poor realization of the distribution of cash flow. Conditional on observing a run, a higher θ_1^* actually suggests that the run is less likely to be triggered by a poor realization of state. On the other hand, consider the partial equilibrium condition (6) in the bank's credit market. An increase in the secondary market asset price P_1^* leads to a lower θ_1^*.

$$\frac{\partial \theta_1^*}{\partial P_1^*} = -\frac{(D_2 - D_1)qD_1}{(1 - qD_1/P_1^*)^2(P^*)^2} < 0$$

Anticipating a higher secondary market price P_1^*, the coordination problem among creditors alleviates. Consequently, they take less aggressive bank run strategy, which leads to lower critical cash flow θ_1^*.

Then it should be clear why our model admits an unique equilibrium despite the mutually feed back mechanism. Indeed, a higher asset price P_1^* leads to a lower θ_1^*, which in turn pushes the asset price downwards.

3 NSFR Requirement and The Risk of Failure

3.1 NSFR Requirement and The Risk of Failure

Our basic model is parsimonious and could serve as a tool to conduct policy analysis. The policy we are particularly interested in is the recent Net Stable Funding Ratio (NSFR) requirement in Basel III Accord proposed by Basel Committee of Banking Supervision (BCBS).

Our particular concern of the NSFR requirement is it could subject to the famous Lucas Critique: When assigning RSF factors according to assets' liquidity risk, the requirement does not take a forward-looking view of evaluating asset' liquidity. In other words, the requirement fails to take into account the (market) liquidity of assets are, in fact, endogenous to market participants' trading decisions, which in turn could be a response to the requirement itself. Consequently, those RSF factors could be wrongly designed to undermine NSFR requirement to contain banks' risk of failure.

We then illustrate our point using the theoretical model developed above. In the model, a bank's risk of failure can be simply defined as the ex-ante probability that its cash flow is lower than the critical bank run threshold θ_1^*. A bank's probability of failure can be expressed as follows.

$$IS = Pr(\tilde{\theta}_1 \leq \theta_1^*) = \alpha \left(\frac{\theta_1^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} \right) + (1 - \alpha) \left(\frac{\theta_1^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B} \right) \tag{7}$$

Note that from an ex-ante point of view, a bank's cash flow is uniformly distributed on the interval $[\underline{\theta}_G, \overline{\theta}]$ with probability α, and $[\underline{\theta}_B, \overline{\theta}]$ with the complementary probability.

Consider a regulator whose objective is to limit bank's risk of failure to be equal to some probability $\gamma \in (0, 1)$, i.e., $IS = \gamma$.[19] The regulatory objective can be translated into the following constraint imposed on bank.

$$\theta_1^* = VaR_\gamma \equiv E(\underline{\theta}_s) + \gamma \cdot (\overline{\theta} - \underline{\theta}_G)(\overline{\theta} - \underline{\theta}_G) \tag{8}$$

where VaR_γ can be directly calculated given the distribution of bank's asset and the confidence level γ, thus is exogenous. To fulfill this purpose, the regulator then sets ASF and RSF factors on banks' liabilities and assets and impose a requirement on NSFR.

[19] γ can be derived from the regulator's optimal choice between limiting the bank's risk of failure and guaranteeing the credit supply.

In our model, for simplistic reason, bank's balance sheet consists of a single kind of asset and two categories of liabilities. For the liability side, the ASF factors for long-term retail deposits and short-term wholesale debts are respectively 100% and 0%. On the other hand, the (unregulated) market equilibrium price P_1^* divided by the asset's expected return $(E[\underline{\theta}_s] + \theta^*)/2$ can be served as a proxy as 'the amount of bank's assets that would have to be funded because it could not be fully monetised through sale or used as collateral in a secured borrowing transaction.' Thus, we use it to gauge bank asset's marketability. Then according to NSFR requirement, we can categorize bank's asset according to its marketability, i.e., its RSF factor being 0 if the proxy is larger or equal to 1. Without loss of generality, we assume the asset's RSF factor is $l\% \in [0\%, 100\%]$. What is critical is that the regulator takes an empiric perspective, by categorizing of assets and liabilities and assigning respective factors in the current NSFR requirement. Then a bank's NSFR can be computed as follows.

$$\text{NSFR} = \frac{100\% \cdot F + 0\% \cdot (1 - F)}{l\% \cdot 1} = \left(\frac{F}{l}\right)\% \tag{9}$$

Now we analyze how the regulator can fulfill its objective of containing bank's risk of failure (8) by imposing the NSFR requirement. To start, we first analyze a situation where the regulator treat the asset price P_1^* of bank's asset as exogenous. Note that this 'naive' treatment of assets' marketability resembles real-world practice of the current NSFR regulation.

3.2 Exogenous Asset Price and Stable Effect

Consider a situation that the bank's private optimal capital structure choice entails $F = \underline{F}$, and $\underline{F}$ satisfies all the above parametric assumptions. On the other hand, the regulator observes the historical data, and treats the bank's asset price as exogenously given at the level P^e. Accordingly, the regulator forecasts the critical cash flow as follows.

$$\theta_1^*(\underline{F}, P^e) = \frac{D_2(\underline{F}) - D_1(\underline{F})}{1 - qD_1(\underline{F})/P^e}$$

Where $D_1(\underline{F}) = q(1 - \underline{F})r_D$ and $D_2(\underline{F}) = \underline{F} + (1 - \underline{F})r_D$. Without loss of generality, we could set the value of P^e to be equal to $P_1^*(\underline{F})$, where $\theta_1^*(\underline{F})$ and $P_1^*(\underline{F})$ jointly solve (6) and (5). So P^e is actually the equilibrium price for the bank's asset. We consider the case where

$\theta_1^*(\underline{F}, P^c) > VaR_\gamma$, so the regulator forecasts the bank's risk of failure is higher than γ, and would like to set regulation to contain the high risk.

Now suppose the regulator's objective is to reduce the bank's risk of failure to be lower than γ. We show this goal can be accomplished by setting a NSFR requirement. For simplicity, we focus on the case that the bank only make adjustment on its liability side by increasing or decreasing the amount of F.[20] Consider now the regulator's targets are a combination of parameters, $(l\%, \underline{\text{NSFR}}\%)$, the RSF factor of bank's asset and a minimum floor on the bank's NSFR. The regulator can choose these regulatory parameters as follows.

$$l\% \cdot \underline{\text{NSFR}}\% = F_\gamma\%$$

where the level F_γ is defined by

$$\theta_1^*(F_\gamma, P^e) = \frac{D_2(F_\gamma) - D_1(F_\gamma)}{1 - qD_1(F_\gamma)/P^e} = VaR_\gamma \tag{10}$$

Again, $D_1(F_\gamma) = q(1 - F_\gamma)r_D$ and $D_2(F_\gamma) = F_\gamma + (1 - F_\gamma)r_D$. We further focus on $F_\gamma \in (\underline{F}, 1]$.

Then the regulator imposes the NSFR requirement, under which the bank's NSFR has to be higher or equal to $\underline{\text{NSFR}}$. Given the bank's NSFR calcualted by (9), the regulator expects the effect of imposing such NSFR requirement as: The bank is forced to increase its amount of retail deposits at least to a level F_γ. This is because

$$\text{NSFR} = \left(\frac{F}{l}\right)\% \geq \underline{\text{NSFR}}\% \Rightarrow F \geq l \cdot \underline{\text{NSFR}} = F_\gamma$$

As a result of the NSFR requirement, the bank's critical cash flow reduces to $\theta_1^*(F_\gamma, P^e)$, and its risk of failure will be lower than q.

It is more intuitive to analyze the marginal effect of increasing F on the bank's risk of failure. Note that $\theta_1^*(F_q, P^c)$ can be rewritten as

$$\theta_1^*(F_\gamma, P^e) = \theta_1^*(\underline{F}, P^e) + \int_{\underline{F}}^{F_\gamma} \frac{\partial\theta_1^*(F, P^e)}{\partial F} dF = \theta_1^*(\underline{F}, P^e) + [\theta_1^*(F_\gamma, P^e) - \theta_1^*(\underline{F}, P^e)] \tag{11}$$

[20]We will allow the bank to adjust its asset portfolio to meet the NSFR requirement and provide the analysis in the Extension and Discussion section.

The regulator consider the derivative $d\theta_1^*(F, P^c)/dF$ has the following form.

$$\frac{\partial\theta_1^*(F, P^e)}{\partial F} = \frac{(1 - r_D)(1 - \frac{qD_1}{P^e}) + qr_D(1 - \frac{qD_2}{P^e})}{(1 - \frac{qD_1}{P^e})^2} < 0. \tag{12}$$

This derivative is negative because the equilibrium price $P^e = P_1^*(\underline{F}) \in [\overline{P}, qD_2)$ where $\overline{P} > D_1$. A marginal increase in F reduces the critical cash flow $\theta_1^*(F, P^e)$. Intuitively, one can check $dD_1/dF = -qr_D < 0$, the (maximum) face value of wholesale debts decreases as F increases. So for a given fraction $\lambda \in [0, 1]$ of the wholesale creditors who choose to withdraw early, the fraction of bank's assets $f = (\lambda D_1)/P^e$ being liquidated reduces as the face value of wholesale debts D_1 reduces. On the other hand, one can check that the bank remaining claim $F + (1 - \lambda)(1 - F)r_D$ also reduces when F increases under the condition $D_1 < P^e < qD_2$. The bank 1 has higher probability to survive at $t = 2$. So its wholesale creditors have less incentive to withdraw early, the coordination problem alleviates.

When the regulator treats the asset price P^e as exogenous, an increase of the retail deposits F makes the bank more resilient to run. The result confirms Basel committee's intention of reducing a bank's risk of failure by demanding it increases its amount of long-term retail funding. We label this marginal effect as th *stable* effect of increasing F. We summarize the above results in the following Proposition 1.

Proposition 1. *When the price (market liquidity) of bank's asset is considered as exogenous, the regulator can set a NSFR requirement as a combination of a RSF factor and a minimum floor on bank's NSFR* ($l\%, \underline{NSFR}\%$) *such that* $l\% \cdot \underline{NSFR}\% = F_q\%$. *Constrained by such NSFR requirement, the bank has to increase its retail deposits to be higher or equal to* F_γ, *where* F_γ *solves the equation* (10).

Suppose the bank incurs costs by increasing retail deposits over its private optimial level $\underline{F}$, and the costs are monotonic increasing as the amount of additional retail funding increases, the bank will just choose $F = F_\gamma$. So the NSFR requirement for the bank is binding. To ease discussion, we will assume this is the case.

In practice, the regulator can set $\underline{NSFR}\% = 100\%$ and the RSF factor of bank's asset to be $l\% = F_\gamma\%$, or alternatively it can set the RSF factor $l\%$, say $l\% = 65\%$, and the $\underline{NSFR}$ to be $(F_\gamma/0.65)\%$. This result confirms the BCBS' intention of reducing a bank's risk of failure by imposing the NSFR requirement.

However, as pointed out by Lucas, it is problematic to evaluate the effect of a policy without taking a forward-looking point view by considering economic agents' response to the policy. In the following section, we show that the NSFR requirement discussed above can not fulfill its goal to limit the bank's risk of failure to q when the asset buyers rationally respond to the policy.

3.3 A Lucas Critique on NSFR Requirement

Different from the regulator's point of view that the asset price is maintained at its equilibrium value P^e, the asset price is actually endogenously determined. Now the bank's adjustment of retail funding F due to the NSFR requirement will take effect through the feed back mechanism analyzed in Section 3.1. To evaluate the effect of NSFR requirement policy involves additional complications. In the following analysis, we show first an increase in F will have a countervailing effect on the bank's risk of failure through the asset buyers' rational response. Second, the effectiveness of stable effect on reducing the risk of failure is also influenced by the endogenous asset price. A NSFR requirement as in Proposition 1 could lead to under-regulation or over-regulation.

To illustrate these points, we conduct the same exercise as (12) to examine the marginal effect on changing F on the equilibrium critical threshold θ_1^*.

$$\frac{d\theta_1^*}{dF} = \frac{(1-r_D)(1-\frac{qD_1}{P_1^*}) + qr_D(1-\frac{qD_2}{P_1^*})}{(1-\frac{qD_1}{P_1^*})^2} - \frac{(D_2-D_1)\frac{qD_1}{(P_1^*)^2}}{(1-\frac{qD_1}{P_1^*})^2}\frac{dP_1^*}{dF} = \frac{\partial\theta_1^*}{\partial F} + \frac{\partial\theta_1^*}{\partial P_1^*}\frac{dP_1^*}{dF} \tag{13}$$

Notice that we now consider the general equilibrium of bank's credit market and asset market.

Besides the stable effect of increasing stable funding remains as the first term $\partial\theta_1^*/\partial F$, there exists a second effect

$$\frac{\partial\theta_1^*}{\partial P_1^*}\frac{dP_1^*}{dF} = -\frac{(D_2-D_1)qD_1}{(P_1^*-qD_1)^2}\frac{dP_1^*}{dF}. \tag{14}$$

A change in the critical cash flow θ_1^* affects buyers' expectation of the asset qualities of the bank, and their equilibrium bid P_1^*. And this change in P_1^* will be correctly anticipated by creditors who will further adjust θ_1^*. This second effect then consider the change in F on P_1^*, and its effect on θ_1^* through the feedback mechanism between bank run and asset sale. It can be seen that this effect through the endogenous price is actually a countervailing force as opposed to the stable effect. We summarize this result in the following lemma 2.

Lemma 2. *From equilibrium conditions* (5) *and* (6)*, we have* $dP_1^*/dF < 0$*. A marginal increase in stable funding F reduces the equilibrium asset price of the bank. An increase of F then increases* θ_1^* *through the feedback mechanism:* $(\partial\theta_1^*/\partial P_1^*)(dP_1^*/dF) > 0$.

Proof. See Appendix A.3 □

The intuition is as follows. Recall the direct effect of NSFR, an increase in F reduces θ_1^* through the reduction of wholesale/total liabilities. When price is endogenous, asset buyers expect the qualities of the bank's assets on sale reduces accordingly, thus they bid lower P_1^*. Rationally anticipating this reduction in equilibrium asset price P_1^*, creditors' coordination problem intensifies. They tend to take even more aggressive bank run strategy leading to a higher θ_1^*. We label this countervailing effect of increasing F through feedback (general equilibrium) channel as the *feedback* effect.

When the asset price is endogenous, the stable effect of increasing F as in the first term of (13) is also different from that when the price is exogenously fixed at P^e. To ease discussion, we label $\theta_1^*(F, P_1^*)$ as the critical cash flow in the general equilibrium framework, where it depends on F both directly through the face value of liabilities and indirectly through the endogenous P_1^*. We also label $\theta_1^*(F, P^e)$ as the critical cash flow in the partial equilibrium framework where P^e is assumed to be fixed at the level $P_1^*(\underline{F})$, i.e., the equilibrium price of Bank 1's asset before imposing the liquidity requirement. The following Lemma then characterizes the comparison.

Lemma 3. $\partial\theta_1^*(F, P_1^*)/\partial F < \partial\theta_1^*(F, P^e)/dF$ *at each level of* $F \in (\underline{F}, F_q]$*. An increase in F is more effective in alleviating creditors' coordination failure through the direct reduction in liabilities when the asset price is endogenous.*

Proof. See Appendix A.4 □

The effect of reduction in liabilities (D_1 and D_1) in curbing creditors' coordination failure and making the bank stable is also related to the asset price P_1^*. One can calculate that $\frac{\partial^2\theta_1^*}{\partial F \partial P_1^*} > 0$, i.e., an increase in F is more effective when the asset price is lower. In addition, we know that P^e equals to P_1^* when $F = \underline{F}$ by our assumption, and it is higher than P_1^* for all other $F \in (\underline{F}, F_q]$ by Lemma 2. So an increase F is more effective in curbing bank run and reducing the critical cash flow as stated in Lemma 3.

Before analyzing the effect of NSFR requirement when the asset price is endogenous, we determine the overall effect of increasing F on θ_1^* and the bank's risk of failure, we should consider the overall effect of stability and feedback. The following Proposition gives the result.

Proposition 2. *When the price of bank's asset is endogenously determined, an increase in F still reduces the bank's critical cash flow θ_1^*, namely $d\theta_1^*/dF < 0$.*

Proof. See Appendix A.5 □

The intuition that stable effect still dominates feedback effect is as follows. The source of the countervailing feedback effect is because asset buyers anticipate a reduction in θ_1^* caused by the stable effect, and bid less. This makes creditors' coordination problem intensifies and results in an otherwise incease in θ_1^*, which pushes asset price upwards. So compared to the stable effect, the feedback effect is both caused by stable effect and limited, so it is of second order. For Bank 1's risk of failure, it is easy to see IS_1 is an increasing function of θ_1^* from (7). The risk decreases as F increases.

Lastly, we analyze the effect of the NSFR requirement, as stated in Proposition 1, on the bank's risk of failure when taking into full consideration of the players rational response to the policy. Given the NSFR requirement ($l\%, \underline{\text{NSFR}}\%$), the bank has to increase its current amount of retail deposits $\underline{F}$ to the level $F_\gamma > \underline{F}$. The critical bank run cash flow will become

$$\theta_1^*(F_\gamma) = \theta_1^*(\underline{F}) + \int_{\underline{F}}^{F_\gamma} \frac{d\theta_1^*(F)}{dF} dF = \theta_1^*(\underline{F}, P_1^*(\underline{F})) + \int_{\underline{F}}^{F_\gamma} \left(\frac{\partial\theta_1^*(F, P_1^*)}{\partial F} + \frac{\partial\theta_1^*(F, P_1^*)}{\partial P_1^*} \frac{dP_1^*(F)}{dF} \right) dF \tag{15}$$

Recall that $\theta_1^*(\underline{F})$ is the critical cash flow when bank's optimal capital structure is $\underline{F}$. The the effect of the NSFR requirement on the bank's risk of failure is determined by the value of $\theta_1^*(F_q)$. By comparing (15) with (11), it can be seen that an increase in F under endogenous asset price makes the bank more stable than that in the exogenous asset price. However, this more significant stable effect has to be weighed against a countervailing feed back effect. The final value of $\theta_1^*(F_\gamma)$ can be either higher or lower than $\theta_1^*(F_\gamma, P^e)$ depends on the relative strength of the two effects. Accordingly, the bank's risk of failure will be higher than F_γ (when $\theta_1^*(F_\gamma) > \theta_1^*(F_\gamma, P^e)$) or lower than F_γ (when $\theta_1^*(F_\gamma) < \theta_1^*(F_\gamma, P^e)$). But recall Proposition 2, the overall effect of the NSFR requirement is it still contains the bank's risk of failure. We summarize the discussion in this section with Proposition 3.

Proposition 3. *When the price of bank's asset is endogenously, a NSFR requirement as in Proposition 1 targeting to limit the bank's risk of failure to F_γ can lead to under-regulation or over-regulation: The bank's risk of failure IS will be either higher than F_γ or lower than F_γ*

4 NSFR Requirement and The Risk of Contagion

Another important target of the NSFR requirement is to limit the risk of contagion in the banking system. The main theme of this section is that a bank's adjustment of balance sheet due to the NSFR requirement can create negative spillovers to the other bank through a new inferencing channel. To formally analyze this implication of the NSFR requirement on the risk of contagion, we extend our model by introducing a second bank, Bank 2. To be as parsimonious as possible, we modify our basic model as follows.

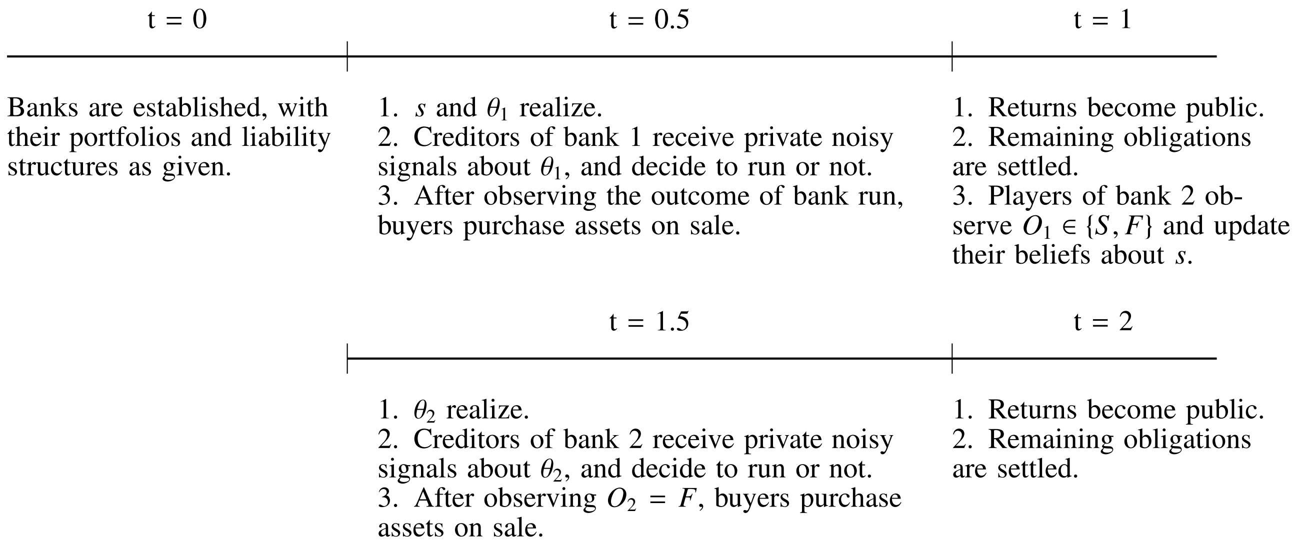
We introduce a second period game starting from $t = 1$ as well as a second bank, Bank 2, with its asset and liability structure private optimally chosen enters into our model. Bank 2's unit portfolio of assets also takes one period to mature, i.e., it matures at $t = 2$. Its return $\tilde{\theta}_2$ has the same distribution function as Bank 1, i.e., $\tilde{\theta}_2 \sim U(\underline{\theta}_s, \overline{\theta})$. However we assume its optimal capital structure entails it use higher amount of retail deposits $f > F$ to finance the unit portfolio.[21] Consequently, we can calculate Bank 2's NSFR as $\text{NSFR}_2 = (f/l)\%$. We further assume $\text{NSFR}_2 > (F_\gamma/l)\% = \underline{\text{NSFR}}\%$, so that Bank 2 does not need to make any adjustment of its balance sheet.[22] Second, we also assume Bank 2 issues the rest $1 - f$ to a measure 1 of wholesale creditors, each of which observes a noisy signal $x_2^j = \theta_2 + \epsilon_2^j$, where ϵ_2^j is independently draw from $U(-\epsilon, \epsilon)$. Different from retail depositors who hold their claims (with unit face value 1) to $t = 2$, each wholesale creditor j also takes two actions, 'withdraw' at $t = 1.5$ to obtain qr_D or 'wait' to $t = 2$ to get r_D if Bank 2 remains. We further assume that this measure of creditors are *separate* with the one of Bank 1. So the first period game will not be affected. On the other hand, we also assume there are many deep-pocketed asset buyers who make bid to purchase upon Bank 2 liquidates its long-term asset at $t = 1.5$. One important assumption we make is that the first period bank run outcome $O_1 \in \{R, S\}$ is observed by the players of the second period game. Accordingly, the players' belief about state s will no longer be $Pr(s = G) = \alpha$

[21] One may think Bank 2 as a retail bank, which has a comparative advantage to collect retail funding and makes consumer/commercial loans. And Bank 1 as a wholesale bank, which has comparative advantage to issue wholesale debts and invest. Our assumption then normalize banks' asset sides.

[22] This assumption is innocuous, we make it for simplicity. We will relax it to consider symmetric banks in the discussion and show that our main result still holds.

as the bank run outcome generates new information about s. This information affects players' decisions in the second period since banks' return is subject to the same aggregate risk. We will also look for a symmetric REE as our solution concept. A symmetric REE of the second period game is characterized by: (i) An equilibrium strategy profile $\left\{(x^*_{R,2}, \theta^*_{R,2}, x^*_{S,2}, \theta^*_{S,2}), (P^*_{R,2}, P^*_{S,2})\right\}$, where R and S denote the first period bank run outcomes. (ii) The respect beliefs of players. In the equilibrium, players' beliefs are 'correct' and their strategies are optimal. [23]The new timeline is as follows.

Figure 2: Timing of the sequential game



4.1 REE of the Second Period Game

We briefly characterize the equilibrium of the second period game. At the beginning of $t = 1$, players of bank 2 observe neither the realized cash flow θ_1 nor State s, but nevertheless form rational belief about s depending on the bank run outcome $O_1 \in \{R, S\}$of first period game. In particular, they understand the bank run game as well as the competitive bidding game, and know that run happens in Bank 1 ($O_1 = R$) if and only if bank 1's realized cash flow is lower than θ^*_1. Note that players' beliefs about State s upon observing an outcome O_1, $\omega^G_R\left(\theta^*_1\right)$ and $\omega^B_R\left(\theta^*_1\right)$ are already derived in (4). On the other hand, the beliefs when observing $O_1 = S$ can

[23] We can also deinfe the REE for the whole game.

be expressed as follows.

$$\omega_S^G \equiv Prob(s = G|O_1 = S) = \frac{1}{1+\kappa_0} = 1 - Prob(s = B|O_1 = S) \equiv 1 - \omega_S^B, \tag{16}$$

where κ_0 has the same definition as before. Note that ω_S^G and ω_S^B are not functions of θ_1^*. This is because now bank 1's realized cash flow is higher than θ_1^* while lower than $\overline{\theta}$. We have assumed $\overline{\theta}$ is fixed across state to model the maximum loan repayment is fixed across states. The same upside potential $(\overline{\theta} - \theta_1^*)$ cancels out during the calculation. But the beliefs when $O_1 = R$ are functions of θ_1^*. It is fairly easy to check that $\omega_S^G > \alpha > \omega_F^G(\theta_1^*)$. Intuitively, banks' fundamental has worse distribution in State B, then observing bank run (cash flow being lower than the equilibrium threshold) and corresponding asset sale suggests it is more likely State $s = B$.

With the priors being replaced by the beliefs characterized above, we solve the second period game starting by characterizing the equilibrium bid of asset buyers who move last. Again, asset buyers move only if bank 2 is forced to liquidate its assets in the secondary asset market. In particular, asset buyers understand the creditors' bank run game and rationally expect the assets on sale have quality lower than $\theta_{O_1,2}^*$, where $O_1 \in \{R, S\}$. Based on the beliefs characterized above, asset buyers Bayesian update again their final posterior beliefs about State s. We can calculate buyers' posterior beliefs of State s as follows.

$$\omega_{O_1,R}^G(\theta_{O_1,2}^*) \equiv Prob(s = G|O_1, O_2 = R) = \frac{1}{1 + \left(\frac{\omega_{O_1}^B}{\omega_{O_1}^G}\right)\left(\frac{\overline{\theta}-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_B}\right)\left(\frac{\theta_{O_1,2}^*-\underline{\theta}_B}{\theta_{O_1,2}^*-\underline{\theta}_G}\right)} = 1 - \omega_{O_1,R}^B(\theta_{O_1,2}^*). \tag{17}$$

Notice that asset buyers update their 'final' beliefs about s based on the first period game outcome O_1 as well as the run/forced asset sale, i.e., $O_2 = R$ of Bank 2. Recall that only the posteriors $\omega_{R,R}^G$ and $\omega_{R,R}^B$ are functions of θ_1^*. Similar to the first period game, the equilibrium secondary market asset price can be expressed explicitly as follows.

$$P_{O_1,2}^* = \omega_{O_1,R}^G\left(\theta_{O_1,2}^*\right)\frac{\underline{\theta}_G + \theta_{O_1,2}^*}{2} + \omega_{O_1,R}^B\left(\theta_{O_1,2}^*\right)\frac{\underline{\theta}_B + \theta_{O_1,2}^*}{2} = \frac{E(\underline{\theta}_s|O_1, O_2 = F) + \theta_{O_1,2}^*}{2} \tag{18}$$

Again the expression $E(\underline{\theta}_s|O_1, O_2 = R) = \omega_{O_1,F}^G\left(\theta_1^*, \theta_{O_1,2}^*\right) \cdot \underline{\theta}_G + \omega_{O_1,R}^B\left(\theta_1^*, \theta_{O_1,2}^*\right) \cdot \underline{\theta}_B$ represents the expected lower-bound value of θ, based on observed O_1 and run in bank 2, i.e., $O_2 = R$. Paralleled to that in the first period game, we define $d_1 \equiv q(1-f)r_D$, $d_2 \equiv f + (1-f)r_D$ and $\underline{P} = (\underline{\theta}_B + d_2)/2$. It can be showed, again, a candidate equilibrium price of Bank 2's assets on sale must belong to the region $\left[\underline{P}, d_2\right)$, and Bank 2 will not directly fail at $t = 1.5$ because of run. However, recall that runs on the intermediate date *do* lead to bank failure at $t = 2$ in equilibrium.

We similarly solve Bank 2 creditors' run game following the outcome O_1 by first characterizing the upper and lower dominance regions as $\left(\theta^U(P^*_{O_1,2}), \overline{\theta}\right]$ and $\left[\underline{\theta}_s, \theta^L\right)$, where $\theta^U(P^*_{O_1,2}) = F/(1 - D_1/P^*_{O_1,2})$ and $\theta^L = D_2$. With the global games refinement for the intermediate cash flows, we obtain the threshold bank run equilibrium $(x^*_{O_1,2}, \theta^*_{O_1,2})$ with the functional form of $\theta^*_{O_1,2}$ explicitly given by

$$\theta^*_{O_1,2} = \frac{d_2 - d_1}{1 - qd_1/P^*_{O_1,2}} \tag{19}$$

Then the REE of the second period game following an outcome $O_1 = R$ or S is jointly determined by (18) and (19). We prove the existence and uniqueness in the following Lemma.

Lemma 4. *Following an outcome $O_1 \in \{R, S\}$, the only REE of the second period game is $\left((x^*_{O_1,2}, \theta^*_{O_1,2}), P^*_{O_1,2}\right)$ and $O_1 \in \{S, F\}$. The equilibrium cash flow $\theta^*_{O_1,2}$ and the asset price $P^*_{O_1,2}$ are explicitly given by* (19) *and* (18). *In the limiting case $\epsilon \to 0$, $x^*_{O_1,2} = \theta^*_{O_1,2}$, $O_1 \in \{S, F\}$.*

Proof. See Appendix A.6. □

Note that depending on the first period's outcome, equation (19) may characterize two distinct critical thresholds $\theta^*_{R,2}$ and $\theta^*_{S,2}$. This condition prescribes a symmetric equilibrium of bank 2 creditors' bank run game for an anticipated equilibrium secondary market price $P^*_{R,2}$ or $P^*_{S,2}$.

With the equilibrium of our two-period model being fully characterized, we present two main properties in the following Proposition.

Proposition 4. *(i) The critical cash flows and equilibrium asset prices have the following ranking: $\theta^*_{R,2} > \theta^*_{S,2}$ and $P^*_{R,2} < P^*_{S,2}$. (ii) $d\theta^*_{R,2}/d\theta^*_1 < 0$, a lower first period critical cash flow leads to a higher second period critical cash flow.*

Proof. See Appendix A.7. □

Asset buyers form rational beliefs about s and bid accordingly. The intuition of result (i) of Proposition 4 hinges on the monotonic ranking of beliefs $\omega^G_{S,R}(\theta^*_{S,2}) > \omega^G_{R,R}(\theta^*_1, \theta^*_{R,2})$. Compared to the single observation $O_1 = R$, the buyers are more optimistic about s when they observe a good outcome in Bank 1 ($O_1 = R$). Their bids are higher when beliefs about s become more optimistic because it is more likely banks' assets have a good distribution. Creditors will take less aggressive run strategy leading to lower critical cash flow anticipating buyers' higher bids. This will in turn force the rational buyers to lower the bid as the maximum return of assets sold gets poorer. Our result (i) shows in equilibrium the expected quality improvement due to the belief of better distribution dominates. Higher asset prices and lower critical cash flows are associated with better beliefs about s.

The result (*ii*) identifies the financial contagion between two banks. When observing $O_1 = R$, players in the second period form beliefs about s based on the Bank 1 creditors' optimal strategy θ_1^* (x_1^*).[24] Through belief updating, there comes the interdependence of the optimal run strategy $\theta_{R,2}^*$ on θ_1^*. To see why a less aggressive first period run strategy will lead to a more aggressive second period run strategy, we take derivative on $\omega_R^G(\theta_1^*)$ with respective to θ_1^* and obtain

$$\frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*} = \frac{(\underline{\theta}_G - \underline{\theta}_B)\kappa_0}{(\theta_1^* - \underline{\theta}_G) + (\theta_1^* - \underline{\theta}_B)\kappa_0} > 0.$$

This means, conditional on observing the run outcome, $O_1 = R$, players anticipate state is less likely to be G when θ_1^* decreases. Intuitively, the failure of Bank 1 can be more easily triggered either if creditors take more aggressive run strategy, i.e., resulting in higher θ_1^*, given the distribution of cash flows, or if the distribution of cash flows gets worse, i.e., $s = B$, given the bank run strategy. When θ_1^* decreases, players tend to make worse inference about s, it is the worse cash flow distribution that contributes to the run. In the second period game, asset buyers update once again the belief about s and

$$\frac{\partial\omega_{R,R}^G(\theta_1^*, \theta_{F,2}^*)}{\partial\theta_1^*} > 0.$$

Accordingly, the buyers bid lower and Bank 2 creditors' coordination problem intensifies, leading to higher $\theta_{R,2}^*$.

Having solving the equilibrium of the second period game, and characterizing its main properties, we then use the model to study the implicationss of the NSFR requirement on the risk of contagion.

[24]On the contrary, when $O_1 = S$, the buyers' posteriors about s when observing a run on Bank 2 are not functions of θ_1^*.

4.2 NSFR Requirement and Risk of Contagion

In this section, our aim is to analyze how the NSFR requirement as in Proposition 1 will affect the risk of contagion. We show that Bank 1's forced increase of its retail deposits due to the NSFR requirement will exert negative externalities on Bank 2. The risk of contagion increases monotonically with the amount of retail being increased. We further identify an informational channel of contagion as the asset buyers' inferencing about aggregate state s.

In our model, we define financial contagion as the spill-over effect of Bank 1's failure on Bank 2's probability of failure. A natural measure of exposure to contagion in the model is the length of interval $[\theta^*_{S,2}, \theta^*_{R,2}]$. Recall that we have pointed out in Proposition 4, this interval is non-empty. When its cash flow realizes in this interval, Bank 2 will fail if Bank 1 have failed in the first period, but it will survive when Bank 1 have survived. Consequently, the risk of contagion is defined as the ex-ante probability that Bank 2's cash flow being realized in the interval $[\theta^*_{S,2}, \theta^*_{R,2}]$.

$$CT = Pr\left\{\tilde{\theta}_2 \in [\theta^*_{S,2}, \theta^*_{R,2}]\right\} = \alpha\left(\frac{\theta^*_{R,2} - \theta^*_{S,2}}{\overline{\theta} - \underline{\theta}_G}\right) + (1-\alpha)\left(\frac{\theta^*_{R,2} - \theta^*_{S,2}}{\overline{\theta} - \underline{\theta}_B}\right) \tag{20}$$

We then analyze the effect of NSFR requirement on risk of contagion. Note that when the NSFR requirement as in Proposition 1 is imposed on the banking system, only Bank 1 has to increase its amount of retail deposits from $\underline{F}$ to F_γ, which decreases the critical cash flow from $\theta^*_1(\underline{F})$ to $\theta^*_1(F_\gamma) < \theta^*_1(\underline{F})$. Bank 2, however, need not adjust its balance sheet as a response to the regulation as $\text{NSFR}_2 > (F_\gamma/l)\% = \underline{\text{NSFR}}\%$, the minimum regulatory ratio. We assume it indeed does not make an adjustment. Consequently, the NSFR requirement will affect Bank 2 solely through informational spill-over. Note that as a robustness check, we also analyze the case where the NSFR requirement is also binding for Bank 2 in next section.

We can similarly define $CT(\underline{F})$ as the risk of contagion when the amount of Bank 1's retail deposits is $\underline{F}$. Note that $\underline{F}$ enters into CT through the inferencing channel, i.e., $\theta^*_{R,2}$ is a function of $\theta^*_1(\underline{F})$. Then the final value of the risk of financial contagion after Bank 1 increasing $\underline{F}$ to F_γ can be similarly calculated as

$$CT(F_q) = CT(\underline{F}) + \int_{\underline{F}}^{F_q} \frac{dCT(F)}{dF} dF \tag{21}$$

We first determine the sign of $dCT(F)/dF$. Take derivative on (20) with respect to F, we obtain

$$\frac{dCT}{dF} \propto \frac{d\left(\theta^*_{R,2} - \theta^*_{S,2}\right)}{dF} = \frac{d\theta^*_{R,2}}{dF} = \frac{d\theta^*_{R,2}}{d\theta^*_1}\frac{d\theta^*_1}{dF} > 0 \tag{22}$$

By Proposition 2 and 4, we have $\frac{d\theta^*_1}{dF} < 0$ and $\frac{d\theta^*_{R,2}}{d\theta^*_1} < 0$. So the risk of contagion increases unambiguously after the imposing of the NSFR requirement. Intuitively, Bank 2 does not need adjust its own stable funding. And the risk of contagion measures the increase in probability of failure of Bank 2 due to the failure of Bank 1, which is proportional to the distance between $\theta^*_{F,2}$ and $\theta^*_{S,2}$. And this distance is solely affected by the inferencing effect. We have the following Proposition.

Proposition 5. *Bank 1's increase $\underline{F}$ to F_γ due to the NSFR requirement leads to an unambiguously increase in the risk of contagion $CT(F_\gamma) > CT(\underline{F})$. The negative informational spill-over is due to players' inferencing about state s.*

5 Extensions

5.1 Adjustment in Asset Portfolio

In this section, we consider the possibility that banks increase NSFR by also adjusting their assets composition. We still maintain the assumption that $NSFR_1 < 100\% < NSFR_2$, so only Bank 1 needs to make adjustment of its balance sheet. A typical Bank 1's balance sheet at $t = 0$ is now as follows.

Assets	Liabilities
M	F
$1 - M$	$1 - F$

Now Bank 1's assets consist of M units of cash and $1 - M$ units of long-term risky portfolio, the $t = 1$ unit return of which is still $\tilde{\theta}_1$. To calculate Bank 1's NSFR, we assume that the RSF of cash is 0%. All other assumptions and notations remain as in our basic model. Now Bank 1's NSFR is calculate as follows.

$$NSFR_1 = \frac{100\% \cdot F + 100\% \cdot (1 - F)}{0\% \cdot M + l\% \cdot (1 - M)} = \frac{F}{1 - M}\%.$$

Besides increasing its retail funding, Bank 1 can also increase M, the amount of its liquid assets to attain a higher NSFR. We then consider the effect of increasing M on the bank risks defined in the main text.

For illustrative purpose, we derive the threshold equilibrium of the bank run game when creditors rationally expecting P_1^* to be the equilibrium asset price. In the complete information setup, it is easy to show that the lower and upper dominance regions for Bank 1's cash flow become $\left[\underline{\theta}_s, \frac{D_2-M}{1-M}\right)$ and $\left(\frac{F}{(1-M)-(D_1-M)/P_1^*}, \overline{\theta}\right]$, with $\theta^L = \frac{D_2-M}{1-M}$ and $\theta^U(P_1^*) = \frac{F}{(1-M)-(D_1-M)/P_1^*}$. We again focus on the intermediate region $[\theta^L, \theta^U(P_1^*)]$, and derive a creditor's payoff difference as a function of the total withdrawal L.

We assume that Bank 1 first uses its liquid assets M to meet the withdrawal L. It can be shown that for any realized cash flow $\theta_1 \in [\theta^L, \theta^U(P_1^*)]$, Bank 1 will never fail at $t = 1$ if it can meet its early withdrawal LD_1 solely with liquid assets M.[25] Consequently, we calculate the critical withdrawal proportion $L^c(\theta_1, P_1^*)$ such that as follows. Facing a withdrawal $L > M/D_1$, Bank 1 needs to liquidate $\lambda = (LD_1 - M)/[(1-M)P_1^*]$ unit of long-term assets at $t = 0.5$. It is on the verge of failure if the realized cash flow if

$$(1-\lambda)(1-M)\theta_1 = F + (1-L)(1-F)r_D = D_2 = \frac{LD_1}{q}.$$

Then the critical withdrawal is

$$L^c(\theta_1, P_1^*) = \frac{P[(1-M+M/P_1^*)\theta_1 - D_2]}{D_1(\theta - P_1^*/q)}$$

A representative creditor j's payoff in the complete information game is then D_1 if she always withdraws early, or $D_1/q > D_1$ if she waits and $L < L^c$ and 0 if she waits and $L \geq L^c$.

We then introduce noise $x_1^j = \theta_1 + \epsilon_1^j$ and study, again, the incomplete information game among creditors. The global-games refinement again predicts an equilibrium cash flow of Bank 1, under which the bank fails.

$$\theta_1^*(P_1^*) = \frac{D_2 - D_1}{(1 - qD_1/P_1^*) - M(P_1^* - 1)/P_1^*}$$

[25] Indeed, one can calculate, if $LD_1 \leq M$, Bank 1's assets return is $(M - LD_1) + (1-M)\theta_1$ larger than its remaining liabilities $F + (1-L)(1-F)r_D$.

To analyze the effect of an increase of M on θ_1^*, we first consider its first order effect on Bank 1.

$$\frac{\partial\theta_1^*(P_1^*)}{\partial M} = \frac{D_2 - D_1}{[(1 - qD_1/P_1^*) - M(P_1^* - 1)/P_1^*]^2} \cdot \frac{P_1^* - 1}{P_1^*}$$

That is, the sign of direct effect actually depends on the value of endogenous asset price P_1^*. If $P_1^* < 1$, $\partial\theta_1^*(P_1^*)/\partial M < 0$, the direct effect of an increase in liquid assets M is that Bank 1's risk of failure being reduced. Then as we have already discussed, its effect on the endogenous asset price is $dP_1^*/dF < 0$. Bank 1's risk of failure increases when F increases due to the feedback effect, however, this effect is again a second order effect. An increase of M is again reduces Bank 1's risk of failure when $P_1^* < 1$. A direct corollary is that increasing M increases the risk of contagion on Bank 2 otherwise as in our main text.

It is interesting to look at the case when the equilibrium asset price $P_1^* > 1$. When this occurs, all our previous results will be reversed. The direct effect and the overall effect of an increase in liquid assets M is that it makes Bank 1 more *prone to runs*, while it reduces Bank 2's risk of contagion. The intuition is simple and can be traced back to the Diamond and Dybvig (1983), where bank's long-term portfolio dominates cash when its interim resale price P_1^* is even higher than the price of cash 1. If this occurs, an increases in M actually makes Bank 1 more illiquid. Yet, cash M is still not completely dominated by long-term assets in our model from a contagion-reduction point of view. We summarize the above discussion in the following Proposition.

Proposition 6. *If $P_1^* < 1$, an increase in M to satisfy NSFR requirement has the same effect of an increase in F. If $P_1^* > 1$, long-term portfolio dominates cash for Bank 1, an increase in M increases Bank 1's risk of failure, however, reduces the risk of contagion of Bank 2.*

Koenig's paper has the similar result as ours.

5.2 Symmetric Banks

We then relax our assumption that only Bank 1 is constrained by NSFR requirement. Once both Bank 1 and 2 needs to increase NSFR, there is no loss of generality to assume homogenous banks. So we assume that $F = f$ and $NSFR_1 = NSFR_2 < 100\%$ in this section. For simplicity reason, we still assume away cash M, so that banks adjust their balance sheets through an increase in F to meet NSFR requirement.

Note that the analyses of Bank 1 remain the same. For Bank 2, we focus on the contingency when $O_1 = F$. We derive $\theta^*_{F,2}$ with respect to F to study the effects of an increase in F.

$$\frac{d\theta^*_{F,2}}{dF} = \underbrace{\underbrace{\frac{\partial\theta^*_{F,2}}{\partial F}}_{\text{stable effect}} + \underbrace{\frac{\partial\theta^*_{F,2}}{\partial P^*_{F,2}}\frac{dP^*_{F,2}}{dF}}_{\text{feedback effect}}}_{(-)} + \underbrace{\underbrace{\frac{d\theta^*_{F,2}}{d\theta^*_1}\frac{d\theta^*_1}{dF}}_{\text{inferencing effect}}}_{(+)}$$

Now an increase in F imposes both negative and positive effect on $\theta^*_{F,2}$. As discussed in the main text, an increase in F stabilizes a bank as its direct effect of reducing total liabilities surpass the indirect feedback effect channelled through endogenous asset price. However, the failure of a stable bank also exerts greater externality on Bank 2, which is the third term. Now, the overall effect on $\theta^*_{F,2}$ is also ambiguous as it is hard to compare the sensitivities of the three effects with respect to an in crease of F.

We can also compute the approximation of the risk of contagion as follows.

$$\frac{d(\theta^*_{F,2} - \theta^*_{F,2})}{dF} = \left(\frac{\partial\theta^*_{F,2}}{\partial F} - \frac{\partial\theta^*_{S,2}}{\partial F}\right) + \left(\frac{\partial\theta^*_{F,2}}{\partial P^*_{F,2}}\frac{dP^*_{F,2}}{dF} - \frac{\partial\theta^*_{S,2}}{\partial P^*_{S,2}}\frac{dP^*_{S,2}}{dF}\right) + \frac{d\theta^*_{F,2}}{d\theta^*_1}\frac{d\theta^*_1}{dF}$$

When the differences in stable effect and feedback effect between two contingencies $O_1 = F$ and S are small, the risk of contagion is still driven by the inferencing effect. An increase in F increases the risk of contagion. Otherwise, we again have the ambiguous result. To summarize, a change to symmetric banks does no alter our qualitative results, we still identify the three effects of interests. However, we may not have a clear result on the risk of contagion.

5.3 The Order of Bank Runs

Lastly, we briefly discuss the change in order of bank runs. Suppose we let run in Bank 2 happen first. It is easy to see now players in Bank 1 update their beliefs about s when observing Bank 2's outcome. However, an increase of F will not have any impact on risk of contagion simply because Bank 2 satisfies NSFR requirement and the equilibrium critical cash flow now is not affected by an increase of F. An increase of F will solely affect Bank 1 through the stable and feedback effect identified in the main text.

6 Conclusion

In this paper, we examine the latest Net Stable Funding Ratio requirement in Basel III Accord. In a general equilibrium framework where both bank's asset market and credit market are considered, we derive the equilibrium secondary market price of bank's assets and the equilibrium critical cash flow. Bank run and asset sale feeds back with each other in our model. Specifically, anticipating the liquidation price is at the equilibrium level, a run occurs if and only if a bank's realized cash flow is lower than the equilibrium critical value. And anticipating the quality of asset on sale upon a run, the secondary asset market clears at the equilibrium price. Then we show that an increase of bank's stable funding due to the NSFR requirement has unintended consequence on bank's risk of failure through the endogenous secondary market asset price. Indeed, increasing the stable funding has a positive first order effect on curbing the bank's risk of failure. Then rational asset buyers fully anticipate this effect, and bid less to purchase the asset because they think the fundamental of the bank must be unusually poor. However, the lower secondary market asset price will feed back to bank run as creditors' coordination problem precipitates in the first place. Fortunately, we show this negative feed back effect of increasing stable funding is second order to its first order stable effect. A NSFR requirement fails to consider the rational response of the market participants will lead to either bank's excessive risk taking or imposing unnecessary high funding costs on banks.

In a dynamic version of our model with two banks and an aggregate risk exposure, we examine the informational contagion among banks. We show that a bank failure imposes negative externalities on other banks. In particular, a bank is more prone to run when a run has already occurred to another bank. This is because the creditors and asset buyers of the second bank make pessimistic inference about the aggregate risk factor upon observing a run to the first bank. We show that an increase of of bank's stable funding due to the NSFR requirement unambiguously increases this risk of informational contagion. A run occurred to a bank with a more stable funding structure has negative spillover on the second bank. However, we show that more stable funding structure does not exert a positive spillover on the second bank. A NSFR requirement targeting at curbing the risk of contagion, however, impose a greater danger of failure to the second bank through this information contagion channel of inferencing about the aggregate risk.

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Appendix A Proofs

Appendix A.1 Proof of $P_1^* \in \left[\overline{P}, D_2\right)$

Proof. Since buyers' equilibrium bid cannot be negative, an equilibrium price P_1^*, if exits, must be in one of the three regions: $\left[0, \overline{P}\right)$, $\left[\overline{P}, qD_2\right)$, or $[qD_2, +\infty)$. We show that it cannot be greater than or equal to qD_2, nor can it be lower than $\underline{P}$.

Suppose $P_1^* \geq qD_2$ for any runs $M \in \{1, 2, ..., N\}$ observed, then it is not sequentially rational for the wholesale creditors to withdraw from a solvent bank, i.e., $\theta > D_2$. To see so, one can take the perspective of a representative creditor j. Even when all other creditors withdraw, the bank needs to liquidate no more than D_1/qD_2 fraction of its asset, for $P_1^* \geq qD_2$. While the bank's $t = 2$ liability drops to F, its residual cash flow is $\left(1 - \frac{D_1}{P_1^*}\right)\theta \geq \left(1 - \frac{D_1}{qD_2}\right)D_2 \geq F$ as $\theta \geq D_2$. As a result, by running on the bank, creditor j will only incur a penalty for early withdrawal. This implies that whenever a run happens when $P_1^* \geq qD_2$, the bank must be fundamentally insolvent with $\theta < D_2$. Therefore, buyers must expect asset quality to be lower than $(D_2 + \underline{\theta}_G)/2$, which is in turn lower than qD_2 given our parametric assumption (3). Buyers would make a loss by offering $P_1^* \geq qD_2$, a contradiction.

An equilibrium price P_1^* cannot be smaller than $\overline{P}$ either. Note that when a bank is fundamentally insolvent with cash flow $\theta < D_2$, it is a dominant strategy for its wholesale creditors to run, independently of the asset price. To see so, notice that if $P_1^* \geq D_1$ and the bank does not fail at $t = 1$, a creditor is better off to run and receive qr_D than to wait and receive 0.[26] On the other hand, if $P_1^* < D_1$, a creditor will receive a zero payoff for his claim whether he runs or not, but can still obtain an arbitrarily small reputational benefit by running on a bank that is doomed to fail. This implies that runs must happen to those banks with $\theta < D_2$, and the expected quality of assets on sale is at least $\left(\underline{\theta}_B + D_2\right)/2 = \overline{P}$. As asset buyers break even with their competitive bidding, the price they offer must be greater than or equal to $\overline{P}$. □

Appendix A.2 Proof of Lemma 1

Proof. The proof unfolds in four steps. Steps 1 to 3 use global games technic to solve a unique (partial) equilibrium of the bank run game when creditors rationally anticipating the secondary market asset price to be P_1^*. Eventually we obtain the equilibrium condition (6). Step 4 adds

[26] Note that asset sale will never increase an insolvent bank's solvency as we just proved that $P_1^* \geq qD_2$ could never happen.

the asset market equilibrium condition (5), and proves this system of equations has a unique solution (θ_1^*, P_1^*) such that $\theta_1^* \in [\theta^L, \theta^U(P_1^*)]$ and $P_1^* \in [\overline{P}, qD_2)$.

We start to solve a unique Bayesian Nash Equilibrium of the bank run game assuming there exists an equilibrium asset price $P_1^* \in [\overline{P}, qD_2)$. To do so, we follow the standard procedures of global games by deriving a representative creditor j's best response function to the other creditors' strategy.

Step 1: Lower and upper dominance regions

In this step, we characterize two regions of bank's fundamental where creditor j has respective dominate strategy.

Suppose we were in a complete information setup where the Bank 1's realized fundamental θ_1 is directly observed. When L fraction of creditors withdraw early, the bank will face a liquidity demand of LD_1, $L \in [0, 1]$ and need to liquidate a λ fraction of its assets at the price P_1^*.

$$\lambda = \frac{LD_1}{P_1^*} \in [0, 1)$$

Note that λ is between 0 and 1, since we have established that any equilibrium price P_1^* must be higher than $\underline{P}$, which is higher than D_1. After liquidating a fraction λ of its assets, the bank will fail at $t = 2$ if and only if the value of its remaining assets is lower than its remaining liabilities.

$$(1 - \lambda) \cdot \theta_1 < F + (1 - L)(1 - F)r_D \tag{A.23}$$

In other words, a bank will fail at $t = 2$ if and only if the fraction of creditors' withdrawal exceeds a critical value L^c.

$$L > \frac{P_1^* \cdot [\theta_1 - F - (1 - F)r_D]}{\left[q\theta_1 - P_1^*\right](1 - F)r_D} = \frac{P_1^* \cdot (\theta_1 - D_2)}{D_1 \cdot \left[\theta_1 - P_1^*/q\right]} \equiv L^c(\theta_1) \tag{A.24}$$

Creditor j's payoff, therefore, depends on realized fundamental as well as the actions of other creditors, in particular, the fraction of runs L. With (A.23) and (A.24) defined, we then move to our original incomplete information assumption and characterize two regions where respective unique dominate strategies exist.

First, it is easy to check that $dL^c(\theta_1)/d\theta_1 > 0$ when $P_1^* < qD_2$. This means it requires higher proportion of early withdrawals to trigger a bank failure when the bank's fundamental gets stronger.

We then denote a lower dominance region by $\left[\underline{\theta}, \theta^L\right)$ where creditor j has a unique dominate strategy to withdraw when Bank 1's cash flow realizes in this region. It is easy to see in (A.23) $L^c(D_2) = 0$, so $L^c(\theta_1) \leq 0$ for $\theta_1 \leq D_2$. Consequently, we choose $\theta^L = D_2$. Then by definition, a bank failure is triggered even if all other creditors wait, i.e., $L^c \leq 0$, creditor j would be strictly better off to withdraw. In other words, the bank is fundamentally insolvent and will fail at $t = 2$ even if no premature liquidation takes place at $t = 1$. In this case, a creditor, if chooses to wait, will receive a zero payoff because of the bank failure, but will receive D_1 if he withdraws early. Although we are in the incomplete information setup, creditor j is still sure that $\theta_1 < D_2$ because we assume limiting case of noise. Thus, his best action is to withdraw, independent of his belief about other creditors' actions.

On the other hand, we denote $\theta^U(P_1^*) \equiv F/\left(1 - D_1/P_1^*\right)$, we show that for an expected asset price P_1^*, the upper dominance region is $\left(\theta^U(P_1^*), \overline{\theta}\right]$. Similarly, it is easy to see in (A.23) $L^c\left(F/\left(1 - D_1/P_1^*\right)\right) = 1$, so $L^c(\theta) \geq 0$ for $\theta \geq F/\left(1 - D_1/P_1^*\right)$. Consequently, we choose $\theta^U(P_1^*) = F/\left(1 - D_1/P_1^*\right)$. Then by definition, a bank failure will never be triggered even if all other creditors withdraw, i.e., $L^c \geq 1$, creditor j would be strictly better off to wait. In other words, the bank is fundamentally super-solvent and will always survive at $t = 2$ even if nearly all creditors withdraw at $t = 1$. In this case, a creditor, if chooses to wait, will receive D_1/q, but will only receive D_1 if he join the run at $t = 1$.

Step 2: Creditors' beliefs outside the dominance regions When Bank 1's realized cash flow θ_1 is in the intermediate region $\left[\theta^L, \theta^U(P_1^*)\right]$, creditor j's optimal action depends on his beliefs about other creditors' actions. In this subsection, we characterize such beliefs.

Note first that the fraction of creditors who withdraw early is a function of the bank's fundamental θ_1 and the threshold signal x_1^*. We denote this fraction by $L = L(\theta_1, x_1^*)$ and determine the functional form of $L(\theta, x_1^*)$. For a realized θ_1, we have three cases. (1) When $\theta_1 + \epsilon < x_1^*$, even the highest possible signal is below the threshold x^*. By the definition of the threshold strategy, all creditors will withdraw and $L(\theta_1, x_1^*) = 1$. (2) When $\theta_1 - \epsilon > x_1^*$, even the lowest possible signal exceeds the threshold x^*. All creditors will wait and $L(\theta_1, x_1^*) = 0$. (3) When θ_1 falls into the intermediate range $\left[x_1^* - \epsilon, x_1^* + \epsilon\right]$, the fraction of creditors who withdraw at $t = 1$ is as follows, where x_k denotes the private signal of an arbitrary creditor k other than creditor j.

$$L(\theta_1, x_1^*) = Prob\left(x_1^k < x_1^* \middle| \theta_1\right) = Prob\left(\epsilon_1^k < x_1^* - \theta_1 \middle| \theta_1\right) = \frac{x_1^* - \theta_1 - (-\epsilon)}{2\epsilon} = \frac{x_1^* - \theta_1 + \epsilon}{2\epsilon} \tag{A.25}$$

$L(\theta_1, x_1^*) \in (0,1)$ would look uncertain from the perspective of creditor j, as the creditor only receives a noisy signal x_1^j and perceives θ with uncertainty. In particular, creditor j has a posterior belief $\theta_1 \sim U(x_1^j - \epsilon, x_1^j + \epsilon)$ conditional his private signal x_1^j. Depends on the value of x_1^j, we have five cases.

Case (1) $x_1^j > x_1^* + 2\epsilon$: In this case, creditor j is certain that all other creditors must have received signals higher than x_1^*. As all other creditors choose to wait, creditor j has a posterior belief $Prob\left(L(\theta_1, x_1^*) = 0 \middle| x_1^j\right) = 1$ when observing $x_1^j > x_1^* + 2\epsilon$.

Case (2) $x_1^* < x_1^j \leq x_1^* + 2\epsilon$: Recall that $\theta \sim U(x_1^j - \epsilon, x_1^j + \epsilon)$ conditional x_j. As $x_1^j > x_1^*$, it follows $x_1^j + \epsilon > x_1^* + \epsilon$, so that we can divide the support of θ into two intervals: $\left[x_1^j - \epsilon, x_1^* + \epsilon\right]$ and $\left(x_1^* + \epsilon, x_1^j + \epsilon\right]$. When θ_1 lies in the second interval, all other creditors receive signals higher than x_1^* and chosen to wait, so that $L = 0$. Creditor j, therefore, assigns the event $L(\theta_1, x_1^*) = 0$ with the following posterior probability:

$$Prob\left(L(\theta_1, x_1^*) = 0 \middle| x_1^j\right) = Prob\left(x_1^* + \epsilon < \theta < x_1^j + \epsilon \middle| x_j\right) = \frac{x_1^j + \epsilon - (x_1^* + \epsilon)}{(x_1^j + \epsilon) - (x_1^j - \epsilon)} = \frac{x_1^j - x_1^*}{2\epsilon} \in (0,1]$$

On the other hand, the first interval $\left[x_1^j - \epsilon, x_1^* + \epsilon\right]$ is a subset of $[x_1^* - \epsilon, x_1^* + \epsilon]$ so that $L(\theta_1, x_1^*)$ is given by expression (A.25). Creditor j's posterior belief of $L(\theta_1, x_1^*)$ can be calculated as

$$Prob\left(L(\theta_1, x_1^*) \leq \hat{L} \middle| x_1^j\right) = Prob\left(\frac{x_1^* - \theta_1 + \epsilon}{2\epsilon} \leq \hat{L} \middle| x_1^j\right) = Prob\left(\theta \geq x_1^* + \epsilon - 2\epsilon\hat{L} \middle| x_1^j\right),$$

where $\hat{L} \in (0,1)$. Since $\theta \sim U(x_1^j - \epsilon, x_1^j + \epsilon)$ conditional on x_1^j, we know that θ_1 is still uniformly distributed on $[x_1^j - \epsilon, x_1^* + \epsilon]$. And the probability above can be written explicitly as

$$Prob\left(L(\theta_1, x_1^*) \leq \hat{L} \middle| x_1^j\right) = \frac{(x_1^* + \epsilon) - (x_1^* + \epsilon - 2\epsilon\hat{L})}{(x_1^* + \epsilon) - (x_1^j - \epsilon)} = \frac{2\epsilon\hat{L}}{2\epsilon - (x_1^j - x_1^*)} = \frac{\hat{L}}{1 - (x_1^j - x_1^*)/2\epsilon}.$$

Therefore, for a signal $x_1^j \in \left(x_1^*, x_1^* + 2\epsilon\right]$, creditor j perceives $L(\theta_1, x_1^*)$ having a mixed distribution, with a positive probability mass $(x_1^j - x_1^*)/2\epsilon$ at $L = 0$ and being uniformly distributed on $\left(0, 1 - \left(x_1^j - x_1^*\right)/2\epsilon\right]$ with density 1.

Case (3) $x_1^j = x_1^*$: Creditor j still perceive $L(\theta_1, x_1^*)$ as given by expression (A.25) as $\theta_1 \sim U\left(x_1^* - \epsilon, x_1^* + \epsilon\right)$ conditional on $x_1^j = x^*$. Creditor j calculates the posterior distribution of

$L(\theta_1, x_1^*)$ as follows.

$$Prob\left(L(\theta_1, x_1^*) \le \hat{L} \middle| x_1^j = x_1^*\right) = Prob\left(\frac{x_1^* - \theta + \epsilon}{2\epsilon} \le \hat{L} \middle| x_1^j = x_1^*\right)$$
$$= Prob\left(\theta \ge x_1^* + \epsilon - 2\epsilon\hat{L} \middle| x_1^j = x_1^*\right) = \frac{(x_1^* + \epsilon) - (x_1^* + \epsilon - 2\epsilon\hat{L})}{(x_1^* + \epsilon) - (x_1^* - \epsilon)} = \hat{L}.$$

Therefore, creditor j holds a posterior belief that $L(\theta_1, x_1^*) \sim U(0, 1)$, when observing $x_1^j = x_1^*$.

Case (4) $x_1^* - 2\epsilon \le x_1^j < x_1^*$: This case can be analyzed in the same way as Case (2). When observing a signal $x_1^j \in \left[x_1^* - 2\epsilon, x_1^*\right)$, creditor j perceives $L(\theta_1, x_1^*)$ having a mixed distribution, with a positive probability mass $(x_1^* - x_1^j)/2\epsilon$ at $L = 1$ and being uniformly distributed on $\left[(x_1^* - x_1^j)/2\epsilon, 1\right)$ with density 1.

Case (5) $x_1^j < x_1^* - 2\epsilon$: Similar to Case (1), when observing a signal $x_1^j < x_1^* - 2\epsilon$, creditor j is certain that all other creditors must have received signals lower than x_1^*, and therefore, has a posterior belief $Prob\left(L(\theta_1, x_1^*) = 1 \middle| x_1^j\right) = 1$.

It worth noticing that creditor j becomes more pessimistic about the proportion of early withdrawals when observing a lower signal. That is, the distribution of $L(\theta_1, x_1^*)$ associated with a lower x_1^j first-order-stochastic dominates one associated with a higher x_1^j.

Step 3: Threshold equilibrium of the bank run game

Notice now we can tabulate creditor j's payoffs of playing 'withdraw' or 'wait' as follows

	$L \in [0, L^c]$	$L \in (L^c, 1]$
withdraw	D_1	D_1
wait	D_1/q	0

This is because $L^c \in [0, 1]$. If the creditor withdraws, his payoff will always be $W_{run}(L) = D_1$. Instead, if he waits, his payoff depends on the action of other creditors.

$$W_{wait}(L) = \begin{cases} D_1/q & L \in [0, L^c] \\ 0 & L \in (L^c, 1] \end{cases}$$

Defining the difference between the creditor's payoffs of "wait" and "withdraw" as $DW(L) \equiv W_{wait}(L) - W_{run}(L)$, we have the following expression.

$$DW(L) = \begin{cases} (1-q)D_1/q & L \in [0, L^c] \\ -D_1 & L \in (L^c, 1] \end{cases}$$

Creditor j's expected payoff difference between action 'wait' and 'withdraw' conditional on his signal x_1^j dictates his action. We denote this expected payoff difference by $E[DW(L)|x_1^j]$ and calculate it explicitly using the posterior distribution of $L(\theta_1, x_1^*)$.

By the definition of θ_1^*, the following equality must hold.

$$\left(1 - \frac{LD_1}{P_1^*}\right)\theta_1^* = F + (1-L)(1-F)r_D \tag{A.26}$$

That is, a bank is on the verge of failure if its fundamental equals θ^*. The critical fundamental θ^* then implies a critical run proportion $L^c(\theta^*)$.

$$L^c(\theta_1^*) = \frac{P_1^*(\theta_1^* - D_2)}{D_1\left[\theta_1^* - P_1^*/q\right]} \tag{A.27}$$

As we know $P_1^* \in [\underline{P}, qD_2)$ and focus on $\theta_1 \in \left[\theta^L, \theta^U\right]$, it holds that $L^c(\theta_1^*) \in (0,1)$.

In Case (1) $x_1^j > x_1^* + 2\epsilon$: Creditor j perceives $L(\theta_1, x_1^*) = 0$ with probability 1. As $DW(L) = (1-q)D_1/q$ when $L = 0$, we have

$$E[DW(L)|x_1^j] = (1-q)\frac{D_1}{q}, \text{ for } x_1^j > x_1^* + 2\epsilon.$$

In Case (2) $x_1^* < x_1^j \leq x_1^* + 2\epsilon$: Creditor j believes L has a mixed distribution. Notice that $1-(x_1^j - x_1^*)/2\epsilon$ decreases in x_1^j, with $1-(x_1^j - x_1^*)/2\epsilon = 1$ when $x_1^j = x_1^*$ and $1-(x_1^j - x^*)/2\epsilon = 0$ when $x_1^j = x_1^* + 2\epsilon$. So there exists a $\overline{x} \in (x_1^*, x_1^* + 2\epsilon]$, such that $L^c(\theta_1^*) = 1-(x_1^j - x_1^*)/2\epsilon$. When $x_1^j \in \left(\overline{x}, x_1^* + 2\epsilon\right]$, we have $1-(x_1^j - x_1^*)/2\epsilon < L^c(\theta_1^*)$ and the expected payoff difference

$$E[DW(L)|x_1^j] = \frac{x_1^j - x_1^*}{2\epsilon}(1-q)\frac{D_1}{q} + \int_0^{1-\frac{x_1^j - x_1^*}{2\epsilon}}(1-q)\frac{D_1}{q}dL = (1-q)\frac{D_1}{q}.$$

When $x_1^j \in \left(x_1^*, \overline{x}\right]$, we have $1-(x_1^j - x_1^*)/2\epsilon > L^c(\theta_1^*)$ and the expected payoff difference

$$\begin{aligned} E[DW(L)|x_1^j] &= \frac{x_1^j - x_1^*}{2\epsilon}(1-q)\frac{D_1}{q} + \int_0^{L^c(\theta_1^*)}(1-q)\frac{D_1}{q}dL + \int_{L^c(\theta_1^*)}^{1-\frac{x_1^j - x_1^*}{2\epsilon}}(-D_1)dL \\ &= \frac{D_1}{q}\cdot\left[L^c(\theta_1^*) - q\right] + \frac{D_1}{q}\frac{x_1^j - x_1^*}{2\epsilon}. \end{aligned}$$

In Case (3) $x_1^j = x_1^*$: Creditor j believes $L \sim U(0,1)$. We have

$$E[DW(L)|x_1^j] = \int_0^{L^c(\theta_1^*)} (1-q)\frac{D_1}{q} dL + \int_{L^c(\theta_1^*)}^1 (-D_1) dL = \frac{D_1}{q} \cdot \left[L^c(\theta_1^*) - q\right]$$

In Case (4) $x_1^* - 2\epsilon \leq x_1^j < x_1^*$: The analysis mirrors that of Case (2). There exists a $\underline{x} \in [x_1^* - 2\epsilon, x_1^*)$, such that $\left(x_1^* - \underline{x}\right)/2\epsilon = L^c(\theta_1^*)$. When $x_1^j \in \left(\underline{x}, x_1^*\right)$, we have $\left(x_1^* - x_1^j\right)/2\epsilon < L^c(\theta_1^*)$. The expected payoff difference can be written as

$$\begin{aligned} E[DW(L)|x_1^j] &= \int_{\frac{x_1^* - x_1^j}{2\epsilon}}^{L^c(\theta_1^*)} (1-q)\frac{D_1}{q} dL + \int_{L^c(\theta_1^*)}^1 (-D_1) dL + \frac{x_1^* - x_1^j}{2\epsilon}(-D_1) \\ &= \frac{D_1}{q} \cdot \left[L^c(\theta_1^*) - q\right] - \frac{D_1}{q}\frac{x_1^* - x_1^j}{2\epsilon}. \end{aligned}$$

When $x_1^j \in \left[\theta_1^* - 2\epsilon, \underline{x}\right]$, we have $\left(x_1^* - x_1^j\right)/2\epsilon > L^c(\theta_1^*)$ and

$$E[DW(L)|x_1^j] = \int_{\frac{x_1^* - x_1^j}{2\epsilon}}^1 (-D_1) dL + \frac{x_1^* - x_1^j}{2\epsilon}(-D_1) = -D_1.$$

Lastly, in Case (5) $x_1^j < x_1^* - 2\epsilon$: Creditor j perceives $L(\theta_1, x_1^*) = 1$ with probability 1. As $DW(L) = -D_1$ when $L = 1$, we have

$$E[DW(L)|x_1^j] = -D_1, \text{ for } x_1^j < x_1^* - 2\epsilon.$$

It is straightforward to see that when $x_1^j \in \left(\underline{x}, \overline{x}\right)$, $E[DW(L)|x_1^j]$ strictly increases in x_1^j with a slope $r_D/2\epsilon$. As a result, there must exist a unique $\hat{x}$ such that $E[DW(L)|x_1^j = \hat{x}] = 0$. Therefore, creditor j's best response to the other creditors' threshold strategy is a threshold strategy: to withdraw if $x_1^j < \hat{x}$ and to wait if $x_1^j > \hat{x}$. One can derive explicitly the best response of $\hat{x}$ to x_1^*.

$$\hat{x} = x_1^* - 2\epsilon \left[L^c\left(\theta_1^*\right) - q\right] \tag{A.28}$$

In a symmetric equilibrium, it must hold that $\hat{x} = x_1^*$. Therefore, a condition for a symmetric equilibrium of the bank run game to exist is $L^c(\theta_1^*) = q$. Using (A.27), we can derive the condition explicitly as

$$\theta_1^* = \frac{D_2 - D_1}{1 - qD_1/P_1^*}. \tag{A.29}$$

Provided that θ_1^* exists, the fraction of early withdrawal when the bank's fundamental θ_1 happens to be θ_1^* is given by

$$L(x_1^*, \theta_1^*) = \frac{x_1^* - \theta_1^* + \epsilon}{2\epsilon}.$$

By the definition of θ_1^*, the bank will be exactly on the verge of failure when $L(x_1^*, \theta_1^*)$ fraction of creditors withdraw. This in turn suggests $L(x_1^*, \theta_1^*)$ must exactly equal to $L^c(\theta_1^*)$, the critical run proportion derived in (A.27). Thus, the equilibrium threshold signal x_1^*, if exist, must satisfy

$$\frac{x_1^* - \theta_1^* + \epsilon}{2\epsilon} = \frac{P_1^*(\theta_1^* - D_2)}{D_1 \left[\theta_1^* - P_1^*/q\right]} \tag{A.30}$$

In sum, when creditors expecting the asset price to be P_1^*, the equilibrium threshold signal x_1^* and the critical cash flow θ_1^* jointly satisfy equation (**??**) and (A.30).

It should be noted that θ_1^*, if exists, must belong to the interval $[x_1^* - 2\epsilon, x_1^* + 2\epsilon]$. Otherwise, we would have contradictions. For example, suppose $\theta_1^* > x_1^* + 2\epsilon$. When a bank's fundamental happens to be $\theta_1^* - \epsilon/2$, the bank should fail by the definition of θ_1^*. Yet, all creditors will receive signals greater than $x_1^* + \epsilon/2$ and should not withdraw from the bank according to their equilibrium strategy. Therefore θ_1^* must be no higher than $x_1^* + 2\epsilon$. Similarly, one can argue that θ_1^* must be no lower than $x_1^* - 2\epsilon$. In the limiting case where noise ϵ approaches to zero, θ_1^* and x_1^* converge.

Finally, we show that the symmetric equilibrium, if exists in the interval $[\theta^L, \theta^U\left(P_1^*\right)]$, is the only one that survives iterated elimination of strictly dominated strategies.

We construct a sequence $\{\underline{x}_j\}_{j=0}^{\infty}$ starting by $\underline{x}_0 = \underline{\theta}_s$, $s = G$ or B. We let

$$\underline{x}_{j+1} = \underline{x}_j - 2\epsilon\left[L^c\left(\underline{\theta}_j\right) - q\right] \tag{A.31}$$

where $L^c(\underline{\theta}_j) = \frac{P_1^*(\underline{\theta}_j - D_2)}{D_1\left[\underline{\theta}_j - P_1^*/q\right]}$. Note that θ^* enters into the expression of (A.31) because creditors have to form belief that asset price to be P_1^* before they play the global games. And it can be checked that $\frac{\partial L^c(\underline{\theta}_j)}{\partial \underline{\theta}_j} > 0$ when $P_1^* \in [\underline{P}, qD_2)$. We further let $\underline{\theta}_j$, $j \geq 0$ satisfies the equation $\frac{\underline{x}_j - \underline{\theta}_j + \epsilon}{2\epsilon} = L^c(\underline{\theta}_j)$. Or equivalently,

$$\underline{\theta}_j + 2\epsilon L^c(\underline{\theta}_j) = \underline{x}_j + \epsilon \tag{A.32}$$

It is easily seen that $\underline{\theta}_j$ increases as $\underline{x}_j$ increases. Given $\underline{x}_0 = \underline{\theta}_s$, $\underline{\theta}_0$ solves the equation $\underline{\theta}_0 + 2\epsilon L^c(\underline{\theta}_0) = \underline{x}_0 + \epsilon$. By the monotonicity of L^c with respect to $\underline{\theta}_j$, there exists a unique solution of $\underline{\theta}_0$. The solution is sufficiently close to $\underline{x}_0 = \underline{\theta}_s$ because ϵ is sufficiently small. Recall that we claimed $\theta_1^* \geq D_2$, if exists, solves $L^c(\theta_1^*) = q$. By $\underline{\theta}_0 < D_2$ and $\frac{\partial L^c(\underline{\theta}_j)}{\partial \underline{\theta}_j} > 0$, we have $L^c(\underline{\theta}_0) < q$. Consequently, we obtain

$$\underline{x}_1 = \underline{x}_0 - 2\epsilon \left[L^c \left(\underline{\theta}_0 \right) - q \right] > \underline{x}_0$$

This in turn means $\underline{\theta}_1 > \underline{\theta}_0$. We can iterate this process and claim both sequences $\{\underline{x}_j\}_{j=0}^N$ and $\{\underline{\theta}_j\}_{j=0}^N$ are increasing for finite number N.

On the other hand, it is easily seen that $\{\underline{\theta}_j\}_{j=0}^\infty$ has a upper limit of θ_1^*. To see why, because each incremental value from $\underline{\theta}_j$ to $\underline{\theta}_{j+1}$ is small enough, there exists a value of $\underline{\theta}_k$ such that $\underline{\theta}_k = \theta_1^*$. Then by (A.31), we have $\underline{x}_{k+1} = \underline{x}_k$ as θ_1^* makes $L^c(\theta^*) = q$. Consequently, we have $\underline{\theta}_{k+1} = \underline{\theta}_k = \theta_1^*$ by (A.32). By iteration, we have $\underline{\theta}_k = \underline{\theta}_{k+1} = \underline{\theta}_{k+2} = ... = \theta_1^*$, θ_1^* is the upper bound of the sequence. We then know that both $\{\underline{x}_j\}_{j=0}^\infty$ and $\{\underline{\theta}_j\}_{j=0}^\infty$ are increasing. By (A.32), $\{\underline{x}_j\}_{j=0}^\infty$ has a upper limit of $\theta_1^* - \epsilon$.

Similarly, we can construct sequences $\{\overline{x}_j\}_{j=0}^\infty$ and $\{\overline{\theta}_j\}_{j=0}^\infty$ starting by $\overline{x}_0 = \overline{\theta}$. It takes the same procedure to show both sequences are decreasing, and with lower bounds $\theta_1^* - \epsilon$ for the former and θ_1^* for the latter.

To summarize, $x_1^* = \theta_1^*$ when ϵ approaches to zero is the only strategy that survives iterated elimination of strictly dominated strategies.

Step 4: General equilibrium with asset market

Once run occurs in Bank 1, i.e., $O_1 = R$, asset buyers are called upon to move. In particular, they compete in price to purchase Bank 1's asset on sale. The buyers understand the creditors' game, thus they know that, in equilibrium, the return of the asset on sale will be bounded above by θ_1^*.[27] In addition, they rationally update their beliefs about s as

$$\begin{aligned}
\omega_R^G(\theta_1^*) = Pr(s = G|O_1 = R) &= \frac{Pr(O_1 = R|s = G)Pr(s = G)}{Pr(O_1 = R|s = G)Pr(s = G) + Pr(O_1 = R|s = B)Pr(s = B)} \\
&= \frac{\alpha \left(\frac{\theta_1^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} \right)}{\alpha \left(\frac{\theta_1^* - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_G} \right) + (1-\alpha) \left(\frac{\theta_1^* - \underline{\theta}_B}{\overline{\theta} - \underline{\theta}_B} \right)} = \frac{1}{1 + \left(\frac{\theta_1^* - \underline{\theta}_B}{\theta_1^* - \underline{\theta}_G} \right) \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{\overline{\theta} - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_B} \right)} \\
&= \frac{1}{1 + \left(\frac{\theta_1^* - \underline{\theta}_B}{\theta_1^* - \underline{\theta}_G} \right) \kappa_0} = 1 - \omega_R^B(\theta_1^*).
\end{aligned}$$

[27]They also know that the creditors take a symmetric switching strategy x_1^*, and $x_1^* \to \theta_1^*$ as $\epsilon \to 0$ in our model.

So the expected quality of asset is

$$\omega_R^G(\theta_1^*) \cdot E_G\left[\tilde{\theta}_1 \middle| \tilde{\theta}_1 < \theta_1^*\right] + \omega_R^B(\theta_1^*) \cdot E_B\left[\tilde{\theta}_1 \middle| \tilde{\theta}_1 < \theta_1^*\right].$$

The asset buyers then bid in price P against each other to make a profit. It can be checked that

$$\frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*} = \frac{\kappa_0(\underline{\theta}_G - \underline{\theta}_G)}{[(\theta_1^* - \underline{\theta}_G) + \kappa_0(\theta_1^* - \underline{\theta}_B)]^2} > 0.$$

Consequently, suppose there exists a unique $\theta_1^* \in [\theta^L, \overline{\theta}]$, this bidding game features a unique symmetric Nash Equilibrium P_1^*. It must satisfy the following condition.

$$P_1^* = \omega_R^G(\theta_1^*) \cdot E_G\left[\tilde{\theta}_1 \middle| \tilde{\theta}_1 < \theta_1^*\right] + \omega_R^B(\theta_1^*) \cdot E_B\left[\tilde{\theta}_1 \middle| \tilde{\theta}_1 < \theta_1^*\right] \tag{A.33}$$

It is easy to check that there will be no unilateral profitable deviation given that all other buyers bid this price.

We have the general equilibrium of our model incorporating both bank's credit market and asset market is then characterized as follows.

$$\left(\theta_1^*, P_1^*\right) \quad \Bigg| \quad \begin{cases} \theta^* = (D_2 - D_1) \,/\, \left(1 - qD_1/P_1^*\right) \\ P_1^* = \omega_R^G(\theta_1^*) \cdot E_G\left[\tilde{\theta}_1 \middle| \tilde{\theta}_1 < \theta_1^*\right] + \omega_R^B(\theta_1^*) \cdot E_B\left[\tilde{\theta}_1 \middle| \tilde{\theta}_1 < \theta_1^*\right] \\ st. \quad \theta_1^* \in [\theta^L, \theta^U(P_1^*)], \quad P_1^* \in [\overline{P}, qD_2) \end{cases}$$

We then show the existence of uniqueness of the equilibrium. Note that (A.33) can be further rewritten as

$$P_1^* - \frac{\theta_1^* + \underline{\theta}_B + \omega_R^G(\theta_1^*)(\underline{\theta}_G - \underline{\theta}_B)}{2} = 0.$$

Thus, we insert (A.29) into the above equation and reformulate it as

$$2P_1^* - \frac{D_2 - D_1}{1 - qD_1/P_1^*} - \underline{\theta}_B - \omega_R^G\left(\frac{D_2 - D_1}{1 - qD_1/P_1^*}\right)(\underline{\theta}_G - \underline{\theta}_B) = 0,$$

a function solely depends on P_1^*. We thus define an auxiliary function $\eta(P)$ as the follows.

$$\eta(P) = 2P - \frac{D_2 - D_1}{1 - qD_1/P} - \underline{\theta}_B - \omega_R^G\left(\frac{D_2 - D_1}{1 - qD_1/P}\right)(\underline{\theta}_G - \underline{\theta}_B) \tag{A.34}$$

For the existence part, we need to show that $\eta(\overline{P}) < 0$ and $\eta(qD_2) > 0$. This establish at least a $P_1^* \in [\overline{P}, qD_2)$ such that $\eta(P_1^*) = 0$. To show $\eta(\overline{P}) < 0$, we have

$$2\overline{P} = \underline{\theta}_B + D_2 < \underline{\theta}_B + \frac{D_2 - D_1}{1 - qD_1/\overline{P}} < \underline{\theta}_B + \frac{D_2 - D_1}{1 - qD_1/\overline{P}} + \omega_R^G \left(\frac{D_2 - D_1}{1 - qD_1/\overline{P}} \right) (\underline{\theta}_G - \underline{\theta}_B).$$

Note that the first inequality is true because $\overline{P} < qD_2$, and the second one holds since $\overline{P} > D_1$ making $\omega_R^G > 0$ well defined. On the other hand, we have

$$2qD_2 > D_2 + \underline{\theta}_G > \frac{D_2 - D_1}{1 - qD_1/(qD_2)} + \underline{\theta}_B + \omega_R^G \left(\frac{D_2 - D_1}{1 - qD_1/(qD_2)} \right) (\underline{\theta}_G - \underline{\theta}_B).$$

The first inequality is true because our parametric assumption (3), while the second one holds because $\omega_R^G(D_2) < 1$. So, we prove $\eta(qD_2) > 0$. The uniqueness is guaranteed.

We then turn to the uniqueness part. Note that it can be calculated that

$$\frac{d\eta(P)}{dP} = 2 - \frac{\partial}{\partial P} \left(\frac{D_2 - D_1}{1 - qD_1/P} \right) - \frac{d\omega_R^G(\hat{\theta})}{d\hat{\theta}} \frac{\partial}{\partial P} \left(\frac{D_2 - D_1}{1 - qD_1/P} \right) (\underline{\theta}_G - \underline{\theta}_B)$$

where $\hat{\theta} = (D_2 - D_1)/(1 - qD_1/P)$. Then we can calculate that

$$\frac{\partial \hat{\theta}}{\partial P} = \frac{\partial}{\partial P} \left(\frac{D_2 - D_1}{1 - qD_1/P} \right) = -\frac{qD_1(D_2 - D_1)}{(P - qD_1)^2} < 0$$

That is, in a partial bank run equilibrium, an increase in the asset liquidation price (value) P will reduce the critical cash flow $\hat{\theta}$. And we have already known that $\frac{d\omega_R^G(\hat{\theta})}{d\hat{\theta}} > 0$. So we have $\frac{d\eta(P)}{dP} > 0$, the $P_1^* \in [\overline{P}, qD_2)$ is also unique.

With the unique $P_1^* \in [\overline{P}, qD_2)$ established, the unique $\theta_1^* = (D_2 - D_1)/(1 - qD_1/P_1^*) \in [D_2, F/(1 - D_1/P_1^*)]$ is obvious.

□

Appendix A.3 Proof of Lemma 2

Proof. In the general equilibrium where both bank's credit market and asset market are considered, an increase in F brings in a feedback effect on θ_1^* through the endogenous asset price, besides the partial equilibrium stable effect. To see this, we take the first order derivative of θ_1^* with respect to F

$$\frac{d\theta_1^*}{dF} = \frac{\partial \theta_1^*}{\partial F} + \frac{\partial \theta_1^*}{\partial P_1^*} \frac{dP_1^*}{dF}.$$

The direct stable effect due to the reduction in the face value of the liabilities is represented as

$$\frac{\partial\theta_1^*}{\partial F} = \frac{(1-r_D)(1-\frac{qD_1}{P_1^*}) + qr_D(1-\frac{qD_2}{P_1^*})}{(1-\frac{qD_1}{P_1^*})^2} < 0$$

Recall that $P_1^* \in [\overline{P}, qD_2)$, so we have $P_1^* > (\underline{\theta}_B + D_2)/2 > D_1$.

To derive the feedback effect of increasing F, we have

$$\frac{\partial\theta_1^*}{\partial P_1^*}\frac{dP_1^*}{dF} = -\frac{(D_2-D_1)qD_1}{(P_1^*-qD_1)^2}\cdot\frac{dP_1^*}{dF}.$$

Note that $\frac{\partial\theta_1^*}{\partial P_1^*} = \frac{\partial}{\partial P_1^*}\left(\frac{D_2-D_1}{1-qD_1/P_1^*}\right) = -\frac{(D_2-D_1)qD_1}{(P_1^*-qD_1)^2} < 0$. A decrease in the secondary market price intensifies creditors' coordination problem, thus increases θ_1^*. To derive the second part dP_1^*/dF, we use again the auxiliary function (A.34) and recall that the equilibrium asset price P_1^* is such that

$$\eta(P_1^*, F) = 0.$$

Note that F enters into η solely through D_1 and D_2. Then we apply the implicit function theorem to do the comparative static analysis, and obtain

$$\frac{dP_1^*}{dF} = -\frac{\partial\eta(P_1^*, F)/\partial F}{\partial\eta(P_1^*, F)/\partial P_1^*}$$

To calculate the nominator, we have

$$\begin{aligned}\frac{\partial\eta(P_1^*, F)}{\partial F} &= -\frac{\partial}{\partial F}\left(\frac{D_2-D_1}{1-qD_1/P_1^*}\right) - \frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}\frac{\partial}{\partial F}\left(\frac{D_2-D_1}{1-qD_1/P_1^*}\right)(\underline{\theta}_G - \underline{\theta}_B)\\ &= -\frac{\partial\theta_1^*}{\partial F}\left[1+\frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G - \underline{\theta}_B)\right] > 0.\end{aligned}$$

Note that we have derived in Appendix A.3, $d\omega_R^G(\theta_1^*)/d\theta_1^* > 0$. Additionally, $\partial\theta_1^*/\partial F < 0$ is due to the stable effect. Then, to calculate the denominator, we also have

$$\begin{aligned}\frac{\partial\eta(P_1^*, F)}{\partial P_1^*} &= 2 - \frac{\partial}{\partial P_1^*}\left(\frac{D_2-D_1}{1-qD_1/P_1^*}\right) - \frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}\frac{\partial}{\partial P_1^*}\left(\frac{D_2-D_1}{1-qD_1/P_1^*}\right)(\underline{\theta}_G - \underline{\theta}_B)\\ &= 2 - \frac{\partial\theta_1^*}{\partial P_1^*}\left[1+\frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G - \underline{\theta}_B)\right] > 2.\end{aligned}$$

Recall that we have just derived $\frac{\partial\theta_1^*}{\partial P_1^*} < 0$. Consequently, we have

$$\frac{dP_1^*}{dF} = \frac{\partial\theta_1^*}{\partial F}\frac{1 + \frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G - \underline{\theta}_B)}{2 - \frac{\partial\theta_1^*}{\partial P_1^*}\left[1 + \frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G - \underline{\theta}_B)\right]} < 0$$

In the general equilibrium, an increase in F leads a decrease in the equilibrium asset price P_1^*. And the feedback effect is

$$\frac{\partial\theta_1^*}{\partial P_1^*}\frac{dP_1^*}{dF} = \frac{\partial\theta_1^*}{\partial P_1^*}\frac{\partial\theta_1^*}{\partial F}\frac{1 + \frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G - \underline{\theta}_B)}{2 - \frac{\partial\theta_1^*}{\partial P_1^*}\left[1 + \frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G - \underline{\theta}_B)\right]} > 0$$

Recall that $\frac{\partial\theta_1^*}{\partial P_1^*} < 0$. So in the general equilibrium setup, an increase in F also intensifies creditors' coordination problem and leads to a higher θ_1^* because the equilibrium asset price decreases.

□

Appendix A.4 Proof of Lemma 3

Proof. We can rewrite the direct stable effect of increasing F when asset price is endogenous as follows

$$\frac{\partial\theta_1^*(F, P_1^*)}{\partial F} = \frac{(1 - r_D)P_1^*}{(P_1^* - qD_1)} + \frac{qr_D(P_1^* - qD_2)P_1^*}{(P_1^* - qD_1)^2} < 0$$

We then analyze how the change in P_1^* affect this partial derivative. To do so, we define an auxiliary function

$$\varphi(P) = \frac{(1 - r_D)P}{(P - qD_1)} + \frac{qr_D(P - qD_2)P}{(P - qD_1)^2}$$

for $P \in [\overline{P}, qD_2)$. We then analyze the following derivative

$$\frac{d\varphi(P)}{dP} = \frac{d}{dP}\left[\frac{(1 - r_D)P}{(P - qD_1)}\right] + \frac{d}{dP}\left[\frac{qr_D(P - qD_2)P}{(P - qD_1)^2}\right].$$

It can be calculated that the first term is

$$\frac{d}{dP}\left[\frac{(1 - r_D)P}{(P - qD_1)}\right] = -\frac{qD_1(1 - r_D)}{(P - qD_1)^2} > 0$$

as $r_D > 1$ is the gross return of the wholesale debts. And the second terms is

$$\frac{d}{dP}\left[\frac{qr_D(P-qD_2)P}{(P-qD_1)^2}\right] = qr_D\frac{qD_1(qD_2-P)+Pq(D_2-D_1)}{(P-qD_1)} > 0$$

We obtain

$$\frac{d\varphi(P)}{dP} > 0.$$

for $P \in [\overline{P}, qD_2)$. Recall we proved in Lemma 2 that $\frac{dP_1^*}{dF} < 0$. Combined with the fact that $P^e = P_1^*$ when F is at the private optimal level $\underline{F}$, we obtain the endogenous equilibrium asset price $P_1^* < P^e$ for all $F \in (\underline{F}, F_q]$. Consequently, we have

$$\frac{\partial\theta_1^*(F,P_1^*)}{\partial F} = \varphi(P_1^*) < \varphi(P^e) = \frac{\partial\theta_1^*(F,P^e)}{\partial F} \Leftrightarrow \left|\frac{\partial\theta_1^*(F,P_1^*)}{\partial F}\right| > \left|\frac{\partial\theta_1^*(F,P^e)}{\partial F}\right|$$

for each level of $F \in (\underline{F}, F_q]$. An marginal increase in F leads to a larger reduction in critical cash flow in the general equilibrium case.

$$\left|\frac{\partial\theta_1^*(F,P_1^*)}{\partial F}\right|\Delta F > \left|\frac{\partial\theta_1^*(F,P^e)}{\partial F}\right|\Delta F$$

at each level of $F \in (\underline{F}, F_q]$.

□

Appendix A.5 Proof of Proposition 2

Proof. To determine the aggregate general equilibrium effect of increasing F on θ_1^*, we need to consider both the ‘positive’ stable effect, which reduces θ_1^*, and the ‘negative’ feedback effect, which increases θ_1^*. We have

$$\begin{aligned}\frac{d\theta_1^*}{dF} &= \frac{\partial\theta_1^*}{\partial F} + \frac{\partial\theta_1^*}{\partial P_1^*}\frac{\partial\theta_1^*}{\partial F}\frac{1+\frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G-\underline{\theta}_B)}{2-\frac{\partial\theta_1^*}{\partial P_1^*}\left[1+\frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G-\underline{\theta}_B)\right]}\\ &= \frac{\partial\theta_1^*}{\partial F}\frac{2}{2-\frac{\partial\theta_1^*}{\partial P_1^*}\left[1+\frac{d\omega_R^G(\theta_1^*)}{d\theta_1^*}(\underline{\theta}_G-\underline{\theta}_B)\right]} < 0\end{aligned}$$

Note that the last inequality is true due to $\frac{\partial\theta_1^*}{\partial F} < 0$. This means an increase in F still reduces θ_1^* in the general equilibrium setup. Intuitively, the stable effect is the first order effect while the feedback effect is second order.

$\square$

Appendix A.6 Proof of Lemma 4

Proof. The proof resembles to that of Lemma 1, thus we just highlight same differences.

First, players' beliefs about the state at the beginning of the second period game differ from the priors. Specifically, we have

$$\omega_S^G = \frac{1}{1+\kappa_0} > \alpha > \frac{1}{1+\left(\frac{\theta_1^*-\underline{\theta}_B}{\theta_1^*-\underline{\theta}_G}\right)\kappa_0} = \omega_R^G(\theta_1^*)$$

where $\kappa_0 = \frac{1-\alpha}{\alpha}\left(\frac{\overline{\theta}-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_B}\right)$ when $O_1 = S$ and F respectively.

We illustrate the more complicated case where $O_1 = R$. Suppose now the equilibrium asset price $P_{R,2}^* \in [\underline{P}, qd_2)$, the equilibrium bank run threshold is again characterized by the equation

$$\theta_{R,2}^* = \frac{d_2 - d_1}{1 - qd_1/P_{R,2}^*}.$$

from the global-games refinement. And we also have $x_{R,2}^* = \theta_{R,2}^*$.

On the other hand, given the belief $\omega_R^G(\theta_1^*)$, the posterior belief of asset buyers when they are called upon to move is given by (17) as

$$\omega_{R,R}^G(\theta_{R,2}^*) = \frac{1}{1+\left(\frac{\omega_{O_1}^B(\theta_1^*)}{\omega_{O_1}^G(\theta_1^*)}\right)\left(\frac{\overline{\theta}-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_B}\right)\left(\frac{\theta_{R,2}^*-\underline{\theta}_B}{\theta_{R,2}^*-\underline{\theta}_G}\right)}$$

So the asset market equilibrium is characterized by

$$P_{R,2}^* - \frac{\theta_{R,2}^* + \underline{\theta}_B + \omega_{R,R}^G(\theta_{R,2}^*)(\underline{\theta}_G - \underline{\theta}_B)}{2} = 0.$$

Given these two equilibrium conditions, one can check that $\omega_{R,R}^G(P) \in (0,1)$ when $P = \underline{P}$ and qd_2. So we can follow the same procedure to establish the existence. To show the uniqueness, the only point needs to be mentioned is that

$$\frac{d\omega_{R,R}^G(\theta_{R,2}^*)}{d\theta_{R,2}^*} = \frac{\kappa_{R,1}(\theta_1^*)}{[(\theta_{R,2}^* - \underline{\theta}_G) + (\theta_{R,2}^* - \underline{\theta}_B)\kappa_{R,1}(\theta_1^*)]^2}(\underline{\theta}_G - \underline{\theta}_B) > 0$$

where $\kappa_{R,1}(\theta_1^*) = \left(\frac{\overline{\theta}-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_B}\right)\left(\frac{\omega_R^B(\theta_1^*)}{\omega_R^G(\theta_1^*)}\right)$. Note that now θ_1^* is taken as given for the players in the second period game. The rest procedures are just replications of Lemma 1. And similar arguments applies when $O_1 = S$.

□

Appendix A.7 Proof of Proposition 4

Proof. Part (i)

Let us define function

$$\pi^s(\hat{\theta}) = \frac{\hat{\theta} + \underline{\theta}_s}{2} - P(\hat{\theta})$$

where $P(\hat{\theta}) = \frac{qd_1\hat{\theta}}{[\hat{\theta}-(d_2-d_1)]}$ and $\hat{\theta} \in [d_2, \overline{\theta}]$. One can also check that $\frac{dP(\hat{\theta})}{d\hat{\theta}} < 0$. Then the general equilibrium condition can be expressed as

$$\omega^G_{O_1,R}(\theta^*_{O_1,2})\pi^G(\theta^*_{O_1,2}) + \omega^B_{O_1,R}(\theta^*_{O_1,2})\pi^B(\theta^*_{O_1,2}) = 0 \tag{A.35}$$

where $O_1 = S$ or R. It will become evident that Part (i) of this proposition hinges on the comparison of the following ratios $\frac{\omega^B_{S,R}(\theta^*_{S,2})}{\omega^G_{S,R}(\theta^*_{S,2})}$ and $\frac{\omega^B_{R,R}(\theta^*_{R,2})}{\omega^G_{R,R}(\theta^*_{R,2})}$.

First, it can be checked that the following inequalities

$$\frac{d}{d\hat{\theta}}\left(\frac{\pi^G(\hat{\theta})}{\pi^B(\hat{\theta})}\right) < 0 \quad \text{and} \quad \frac{d}{d\hat{\theta}}\left(\frac{\omega^B_{O_1,R}(\hat{\theta})}{\omega^G_{O_1,R}(\hat{\theta})}\right) < 0$$

are true for all $\hat{\theta} \in [d_2, \overline{\theta}]$. And we also have

$$\frac{\omega^B_{S,R}(\hat{\theta})}{\omega^G_{S,R}(\hat{\theta})} = \frac{(\hat{\theta} - \underline{\theta}_B)\kappa_{1,S}}{\hat{\theta} - \underline{\theta}_G)} < \frac{(\hat{\theta} - \underline{\theta}_B)\kappa_{1,R}}{\hat{\theta} - \underline{\theta}_G)} = \frac{\omega^B_{R,R}(\hat{\theta})}{\omega^G_{R,R}(\hat{\theta})} \tag{A.36}$$

for a given $\hat{\theta}$ in the interval $[d_2, \overline{\theta}]$. In this inequality, we define

$$\kappa_{1,S} = \left(\frac{\overline{\theta} - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_B}\right)\kappa_0 < \left(\frac{\overline{\theta} - \underline{\theta}_G}{\overline{\theta} - \underline{\theta}_B}\right)\left(\frac{\omega^B_R(\hat{\theta})}{\omega^G_R(\hat{\theta})}\right) = \kappa_{1,R}(\hat{\theta})$$

for simplicity.

Then we prove Part (i) by contradiction, suppose $\theta^*_{2,S} \geq \theta^*_{2,R}$. Then by the monotonicity of $\frac{\pi^G(\hat{\theta})}{\pi^B(\hat{\theta})}$, we obtain

$$\frac{\pi^G(\theta^*_{S,2})}{\pi^B(\theta^*_{S,2})} \geq \frac{\pi^G(\theta^*_{R,2})}{\pi^B(\theta^*_{R,2})}.$$

Recall that both $\theta^*_{S,2}$ and $\theta^*_{R,2}$ satisfy the general equilibrium condition (A.35). Thus, we obtain

$$-\frac{\omega^B_{S,R}(\theta^*_{S,2})}{\omega^G_{S,R}(\theta^*_{S,2})} \leq -\frac{\omega^B_{R,R}(\theta^*_{R,2})}{\omega^G_{R,R}(\theta^*_{R,2})} \Leftrightarrow \frac{\omega^B_{S,R}(\theta^*_{S,2})}{\omega^G_{S,R}(\theta^*_{S,2})} \geq \frac{\omega^B_{R,R}(\theta^*_{R,2})}{\omega^G_{R,R}(\theta^*_{R,2})}.$$

Combine this inequality with the inequality (A.36) when $\hat{\theta} = \theta^*_{S,2}$, we obtain

$$\frac{\omega^B_{R,R}(\theta^*_{R,2})}{\omega^G_{R,R}(\theta^*_{R,2})} \leq \frac{\omega^B_{S,R}(\theta^*_{S,2})}{\omega^G_{S,R}(\theta^*_{S,2})} < \frac{\omega^B_{R,R}(\theta^*_{S,2})}{\omega^G_{R,R}(\theta^*_{S,2})}.$$

That gives us

$$\frac{\omega^B_{R,R}(\theta^*_{R,2})}{\omega^G_{R,R}(\theta^*_{R,2})} < \frac{\omega^B_{R,R}(\theta^*_{S,2})}{\omega^G_{R,R}(\theta^*_{S,2})}$$

Lastly, by the monotonicity of $\frac{\omega^B_{O_1,R}(\hat{\theta})}{\omega^G_{O_1,R}(\hat{\theta})}$ for both $O_1 = S$ and R, we have

$$\theta^*_{R,2} > \theta^*_{S,2},$$

a contradiction. Then the opposite naturally holds $\theta^* S, 2 < \theta^*_{R,2}$, and $P^* S, 2 > P^*_{R,2}$.

Part (ii)

It is obvious that $\theta^*_{S,2}$ is not a function of θ^*_1 as we have already pointed out that ω^G_S and ω^B_S are not functions of θ^*_1. Thus, we focus on the discussion of $\theta^*_{R,2}$.

Recall that the asset market equilibrium condition when $O_1 = R$ can be rewritten as

$$2P^*_{R,2} = \theta^*_{R,2} + \underline{\theta}_B + \omega^G_{R,R}(\theta^*_{R,2}, \theta^*_1)(\theta_G - \theta_B).$$

And the global-games bank run equilibrium threshold can be rewritten as

$$P^*_{R,2} = \frac{qd_1\theta^*_{R,2}}{[\theta^*_{R,2} - (d_2 - d_1)]}.$$

Combine these two equilibrium conditions, we define an auxiliary function

$$\phi(\theta^*_{R,2}, \theta^*_1) = 2\frac{qd_1\theta^*_{R,2}}{[\theta^*_{R,2} - (d_2 - d_1)]} - \left[\underline{\theta}_B + \omega^G_{R,R}(\theta^*_{R,2}, \theta^*_1)(\theta_G - \theta_B)\right] = 0.$$

By the implicit function theorem, we have

$$\frac{d\theta_{R,2}^*}{d\theta_1^*} = -\frac{\partial\phi(\theta_{R,2}^*, \theta_1^*)/\partial\theta_1^*}{\partial\phi(\theta_{R,2}^*, \theta_1^*)/\partial\theta_{R,2}^*}.$$

Note that

$$\begin{aligned}
\frac{\partial\phi(\theta_{R,2}^*, \theta_1^*)}{\partial\theta_1^*} &= -\frac{\omega_{R,R}^G(\theta_{R,2}^*, \theta_1^*)}{\partial\theta_1^*}(\underline{\theta}_G - \underline{\theta}_B) \\
&= -\frac{\partial}{\partial\theta_1^*}\left[\frac{1}{1+\left(\frac{\theta_{R,2}^*-\underline{\theta}_B}{\theta_{R,2}^*-\underline{\theta}_G}\right)\kappa_{1,R}(\theta_1^*)}\right](\underline{\theta}_G - \underline{\theta}_B) \\
&= -\frac{-\left(\frac{\theta_{R,2}^*-\underline{\theta}_B}{\theta_{R,2}^*-\underline{\theta}_G}\right)\frac{\partial\kappa_{1,R}(\theta_1^*)}{\partial\theta_1^*}}{\left[1+\left(\frac{\theta_{R,2}^*-\underline{\theta}_B}{\theta_{R,2}^*-\underline{\theta}_G}\right)\kappa_{1,R}(\theta_1^*)\right]^2}(\underline{\theta}_G - \underline{\theta}_B).
\end{aligned}$$

In the above derivative, recall that $\kappa_{1,R}(\theta_1^*) = \left(\frac{\overline{\theta}-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_B}\right)\left(\frac{\omega_R^B(\theta_1^*)}{\omega_R^G(\theta_1^*)}\right)$. So we have $\frac{\partial\kappa_{1,R}(\theta_1^*)}{\partial\theta_1^*} = -\left(\frac{\overline{\theta}-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_B}\right)\frac{(\underline{\theta}_G-\underline{\theta}_B)}{(\theta_1^*-\underline{\theta}_G)^2} <$ 0. We obtain the following.

$$\frac{\partial\phi(\theta_{R,2}^*, \theta_1^*)}{\partial\theta_1^*} = -\frac{\left(\frac{\theta_{R,2}^*-\underline{\theta}_B}{\theta_{R,2}^*-\underline{\theta}_G}\right)\left(\frac{\overline{\theta}-\underline{\theta}_G}{\overline{\theta}-\underline{\theta}_B}\right)\left(\frac{\underline{\theta}_G-\underline{\theta}_B}{\theta_1^*-\underline{\theta}_G^2}\right)^2}{\left[1+\left(\frac{\theta_{R,2}^*-\underline{\theta}_B}{\theta_{R,2}^*-\underline{\theta}_G}\right)\kappa_{1,R}(\theta_1^*)\right]^2} < 0.$$

On the other hand, we have

$$\frac{\partial\phi(\theta_{R,2}^*, \theta_1^*)}{\partial\theta_{R,2}^*} = -2\frac{qd_2(d_2-d_1)}{[\theta_{R,2}^* - (d_2-d_1)]^2} - 1 - \frac{\partial\omega_{R,R}^G(\theta_{R,2}^*, \theta_1^*)}{\partial\theta_{R,2}^*}(\underline{\theta}_G - \underline{\theta}_B).$$

Recall that $\frac{\omega_{R,R}^G(\theta_{R,2}^*,\theta_1^*)}{\partial\theta_{R,2}^*} > 0$ in the above derivative. Consequently, we also have

$$\frac{\partial\phi(\theta_{R,2}^*, \theta_1^*)}{\partial\theta_{R,2}^*} < 0.$$

Combine both partial derivatives, we get the final result

$$\frac{d\theta_{R,2}^*}{d\theta_1^*} < 0.$$

□